Learning Component Reliability with Reduced Information

Louis J. M. Aslett and Simon P. Wilson

Trinity College Dublin

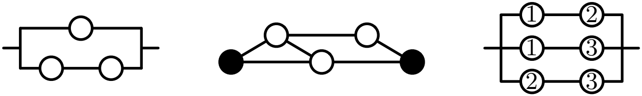
SIMRIDE, 19^{th} March 2013

Trinity College Dublin

Science Foundation Ireland

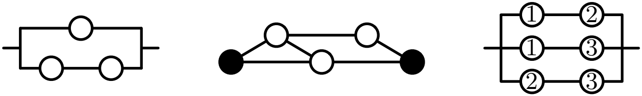
Structural Reliability Theory

- Interest lies in the reliability of 'systems' composed of numerous 'components'.



Structural Reliability Theory

- Interest lies in the reliability of ‘systems’ composed of numerous ‘components’.

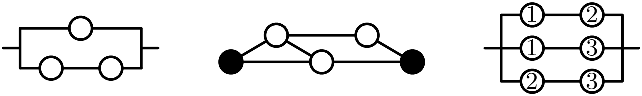


- Lifetime of the system, T, is determined by:
  - the lifetime of the components, $Y_i \sim F_Y(\,\cdot\,; \psi_i)$
  - the structure of the system.
  - the possible presence of a repair process.

via either the *structure function* or *signature*.

Structural Reliability Theory

- Interest lies in the reliability of 'systems' composed of numerous 'components'.

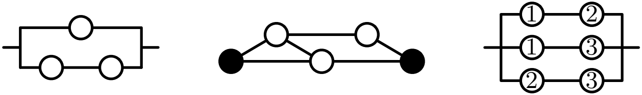


- Lifetime of the system, T, is determined by:
  - the lifetime of the components, $Y_i \sim F_Y(\,\cdot\,; \psi_i)$
  - the structure of the system.
  - the possible presence of a repair process.

~~via either the *structure function* or *signature*.~~
via *survival signature* (Coolen and Coolen-Maturi, 2012)!

Structural Reliability Theory

- Interest lies in the reliability of ‘systems’ composed of numerous ‘components’.



- Lifetime of the system, T, is determined by:
  - the lifetime of the components, $Y_i \sim F_Y(\cdot\,;\psi_i)$
  - the structure of the system.
  - the possible presence of a repair process.

~~via either the *structure function* or *signature*.~~

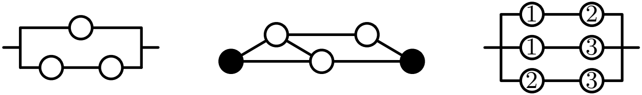
via *survival signature* (Coolen and Coolen-Maturi, 2012)!

Probabilistic Analysis

Statistical Inference

Structural Reliability Theory

- Interest lies in the reliability of 'systems' composed of numerous 'components'.



- Lifetime of the system, T, is determined by:
  - the lifetime of the components, $Y_i \sim F_Y(\,\cdot\,; \psi_i)$
  - the structure of the system.
  - the possible presence of a repair process.

~~via either the *structure function* or *signature*.~~

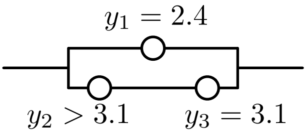
via *survival signature* (Coolen and Coolen-Maturi, 2012)!

Probabilistic Analysis

Statistical Inference

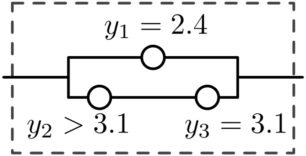
Masked System Lifetime Data (No Repair)

Traditionally, one may have failure time data on components and then infer the parameters ψ of the lifetime distribution.



Masked System Lifetime Data (No Repair)

Traditionally, one may have failure time data on components and then infer the parameters ψ of the lifetime distribution.

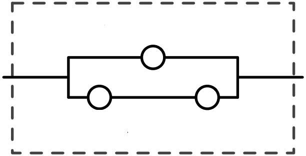


$$t = y_3 = y_{2:3} = 3.1$$

Inference a quite well understood problem here.

Masked System Lifetime Data (No Repair)

Traditionally, one may have failure time data on components and then infer the parameters ψ of the lifetime distribution.

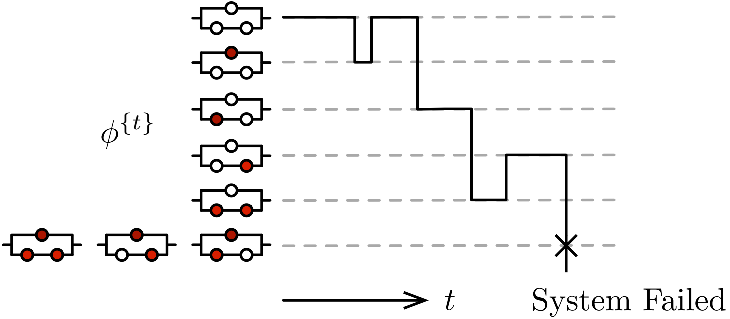


$$t = y_? = y_{?:3} = 3.1$$

Masked system lifetime data means only the failure time of the system as a whole is known, not the component failure times or indeed which components had failed.

