Cryptographically secure multiparty evaluation of system reliability

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General Motivation

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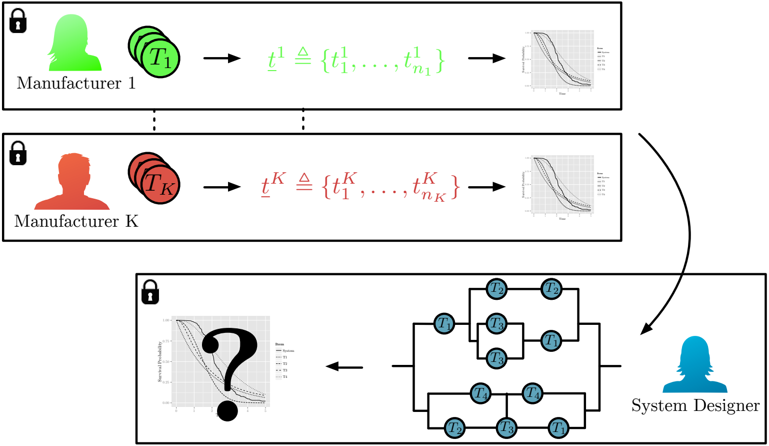
- donation of sensitive/personal data (e.g. medical/genetic studies);
- complex models on constrained devices (e.g. smart watches)
- running confidential algorithms on confidential data (e.g. engineering reliability — topic of this talk)

Motivation in Reliability Theory

Inference on system/network reliability whilst *maintaining privacy requirements* of all parties.



Encryption the solution?

Encryption can provide security guarantees ...

Easy

$$\texttt{Enc}(k_p, m) \rightleftharpoons c \qquad\qquad \texttt{Dec}(k_s, c) = m$$

Hard without k_s

... but is typically 'brittle'.

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$\downarrow \texttt{Enc}(k_p, \cdot) \downarrow$

c_1 $\quad$ c_2

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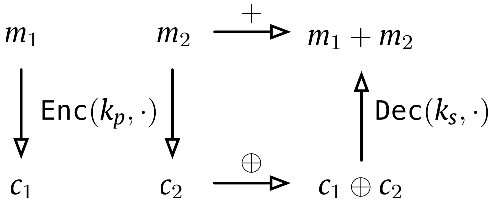

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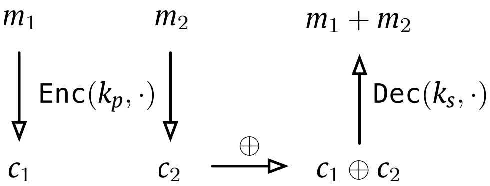


Limitations of homomorphic encryption

1. Message space (what we can encrypt)
   - Commonly only easy to encrypt binary/integers/polynomials
2. Cipher text size (the result of encryption)
   - Present schemes all inflate the size of data substantially (e.g. 1MB $\rightarrow$ 16.4GB)
3. Computational cost (computing without decrypting)
   - 1000's additions per sec
   - $\approx$ 50 multiplications per sec
4. Division and comparison operations (equality/inequality checks)
   - Not possible in current schemes!
5. Depth of operations
   - After a certain depth of multiplications, need to 'refresh' cipher text: hugely time consuming, so avoid!

Survival signature

Coolen & Coolen-Maturi (2012) rethought system signatures (Samaniego 1985) with the objective of retaining separation of structure and component lifetimes for multiple component types.

