Scalability Issues and the Potential for Encrypted Machine Learning

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2nd UCL Workshop on the Theory of Big Data
6th January 2016

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- computing in a ‘hostile’ environment (e.g. cloud computing);
- donation of sensitive/personal data (e.g. medical/genetic studies);
- complex models on constrained devices (e.g. smart watches)
- running confidential algorithms on confidential data (e.g. engineering reliability)

Encryption the solution?

Encryption can provide security guarantees …

Easy

$$\texttt{Enc}(k_p, m) \rightleftharpoons c \qquad\qquad \texttt{Dec}(k_s, c) = m$$

Hard without k_s

… but is typically 'brittle'.

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Limitations of homomorphic encryption

1. Message space (what we can encrypt)
   - Commonly only easy to encrypt binary/integers/polynomials
2. Cipher text size (the result of encryption)
   - Present schemes all inflate the size of data substantially (e.g. 1MB $\rightarrow$ 16.4GB)
3. Computational cost (computing without decrypting)
   - 1000's additions per sec
   - $\approx$ 50 multiplications per sec
4. Division and comparison operations (equality/inequality checks)
   - Not possible in current schemes!
5. Depth of operations
   - After a certain depth of multiplications, need to 'refresh' cipher text: hugely time consuming, so avoid!

Statistics & Machine Learning Encrypted?

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So, want to build a random forest on encrypted data … but,

- No comparisons possible to evaluate splits
- No max possible to find highest class vote
- No division possible to do average votes
- …

Thus random forests (and other methods) need to be tailored for encrypted computation. This is where statistics and machine learning community can get involved!

Completely Random Forests (CRFs) — Data encoding

1

$$x_{ij} \in \mathbb{R} \xrightarrow{B \text{ quantiles}} \boxed{0 \mid 0 \mid 0 \mid 1 \mid 0}$$

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2. Then,

$$\mathbb{I}(x_{ij} \leq b_l) = \sum_{k=1}^{l} x_{ijk} \quad \text{and} \quad \mathbb{I}(x_{ij} > b_l) = \sum_{k=l+1}^{B} x_{ijk}$$

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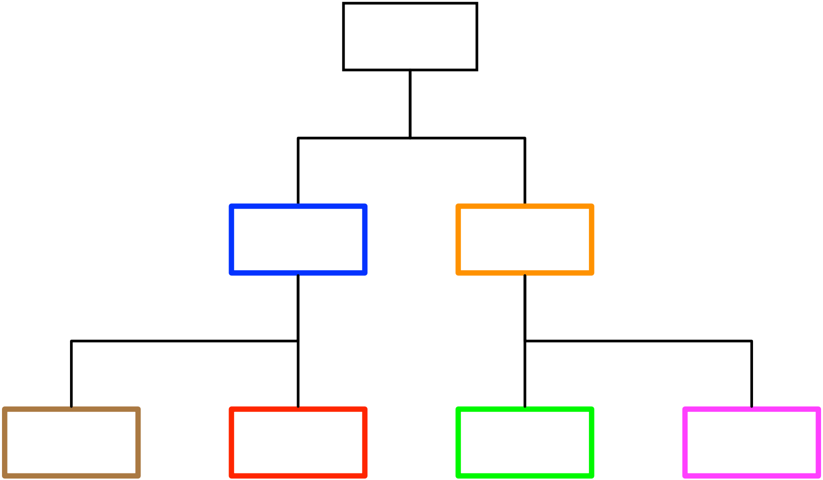
3. Similarly encode response category c, $y_i \to y_{ic} \in \{0, 1\}$.
4. Build a decision tree selecting variable j and split point b_l *completely* at random to a fixed depth.

Completely Random Forests (CRFs) — Tree ‘fitting’, I



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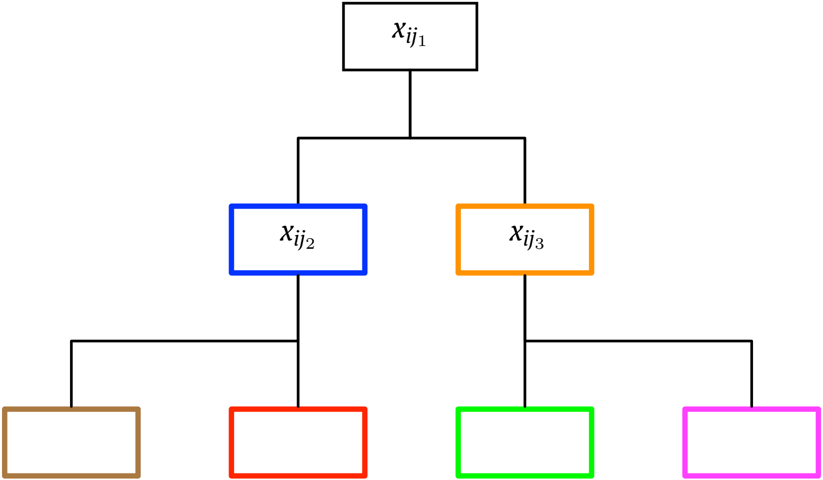