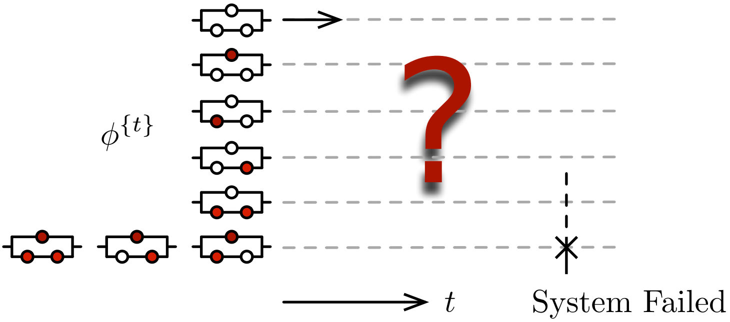
Masked System Lifetime Data (Repair)

Traditionally, one may have full schedule of failure and repair time data on components and then infer the parameters ψ of the lifetime and repair time distributions.



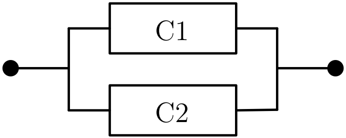
Masked System Lifetime Data (Repair)

Traditionally, one may have full schedule of failure and repair time data on components and then infer the parameters ψ of the lifetime and repair time distributions.



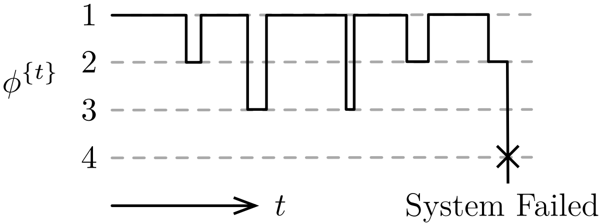
Masked system lifetime data means the schedule of failure and repair is unknown.

Toy Example : Redundant Repairable Components



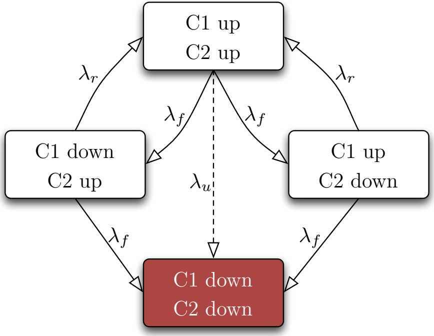
State	Meaning
1	both C1 and C2 work
2	C1 failed, C2 working
3	C1 working, C2 failed
4	system failed

∴ a general stochastic process, e.g.



Continuous-time Markov Chain Model for 

State	Meaning
1	both C1 and C2 work
2	C1 failed, C2 working
3	C1 working, C2 failed
4	system failed



$$\Longrightarrow \boldsymbol{\pi} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} -2\lambda_\mathrm{f} & \lambda_\mathrm{f} & \lambda_\mathrm{f} & 0 \\ \lambda_\mathrm{r} & -\lambda_\mathrm{r} - \lambda_\mathrm{f} & 0 & \lambda_\mathrm{f} \\ \lambda_\mathrm{r} & 0 & -\lambda_\mathrm{r} - \lambda_\mathrm{f} & \lambda_\mathrm{f} \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Definition of Phase-type Distributions

An absorbing continuous time Markov chain is one in which there is a state that, once entered, is never left. That is, the $n+1$ state generator matrix can be written:

$$\mathbf{T} = \begin{pmatrix} \mathbf{S} & \mathbf{s} \\ \mathbf{0} & 0 \end{pmatrix}$$

where $\mathbf{S}$ is $n \times n$, $\mathbf{s}$ is $n \times 1$ and $\mathbf{0}$ is $1 \times n$, with

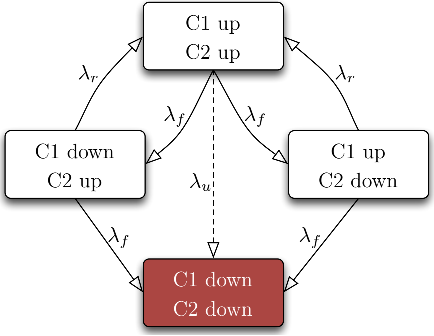
$$\mathbf{s} = -\mathbf{S}\mathbf{e}$$

Then, a *Phase-type distribution* (PHT) is defined to be the distribution of the time to entering the absorbing state.

$$Y \sim \text{PHT}(\boldsymbol{\pi}, \mathbf{S}) \implies \begin{cases} F_Y(y) = 1 - \boldsymbol{\pi}^{\text{T}} \exp\{y\mathbf{S}\}\mathbf{e} \\ f_Y(y) = \boldsymbol{\pi}^{\text{T}} \exp\{y\mathbf{S}\}\mathbf{s} \end{cases}$$

Relating to the Toy Example

State	Meaning
1	both PS working
2	1 failed, 2 working
3	1 working, 2 failed
4	subsystem failed



$$\implies \mathbf{T} = \left(\begin{array}{ccc|c} -2\lambda_\mathrm{f} & \lambda_\mathrm{f} & \lambda_\mathrm{f} & 0 \\ \lambda_\mathrm{r} & -\lambda_\mathrm{r} - \lambda_\mathrm{f} & 0 & \lambda_\mathrm{f} \\ \lambda_\mathrm{r} & 0 & -\lambda_\mathrm{r} - \lambda_\mathrm{f} & \lambda_\mathrm{f} \\ \hline 0 & 0 & 0 & 0 \end{array}\right)$$

$$\mathbf{S} \qquad \mathbf{s}$$

$$f_Y(y) = \boldsymbol{\pi}^\mathrm{T} \exp\{y\mathbf{S}\}\mathbf{s} \qquad F_Y(y) = 1 - \boldsymbol{\pi}^\mathrm{T} \exp\{y\mathbf{S}\}\mathbf{e}$$

Trinity College Dublin

Inferential Setting

Cano *et al.* (2010) provide Bayesian learning results in the context of analysing repairable systems when the stochastic process leading to absorption is observed.