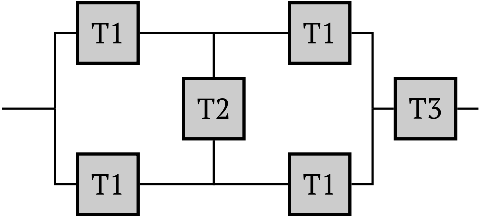
Definition (Survival signature)

Consider a system comprising K component types, with M_k components of type $k \in \{1, \ldots, K\}$. Then the *survival signature* $\Phi(l_1, \ldots, l_K)$, with $l_k \in \{0, 1, \ldots, M_k\}$, is the probability that the system functions given precisely l_k of its components of type k function.

$$\Phi(l_1, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{M_k}{l_k} \right]^{-1} \sum_{\underline{x} \in S_{l_1, \ldots, l_K}} \varphi(\underline{x})$$

where $S_{l_1, \ldots, l_K} = \{\underline{x} : \sum_{i=1}^{M_k} x_i^k = l_k \quad \forall\, k\}$

Survival signature toy example



T1	T2	T3	Φ	T1	T2	T3	Φ
0	0	1	0	0	1	1	0
1	0	1	0	1	1	1	0
2	0	1	0.33	2	1	1	0.67
3	0	1	1	3	1	1	1
4	0	1	1	4	1	1	1

Table 1: Survival signature for a bridge system, omitting all rows with T3 $= 0$, since $\Phi = 0$ for these.

System lifetimes

Let $C_t^k \in \{0, 1, \dots, M_k\}$ be random variable denoting number of components of type k surviving at time t. Then, survival function of system lifetime T_S is:

$$\begin{aligned}\mathbb{P}(T_S > t) &= \sum_{l_1=0}^{M_1} \cdots \sum_{l_K=0}^{M_K} \Phi(l_1, \dots, l_K)\, \mathbb{P}\left(\bigcap_{k=1}^{K} \{C_t^k = l_k\} \right) \\ &= \sum_{l_1=0}^{M_1} \cdots \sum_{l_K=0}^{M_K} \Phi(l_1, \dots, l_K) \prod_{k=1}^{K} \mathbb{P}\left(C_t^k = l_k\right)\end{aligned}$$

if the component types are independent.

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$$= \sum_{l_1=0}^{M_1} \cdots \sum_{l_K=0}^{M_K} \Phi(l_1, \dots, l_K) \prod_{k=1}^{K} \mathbb{P}\left(C_t^k = l_k\right)$$

if the component types are independent.

Note: this is a homogeneous polynomial of degree $K+1$ in the survival signature and component survival probabilities $\implies$ can evaluate encrypted.

Propagating uncertainty as a Bayesian

$$
\begin{aligned}
&P(T_{S^*} > t \mid \underline{y}_1, \dots \underline{y}_K) \\
&= \int \cdots \int P(T_{S^*} > t \mid p_t^1, \dots p_t^K) P(dp_t^1 \mid \underline{y}_1) \dots P(dp_t^K \mid \underline{y}_K) \\
&= \int \cdots \int \left[\sum_{l_1=0}^{M_1} \cdots \sum_{l_K=0}^{M_K} \Phi(l_1, \dots, l_K) P\left(\bigcap_{k=1}^{K} \{C_t^k = l_k \mid p_t^k\} \right) \right] \\
&\qquad\qquad \times P(dp_t^1 \mid \underline{y}_1) \dots P(dp_t^K \mid \underline{y}_K) \\
&= \sum_{l_1=0}^{M_1} \cdots \sum_{l_K=0}^{M_K} \Phi(l_1, \dots, l_K) \prod_{k=1}^{K} \int P(C_t^k = l_k \mid p_t^k) P(dp_t^k \mid \underline{y}_k)
\end{aligned}
$$

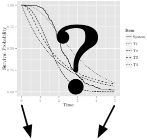
A homogeneous polynomial of degree $K+1$ in the survival signature and posterior predictive component survival probabilities at each time point $\implies$ can still evaluate encrypted.