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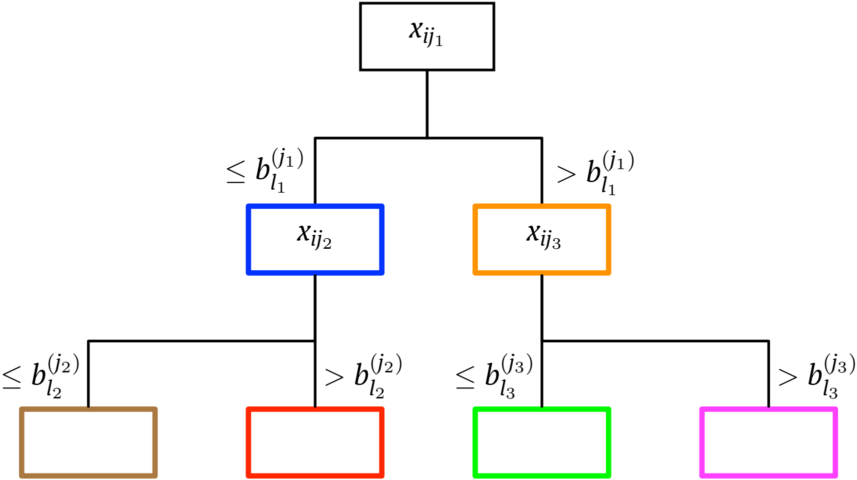

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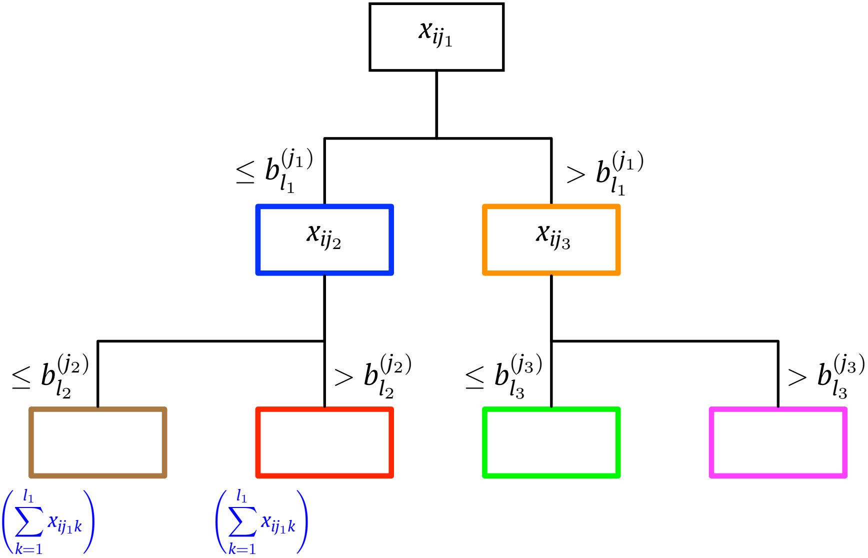

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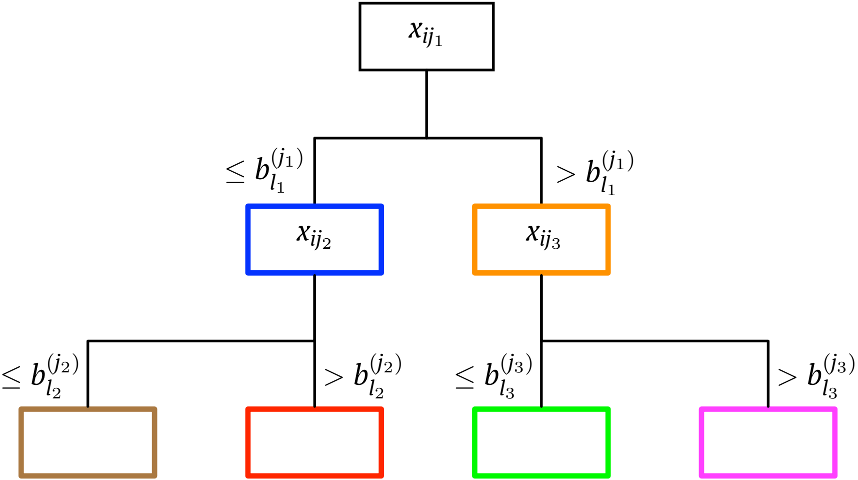
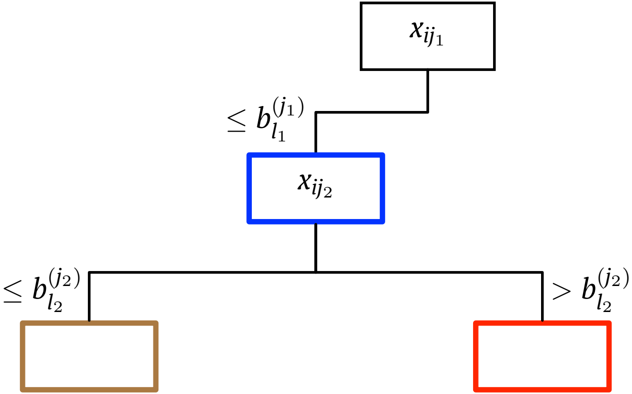


$$\left(\sum_{k=1}^{l_1} x_{ij_1k}\right)\left(\sum_{k=1}^{l_2} x_{ij_2k}\right) \quad \left(\sum_{k=1}^{l_1} x_{ij_1k}\right)\left(\sum_{k=l_2+1}^{B} x_{ij_2k}\right)$$

Exactly one terminal leaf indicator evaluates to 1, encrypted.