Data

For each system failure time, one has:

- Starting state
- Length of time in each state
- Number of transitions between each state
- Ultimate system failure time

Inferential Setting

Cano *et al.* (2010) provide Bayesian learning results in the context of analysing repairable systems when the stochastic process leading to absorption is observed.

Data

For each system failure time, one has:

- Starting state
- ~~Length of time in each state~~
- ~~Number of transitions between each state~~
- Ultimate system failure time

Reduced information scenario $\Longrightarrow$ Bladt *et al.* (2003) provide a Bayesian MCMC algorithm, or Asmussen *et al.* (1996) provide a frequentist EM algorithm.

Slide for Statisticians!

Strategy is a top-level Gibbs step which achieves the goal of simulating from

$$p(\boldsymbol{\pi}, \mathbf{S} \,|\, \mathbf{y})$$

by sampling from

$$p(\boldsymbol{\pi}, \mathbf{S}, \text{paths } \cdot \,|\, \mathbf{y})$$

through the iterative process

$$p(\boldsymbol{\pi}, \mathbf{S} \,|\, \text{paths } \cdot, \mathbf{y})$$

$$p(\text{paths } \cdot \,|\, \boldsymbol{\pi}, \mathbf{S}, \mathbf{y})$$

where $p(\text{paths } \cdot \,|\, \boldsymbol{\pi}, \mathbf{S}, \mathbf{y})$ is achieved by a rejection sampling within Metropolis-Hastings algorithm.

High-level Description of Bladt et al.

The following are key points to note about the MCMC scheme:

- fully dense rate matrix with separate parameters, e.g.

$$\mathbf{T} = \begin{pmatrix} \cdot & S_{12} & S_{13} & s_1 \\ S_{21} & \cdot & S_{23} & s_2 \\ S_{31} & S_{32} & \cdot & s_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- no censored data

- slow computational speed in some common scenarios

- focused on ‘distribution fitting’

High-level Description of Bladt et al.

The following are key points to note about the MCMC scheme:

- fully dense rate matrix with separate parameters, e.g.

$$\mathbf{T} = \begin{pmatrix} \cdot & S_{12} & S_{13} & s_1 \\ S_{21} & \cdot & S_{23} & s_2 \\ S_{31} & S_{32} & \cdot & s_3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

  $\rightarrow$ **we extend to allow structure to be imposed**
- no censored data
  $\rightarrow$ **we accommodate censoring**
- slow computational speed in some common scenarios
  $\rightarrow$ **we provide novel sampling scheme**
- focused on 'distribution fitting'
  $\rightarrow$ **all together shifts focus to stochastic modelling**

Statistical -vs- Stochastic

In other words, we adapt the MCMC algorithm to be fit for performing inference when Phase-types are used for stochastic rather than statistical modelling.

Stochastic Model $\longrightarrow$ **Aslett & Wilson**

"Stochastic models seek to represent an underlying physical phenomenon of interest, albeit often in a highly idealised way, and have parameters that are physically interpretable." — Isham

Statistical Model $\longrightarrow$ **Bladt et al**

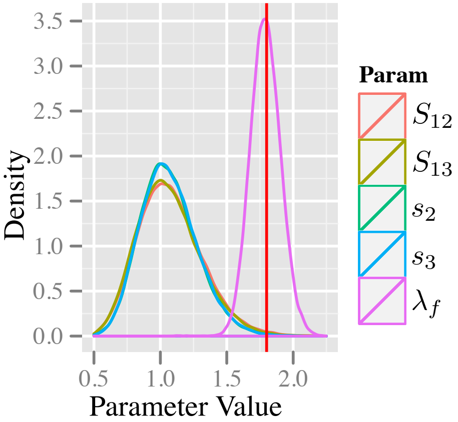
"In contrast, statistical models are descriptive, and represent the statistical properties of data and their dependence on covariates, without aiming to encapsulate the physical mechanisms involved." — Isham

Toy Example Results

100 uncensored observations simulated from PHT with

$$\mathbf{S} = \begin{pmatrix} -3.6 & 1.8 & 1.8 \\ 9.5 & -11.3 & 0 \\ 9.5 & 0 & -11.3 \end{pmatrix}$$

$$\implies \lambda_f = 1.8, \ \lambda_r = 9.5$$

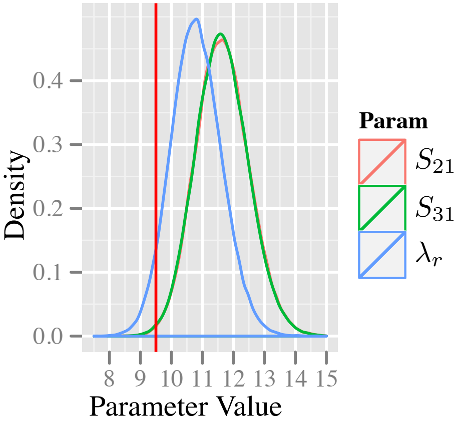


Toy Example Results

100 uncensored observations simulated from PHT with

$$\mathbf{S} = \begin{pmatrix} -3.6 & 1.8 & 1.8 \\ 9.5 & -11.3 & 0 \\ 9.5 & 0 & -11.3 \end{pmatrix}$$

$$\implies \lambda_f = 1.8, \;\; \lambda_r = 9.5$$



Reliability less sensitive to λ_r

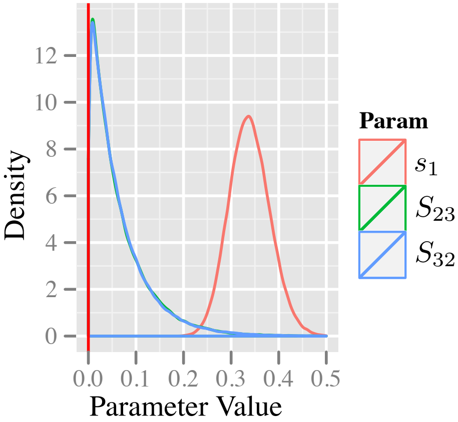
Daneshkhah and Bedford (2008)