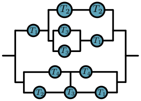
k_s k_p

System Designer

$\Phi(l_1, \ldots, l_K)$

ν



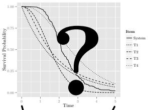


$\underline{t} = \{t_1, \ldots, t_T\}$

k_s 🔒 k_p

System Designer

ν



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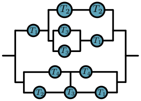

$$\Xi = \begin{pmatrix} 0 & \cdots & 0 & \texttt{Enc}\left(k_p, \lfloor 10^\nu \Phi(0, \dots, 0) \rceil\right) \\ 0 & \cdots & 1 & \texttt{Enc}\left(k_p, \lfloor 10^\nu \Phi(0, \dots, 1) \rceil\right) \\ & \vdots & & \vdots \\ l_1 & \cdots & l_K & \texttt{Enc}\left(k_p, \lfloor 10^\nu \Phi(l_1, \dots, l_K) \rceil\right) \\ & \vdots & & \vdots \\ m_1 & \cdots & m_K & \texttt{Enc}\left(k_p, \lfloor 10^\nu \Phi(m_1, \dots, m_K) \rceil\right) \end{pmatrix}$$

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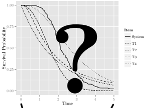

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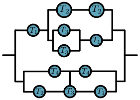

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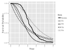
Manufacturer 1

$\rightarrow$ $\underline{t}^1 \triangleq \{t_1^1, \dots, t_{n_1}^1\}$ $\rightarrow$



k_s k_p

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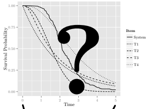
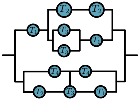


$\Phi(l_1, \dots, l_K)$



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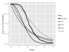
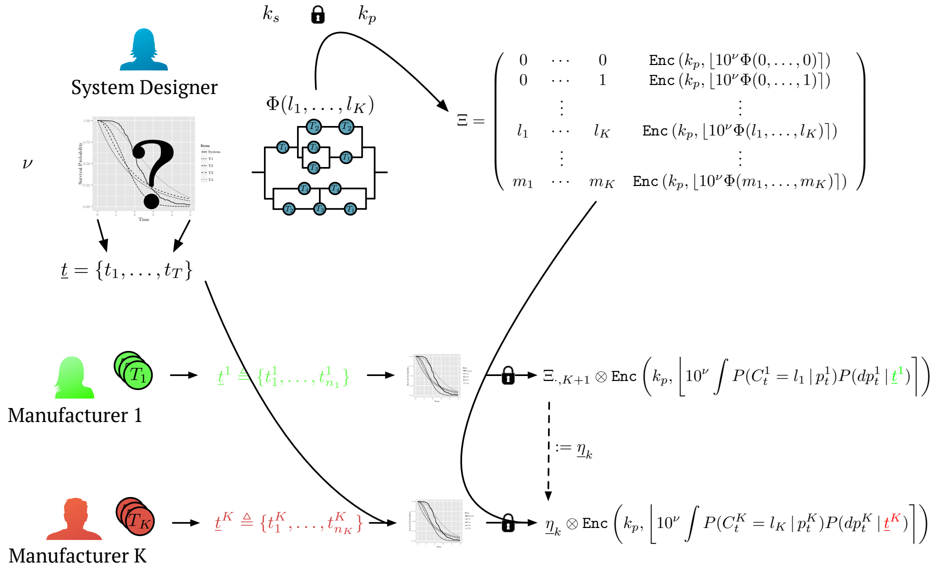
$\underline{t} = \{t_1, \dots, t_T\}$

Manufacturer 1

T_1 → $\underline{t}^1 \triangleq \{t^1_1, \dots, t^1_{n_1}\}$ →



→ $\Xi_{\cdot,K+1} \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C^1_t = l_1 \mid p^1_t) P(dp^1_t \mid \underline{t}^1)\right\rceil\right)$


k_s k_p
System Designer
ν
$\Phi(l_1, \dots, l_K)$
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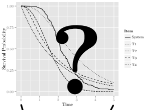
Manufacturer 1
T_1
$\underline{t}^1 \triangleq \{t_1^1, \dots, t_{n_1}^1\}$
$\Xi_{\cdot,K+1} \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^1 = l_1 \,|\, p_t^1) P(dp_t^1 \,|\, \underline{t}^1) \right\rceil\right)$
$:= \underline{\eta}_k$
Manufacturer K
T_K
$\underline{t}^K \triangleq \{t_1^K, \dots, t_{n_K}^K\}$
$\underline{\eta}_k \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^K = l_K \,|\, p_t^K) P(dp_t^K \,|\, \underline{t}^K) \right\rceil\right)$