Completely Random Forests (CRFs) — Tree ‘fitting’, II



$$\left(\sum_{k=1}^{l_1} x_{ij_1k}\right)\left(\sum_{k=1}^{l_2} x_{ij_2k}\right) \qquad \left(\sum_{k=1}^{l_1} x_{ij_1k}\right)\left(\sum_{k=l_2+1}^{B} x_{ij_2k}\right)$$

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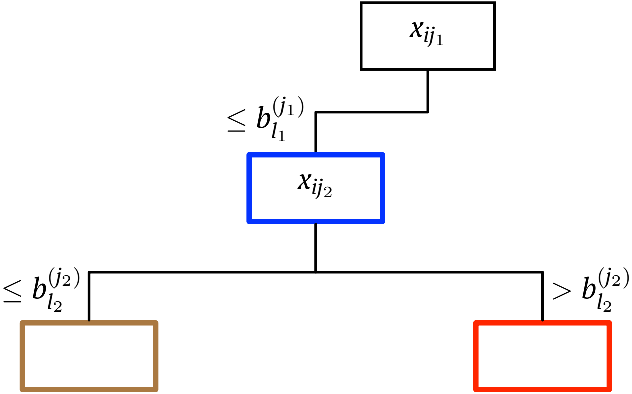

$$\nu_{ic} = y_{ic} \left(\sum_{k=1}^{l_1} x_{ij_1k} \right) \left(\sum_{k=1}^{l_2} x_{ij_2k} \right) \qquad \nu_{ic} = y_{ic} \left(\sum_{k=1}^{l_1} x_{ij_1k} \right) \left(\sum_{k=l_2+1}^{B} x_{ij_2k} \right)$$

NB Must evaluate *all* branches and categories as blindfold.

Completely Random Forests (CRFs) — Prediction

Prediction involves:

- evaluating a new observation through all branches;
- taking product with corresponding vote totals for each class;
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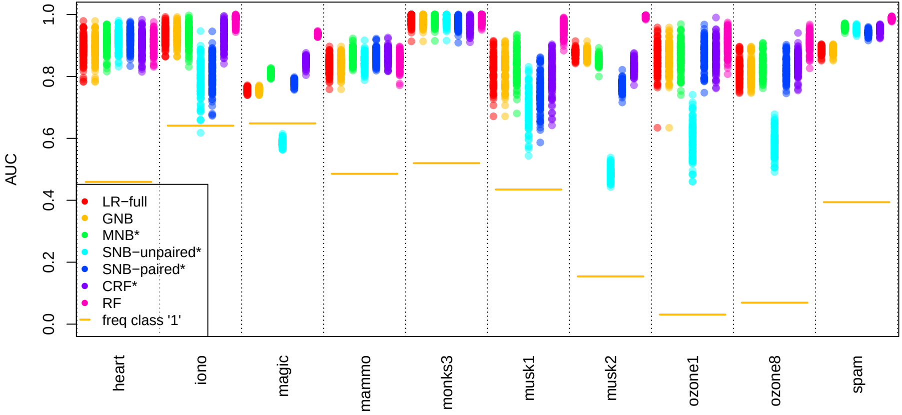
But, confused leaves with many votes can overwhealm certain ones with few. Random Forests usually use:

1. single vote per tree (requires comparison to find max)
2. relative class frequencies (requires division)

… developed novel 'stochastic fraction estimate', an unbiased approximation to 2.

Results


AUC
1.0
0.8
0.6
0.4
0.2
0.0
LR−full
GNB
MNB*
SNB−unpaired*
SNB−paired*
CRF*
RF
freq class '1'
heart
iono
magic
mammo
monks3
musk1
musk2
ozone1
ozone8
spam

HomomorphicEncryption R package

```
library("HomomorphicEncryption")
p <- parsHelp("FandV", lambda=128, L=5)
k <- keygen(p)
c1 <- enc(k$pk, 2); c2 <- enc(k$pk, 3)
cres <- c1 + c2 * c1
dec(k$sk, cres)
```

```
[1] 8
```

```
cmat <- enc(k$pk, matrix(1:9, nrow=3))
cmat2 <- cmat %*% cmat
dec(k$sk, cmat2)
```

```
     [,1] [,2] [,3]
[1,]   30   66  102
[2,]   36   81  126
[3,]   42   96  150
```

References I

Aslett, L. J. M. (2016). HomomorphicEncryption: Fully homomorphic encryption. `http://www.louisaslett.com/HomomorphicEncryption/`.

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