Toy Example Results

100 uncensored observations simulated from PHT with

$$\mathbf{S} = \begin{pmatrix} -3.6 & 1.8 & 1.8 \\ 9.5 & -11.3 & 0 \\ 9.5 & 0 & -11.3 \end{pmatrix}$$

$$\implies \lambda_f = 1.8,\ \lambda_r = 9.5$$



Solution: "Exact Conditional Sampling"

Metropolis-Hastings

$$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \boldsymbol{\pi}, \mathbf{S}, Y = y)$$

$\uparrow$

Rejection Sampling

$$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \boldsymbol{\pi}, \mathbf{S}, Y \geq y)$$

$\uparrow$

CTMC Sampling

$$f_{\Phi \,|\, \Psi}(\phi \,|\, \boldsymbol{\pi}, \mathbf{S})$$

Solution: "Exact Conditional Sampling"

Metropoli…

$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \ldots$

Rejection

$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \ldots$

CTMC S…

$f_{\Phi \,|\, \Psi}(\phi \,|\, \boldsymbol{\pi}, \mathrm{S})$

i) Starting state ~ discrete

$$\phi^{\{0\}} \sim \boldsymbol{\pi}$$

ii) Advance time ~ Exponential

$$\delta \sim \mathrm{Exp}(-G_{\phi^{\{t\}},\phi^{\{t\}}})$$

iii) Select next state ~ discrete

$$\phi^{\{t+\delta\}} \sim \{-G_{\phi^{\{t\}},-\phi^{\{t\}}} / G_{\phi^{\{t\}},\phi^{\{t\}}}\}$$

iv) If not absorbed, $t = t + \delta$, loop to ii

Solution: "Exact Conditional Sampling"

~~Metropoli[s-Hastings]~~

$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \ldots$

~~Rejection~~

$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \ldots$

CTMC S...

$f_{\Phi \,|\, \Psi, Y}(\phi \,|\, \boldsymbol{\pi}, \mathbf{S}, Y = y)$

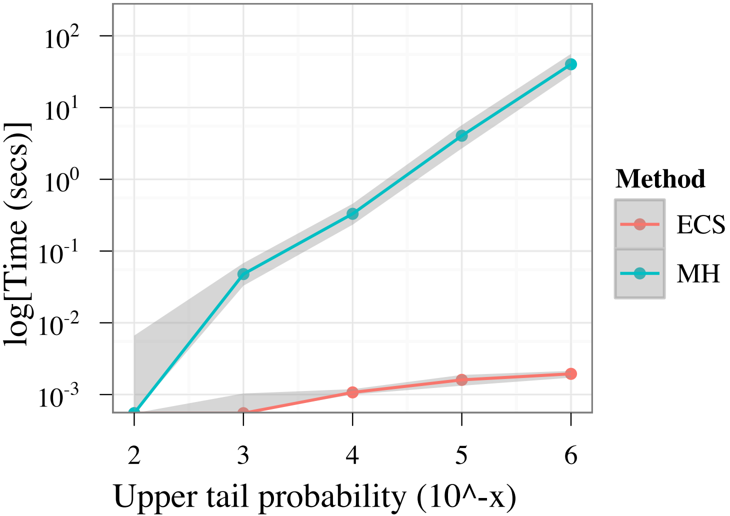
e.g. Starting state mass function changes with conditioning:

$$\mathbb{P}(\phi^{\{0\}} = i \,|\, \boldsymbol{\pi}, \mathbf{S}) = \pi_i$$

$$\downarrow$$

$$\mathbb{P}(\phi^{\{0\}} = i \,|\, \boldsymbol{\pi}, \mathbf{G}, Y = y) = \frac{\mathbf{e}_i^{\mathrm{T}} \exp\{\mathbf{S}y\}\mathbf{s}\ \pi_i}{\boldsymbol{\pi}^{\mathrm{T}} \exp\{\mathbf{S}y\}\mathbf{s}}$$

'Tail Depth' Performance Improvement



Trinity College Dublin

Science Foundation Ireland

Overall Performance Improvement

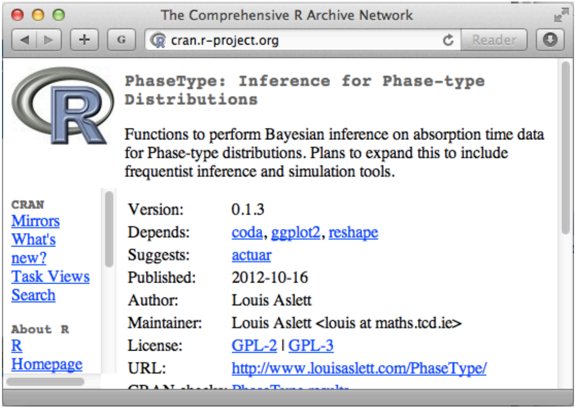
This shows the new method keeping pace in 'nice' problems and significantly outperforming otherwise.