System Designer



ν

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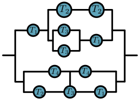
k_s k_p

$\Phi(l_1, \dots, l_K)$



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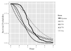
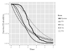


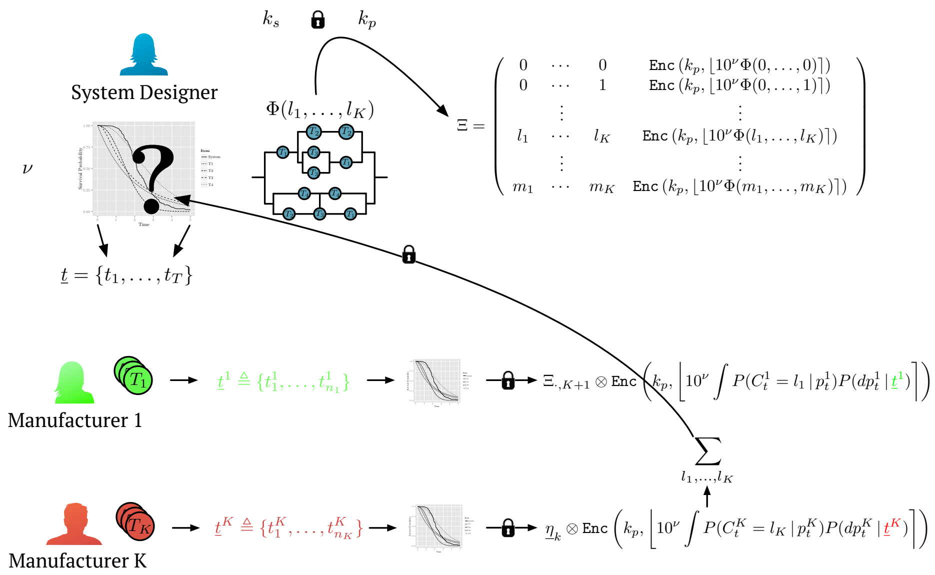
$\Xi_{\cdot, K+1} \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^1 = l_1 \mid p_t^1) P(dp_t^1 \mid \underline{t}^1) \right\rceil\right)$

Manufacturer K

$\underline{t}^K \triangleq \{t_1^K, \dots, t_{n_K}^K\}$



$\underline{\eta}_k \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^K = l_K \mid p_t^K) P(dp_t^K \mid \underline{t}^K) \right\rceil\right)$



k_s
k_p
System Designer
$\Phi(l_1, \dots, l_K)$
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ν
Survival Probability
Time
Item
System
T1
T2
T3
T4
$\underline{t} = \{t_1, \dots, t_T\}$
Manufacturer 1
T_1
$\underline{t}^1 \triangleq \{t_1^1, \dots, t_{n_1}^1\}$
$\Xi_{\cdot,K+1} \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^1 = l_1 \mid p_t^1) P(dp_t^1 \mid \underline{t}^1) \right\rceil\right)$
$\sum_{l_1,\dots,l_K}$
Manufacturer K
T_K
$\underline{t}^K \triangleq \{t_1^K, \dots, t_{n_K}^K\}$
$\underline{\eta}_k \otimes \texttt{Enc}\left(k_p, \left\lfloor 10^\nu \int P(C_t^K = l_K \mid p_t^K) P(dp_t^K \mid \underline{t}^K) \right\rceil\right)$