$\mathbf{T} = \begin{pmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 1 & 1 & -3 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\mathbf{T} = \begin{pmatrix} -2 & 0.01 & 1.99 & 0 \\ 1 & -300 & 0 & 299 \\ 299 & 0 & -300 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
<u>No</u> problems i-iii	<u>All</u> problems i-iii

	MH	ECS	MH	ECS
$\bar{t}$	1.6	7.2	10.2 hours	0.016 secs
s_t	104	19	9.4 hours	0.015 secs

2,300,000 × faster on average in hard problem

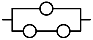
R Package: PhaseType



http://cran.r-project.org/package=PhaseType

Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

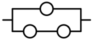
$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:

$$\psi = \psi_1$$

Missing Data

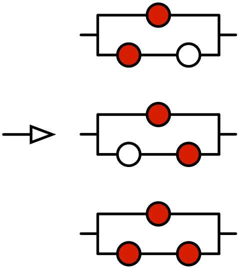
Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

$$\mathbf{t} = \{1.1, 4.2\}$$

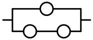
Need realisations concordant with each observation:

$\psi = \psi_1$



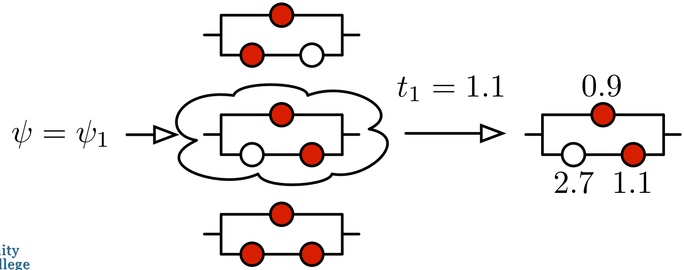
Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

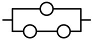
$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:



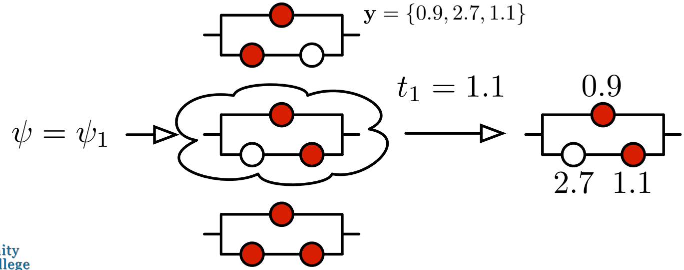
Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

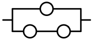
$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:



Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

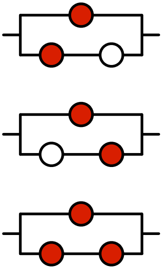
Consider the system  from the introduction, with observed system failure times:

$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:

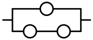
$\mathbf{y} = \{0.9, 2.7, 1.1\}$

$\psi = \psi_1$



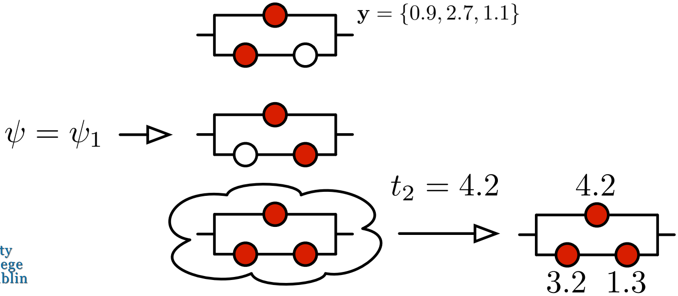
Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

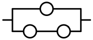
$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:



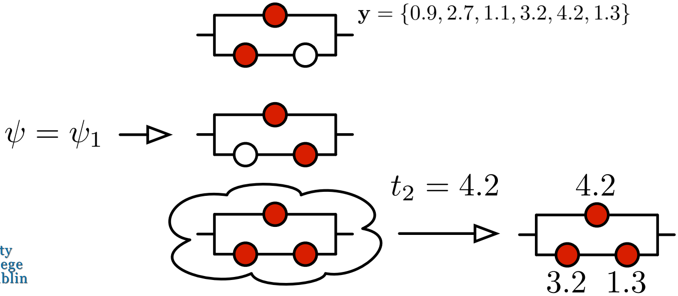
Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

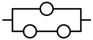
$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:



Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

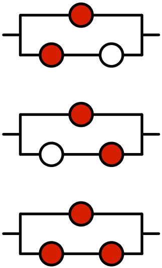
Consider the system  from the introduction, with observed system failure times:

$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:

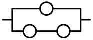
$\mathbf{y} = \{0.9, 2.7, 1.1, 3.2, 4.2, 1.3\}$

$\psi = \psi_1$



Missing Data

Again, the missing data is what makes the inference hard. Tanner and Wong (1987) is a classic solution to this in a Bayesian framework if the missing data can be simulated.