Example system

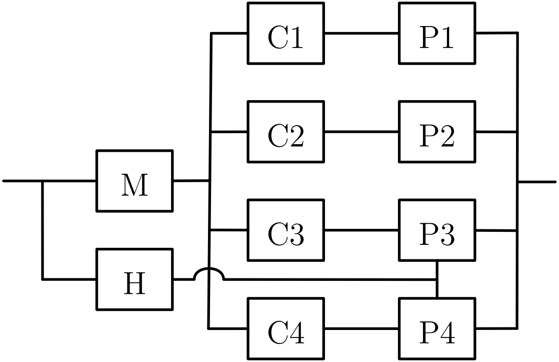

Figure 1: Simple automotive braking system. The master brake cylinder (M) engages all the four wheel brake cylinders (C1 – C4). These in turn each trigger a braking pad assembly (P1 – P4). The hand brake (H) goes directly to the rear brake pad assemblies P3 and P4; the vehicle brakes when at least one of the brake pad assemblies is engaged.

Experimental results

In order to examine the practicality of the problem, perform a full encrypted analysis using Amazon EC2 cloud computing service to mimic a global supply chain.

Role	Physical Server Location	Server Type
System designer	Dublin, Ireland	m4.10xlarge
Manufacturer C	Northern California, USA	m4.10xlarge
Manufacturer H	São Paulo, Brazil	c3.8xlarge
Manufacturer M	Sydney, Australia	r3.4xlarge
Manufacturer P	Tokyo, Japan	i2.8xlarge

Precision was set to $\nu = 5$ and system designer specifies an evenly spaced time grid of 100 points $t \in [0, 5]$.

Computational cost (I)

Role	Action	Timing / Size
System designer Dublin, Ireland	Generation of (k_p, k_s)	0.3 secs
	Encryption of $\Xi^{(\Phi)}$	1 min 41.1 secs
	Saving $\Xi^{(\Phi)}$ to disk	2 min 41.3 secs
	Compressing $\Xi^{(\Phi)}$ on disk	48.0 secs
	Size of $\Xi^{(\Phi)}$ on disk	5.5GB

Computational cost (I)

Role	Action	Timing / Size	
System designer Dublin, Ireland	Generation of (k_p, k_s)		0.3 secs
	Encryption of $\Xi^{(\Phi)}$	1 min	41.1 secs
	Saving $\Xi^{(\Phi)}$ to disk	2 min	41.3 secs
	Compressing $\Xi^{(\Phi)}$ on disk		48.0 secs
	Size of $\Xi^{(\Phi)}$ on disk	5.5GB	
Transfer $\Xi^{(\Phi)}$ to Manufacturer C		11 min	37.5 secs
Manufacturer C Northern California, USA			

Computational cost (I)

Role	Action	Timing / Size	
System designer Dublin, Ireland	Generation of (k_p, k_s)		0.3 secs
	Encryption of $\Xi^{(\Phi)}$	1 min	41.1 secs
	Saving $\Xi^{(\Phi)}$ to disk	2 min	41.3 secs
	Compressing $\Xi^{(\Phi)}$ on disk		48.0 secs
	Size of $\Xi^{(\Phi)}$ on disk	5.5GB	
Transfer $\Xi^{(\Phi)}$ to Manufacturer C		11 min	37.5 secs
Manufacturer C Northern California, USA	Decompress & load $\Xi^{(\Phi)}$ from disk	10 min	22.4 secs
	Update $\Xi^{(\Phi)}$	6 min	18.3 secs
	Saving & compressing $\Xi^{(\Phi)}$ to disk	2 min	9.8 secs

Computational cost (I)

Role	Action	Timing / Size	
System designer Dublin, Ireland	Generation of (k_p, k_s)		0.3 secs
	Encryption of $\Xi^{(\Phi)}$	1 min	41.1 secs
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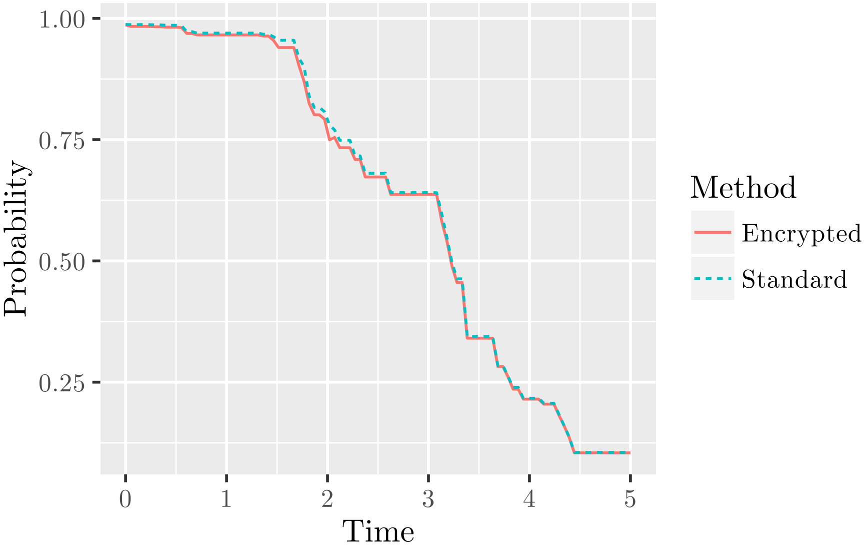
Transfer $\Xi^{(\Phi)}$ to Manufacturer C		11 min	37.5 secs
Manufacturer C Northern California, USA	Decompress & load $\Xi^{(\Phi)}$ from disk	10 min	22.4 secs
	Update $\Xi^{(\Phi)}$	6 min	18.3 secs
	Saving & compressing $\Xi^{(\Phi)}$ to disk	2 min	9.8 secs
Transfer $\Xi^{(\Phi)}$ to Manufacturer H		11 min	24.4 secs
Manufacturer H São Paulo, Brazil	Decompress & load $\Xi^{(\Phi)}$ from disk	10 min	13.2 secs
	Update $\Xi^{(\Phi)}$	7 min	23.1 secs
	Saving & compressing $\Xi^{(\Phi)}$ to disk	4 min	45.2 secs
Transfer $\Xi^{(\Phi)}$ to Manufacturer M		20 min	16.5 secs
Manufacturer M Sydney, Australia	Decompress & load $\Xi^{(\Phi)}$ from disk	9 min	41.0 secs
	Update $\Xi^{(\Phi)}$	11 min	28.2 secs
	Saving & compressing $\Xi^{(\Phi)}$ to disk	2 min	54.2 secs

Computational cost (II)

Role	**Action**	**Timing / Size**		
	Transfer $\Xi^{(\Phi)}$ to Manufacturer P		6 min	40.7 secs
Manufacturer P Tokyo, Japan	Decompress & load $\Xi^{(\Phi)}$ from disk		9 min	57.1 secs
	Update $\Xi^{(\Phi)}$		7 min	13.5 secs
	Compute ξ			6.1 secs
	Saving & compressing ξ to disk			2.5 secs
	Size of ξ on disk		58.4MB	
	Transfer ξ to System Designer			39.5 secs
System designer Dublin, Ireland	Decompress & load ξ from disk			5.9 secs
	Decryption of ξ			8.6 secs
Total:		2 hr	18 min	38.4 secs

Result


1.00
0.75
0.50
0.25
Probability
0
1
2
3
4
5
Time
Method
Encrypted
Standard

HomomorphicEncryption R package (Aslett 2014)

```
library("HomomorphicEncryption")
p <- parsHelp("FandV", lambda=128, L=5)
k <- keygen(p)
c1 <- enc(k$pk, 2); c2 <- enc(k$pk, 3)
cres <- c1 + c2 * c1
dec(k$sk, cres)
```

```
[1] 8
```

```
cmat <- enc(k$pk, matrix(1:9, nrow=3))
cmat2 <- cmat %*% cmat
dec(k$sk, cmat2)
```

```
     [,1] [,2] [,3]
[1,]   30   66  102
[2,]   36   81  126
[3,]   42   96  150
```

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