Consider the system  from the introduction, with observed system failure times:

$$\mathbf{t} = \{1.1, 4.2\}$$

Need realisations concordant with each observation:

$$\mathbf{y} = \{0.9, 2.7, 1.1, 3.2, 4.2, 1.3\}$$

$$\psi = \psi_2$$

Trinity College Dublin

Science Foundation Ireland

Missing Data

For any statisticians, that is:

$$f_{Y \,|\, \Psi, T}(\mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot} \,|\, \psi, \mathbf{t})$$

$$f_{\Psi \,|\, Y, T}(\psi \,|\, \mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot}, \mathbf{t})$$

Missing Data

For any statisticians, that is:

$$f_{Y\,|\,\Psi,T}(\mathbf{y}_{1\cdot},\dots,\mathbf{y}_{m\cdot}\,|\,\psi,\mathbf{t})$$

$$f_{\Psi\,|\,Y,T}(\psi\,|\,\mathbf{y}_{1\cdot},\dots,\mathbf{y}_{m\cdot},\mathbf{t})$$

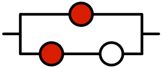
Missing Data

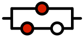
For any statisticians, that is:

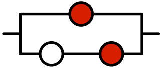
$$f_{Y \,|\, \Psi, T}(\mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot} \,|\, \psi, \mathbf{t})$$

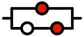
$$f_{\Psi \,|\, Y, T}(\psi \,|\, \mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot}, \mathbf{t})$$

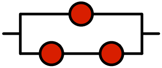
What is the challenge?

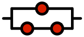


$\mathbb{P}($  $|\, \psi_1, t_1) = \, ?$

$\psi = \psi_1 \longrightarrow$ 

$\mathbb{P}($  $|\, \psi_1, t_1) = \, ?$



$\mathbb{P}($  $|\, \psi_1, t_1) = \, ?$

Trinity College Dublin

System Signatures

The signature (Samaniego, 1985) is less widely used than the structure function, but in some ways more elegant.

Definition (Signature)

The *signature* of a system is the n-dimensional probability vector $\mathbf{s} = (s_1, \dots, s_n)$ with elements:

$$s_i = \mathbb{P}(T = Y_{i:n})$$

where T is the failure time of the system and $Y_{i:n}$ is the ith order statistic of the n component failure times.

System Signatures

The signature (Samaniego, 1985) is less widely used than the structure function, but in some ways more elegant.

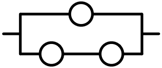
Definition (Signature)

The *signature* of a system is the n-dimensional probability vector $\mathbf{s} = (s_1, \dots, s_n)$ with elements:

$$s_i = \mathbb{P}(T = Y_{i:n})$$

where T is the failure time of the system and $Y_{i:n}$ is the ith order statistic of the n component failure times.

e.g.



$$\implies \mathbf{s} = \left(0, \frac{2}{3}, \frac{1}{3}\right)$$

Sampling Latent Failure Times

It can be shown:

$$
\begin{aligned}
&f_{Y|\,T}(y_{i1}, \dots, y_{in}; \psi \mid t) \\
&\quad \propto \sum_{j=1}^{n} \Bigg[f_{Y|\,Y<t}(y_{i(1)}, \dots, y_{i(j-1)}; \psi) \\
&\qquad\qquad \times f_{Y|\,Y>t}(y_{i(j+1)}, \dots, y_{i(n)}; \psi) \\
&\qquad\qquad \times \mathbb{I}_{\{t\}}(y_{i(j)}) \\
&\qquad\qquad \times \binom{n-1}{j-1} F_Y(t;\psi)^j \, \bar{F}_Y(t;\psi)^{n-j+1} \, s_j \Bigg]
\end{aligned}
$$

Signature based data augmentation

1. With probability

$$\mathbb{P}(j) \propto \binom{n-1}{j-1} F_Y(t_i;\psi)^j \, \bar{F}_Y(t_i;\psi)^{n-j+1} \, s_j$$

it was the jth failure that caused system failure.

2. Having drawn a random j, sample
   - $j-1$ values, $y_{i1}, \dots, y_{i(j-1)}$, from $F_{Y \mid Y<t_i}(\,\cdot\,;\psi)$, the distribution of the component lifetime conditional on failure before t_i
   - $n-j$ values, $y_{i(j+1)}, \dots, y_{in}$, from $F_{Y \mid Y>t_i}(\,\cdot\,;\psi)$, the distribution of the component lifetime conditional on failure after t_i

and set $y_{ij} = t_i$.

Prerequisites

This is a very general method. The prerequisites for use are,

1. The signature of the system;
2. The ability to perform standard Bayesian inference with the full data;
3. The ability to sample from $F_{Y \mid Y<t_i}(\,\cdot\,;\psi)$ and $F_{Y \mid Y>t_i}(\,\cdot\,;\psi)$.

Prerequisites

This is a very general method. The prerequisites for use are,

1. The signature of the system;

   Easy for systems that are not huge

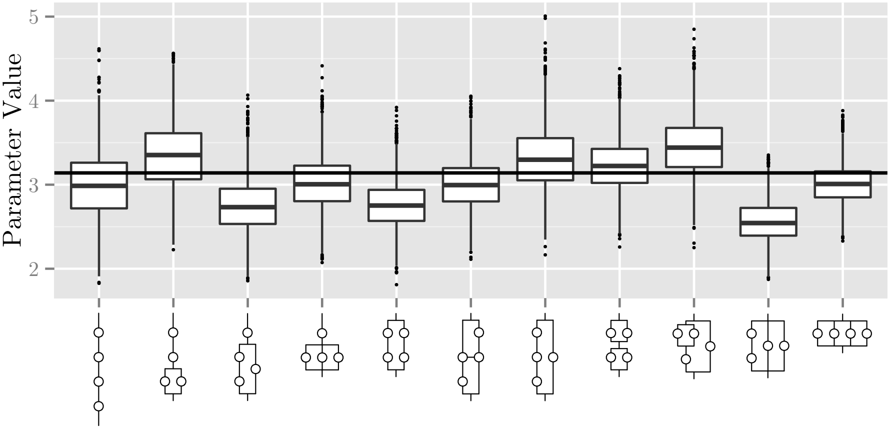
2. The ability to perform standard Bayesian inference with the full data;

   Easy for common lifetime distributions

3. The ability to sample from $F_{Y \mid Y < t_i}(\,\cdot\,; \psi)$ and $F_{Y \mid Y > t_i}(\,\cdot\,; \psi)$.

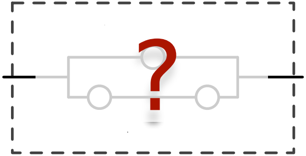
   Depends!

Canonical Exponential Component Lifetime Example



Unknown Topologies

A little 'blue skies' academic thinking ...



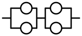
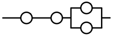
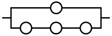
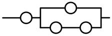
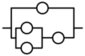
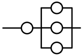
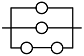
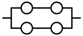
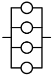
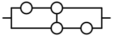
$$t = y_? = y_{?:3} = 3.1$$

Uniqueness of the Signature

Type	Order	Signature repetition							Total
		Unique	2	3	4	5	6	7	
Coherent systems	2	2	0	0	0	0	0	0	2
	3	5	0	0	0	0	0	0	5
	4	14	3	0	0	0	0	0	20
	5	43	15	2	6	2	10	1	180
Coherent systems /w graph	2	2	0	0	0	0	0	0	2
	3	4	0	0	0	0	0	0	4
	4	11	0	0	0	0	0	0	11
	5	27	4	0	0	0	0	0	35

Signature & Topology

Order 4 coherent systems with graph representation.

System Topology	Signature	System Topology	Signature
	$(1, 0, 0, 0)$		$\left(0, \frac{1}{3}, \frac{2}{3}, 0\right)$
	$\left(\frac{1}{2}, \frac{1}{2}, 0, 0\right)$		$\left(0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right)$
	$\left(\frac{1}{4}, \frac{7}{12}, \frac{1}{6}, 0\right)$		$\left(0, \frac{1}{6}, \frac{7}{12}, \frac{1}{4}\right)$
	$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 0\right)$		$\left(0, 0, \frac{1}{2}, \frac{1}{2}\right)$
	$\left(0, \frac{2}{3}, \frac{1}{3}, 0\right)$		$(0, 0, 0, 1)$
	$\left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$		

Jointly Inferring the Topology

$$f_{Y \,|\, \Psi, T}(\mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot} \,|\, \psi, \mathbf{t})$$

$$f_{\Psi \,|\, Y, T}(\psi \,|\, \mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot}, \mathbf{t})$$

Jointly Inferring the Topology

$$f_{Y \,|\, \Psi, T}(\mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot} \,|\, \psi, \mathbf{t}, \mathbf{s})$$

$$f_{\Psi \,|\, Y, T}(\psi \,|\, \mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot}, \mathbf{t}, \cancel{\mathbf{s}})$$

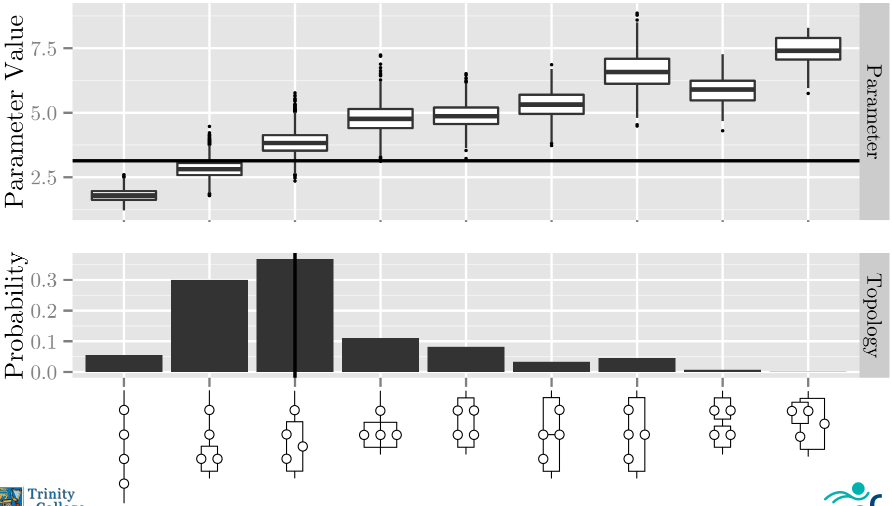
Jointly Inferring the Topology

$$f_{Y \,|\, \Psi, T}(\mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot} \,|\, \psi, \mathbf{t}, \mathbf{s})$$

$$f_{\Psi \,|\, Y, T}(\psi \,|\, \mathbf{y}_{1\cdot}, \dots, \mathbf{y}_{m\cdot}, \mathbf{t}, \cancel{\mathbf{s}})$$

After satisfying a few technical subtleties, implementation is not too difficult.

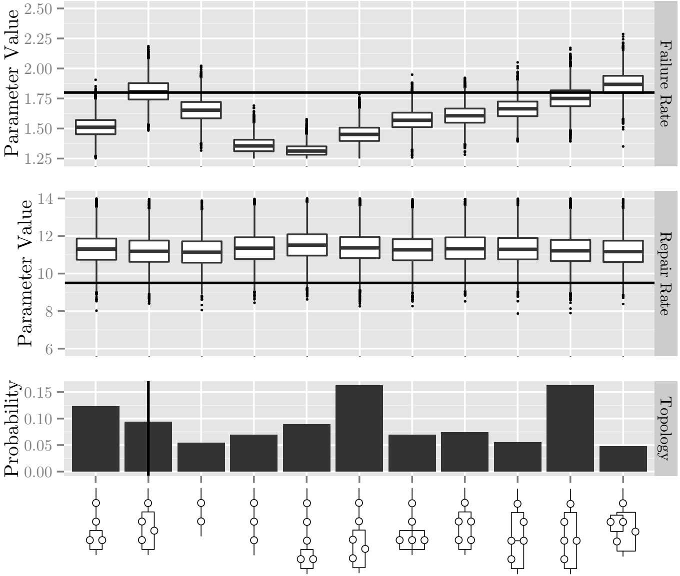
Canonical Exponential Component Lifetime Example



Trinity College Dublin

Science Foundation Ireland

Phase-type Component Lifetime Example

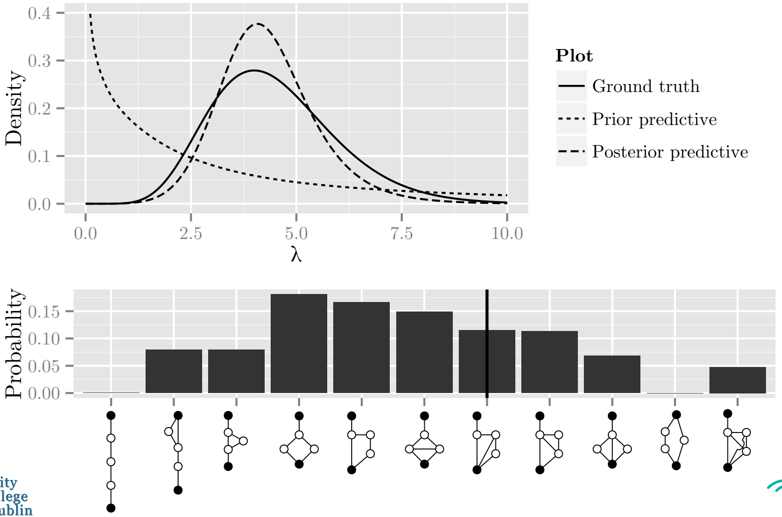


Trinity College Dublin

sfi Science Foundation Ireland

Exchangeable Systems

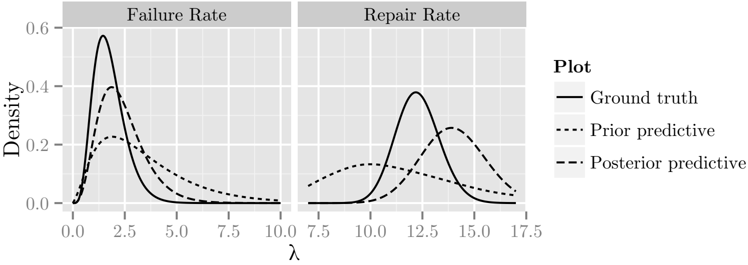
The i.i.d. systems assumption easily relaxed to exchangeability.



Trinity College Dublin

Phase-type Component Lifetimes

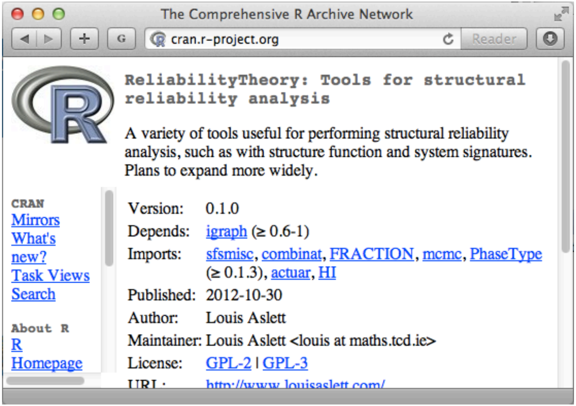
Extreme generality of the solution allows wide variety of component lifetime distributions. Solutions to the prerequisites have been derived for Phase-type distributed components.



May interpret as:

- Repairable redundant subsystems;
- Theoretically dense in function space of all positively supported continuous distributions.

R Package: ReliabilityTheory



http://cran.r-project.org/package=ReliabilityTheory

Future Work

A couple of the many important avenues to be pursued:

- Many partial information scenarios between full information and the extreme presented here.
- Extend the non-repairable work to non-identical components using the survival signature (Coolen and Coolen-Maturi, 2012).

References I

Aslett, L. J. M. (2011), *PhaseType: Inference for Phase-type Distributions*. R package version 0.1.3.

Aslett, L. J. M. (2012*a*), MCMC for Inference on Phase-type and Masked System Lifetime Models, PhD thesis, Trinity College Dublin.

Aslett, L. J. M. (2012*b*), *ReliabilityTheory: Tools for structural reliability analysis*. R package version 0.1.0.

Asmussen, S., Nerman, O. and Olsson, M. (1996), 'Fitting Phase-type distributions via the EM algorithm', *Scandinavian Journal of Statistics* **23**(4), 419–441.

Bladt, M., Gonzalez, A. and Lauritzen, S. L. (2003), 'The estimation of Phase-type related functionals using Markov chain Monte Carlo methods', *Scandinavian Actuarial Journal* **2003**(4), 280–300.

Cano, J., Moguerza, J. M. and Ríos Insua, D. (2010), 'Bayesian reliability, availability, and maintainability analysis for hardware systems described through continuous time Markov chains', *Technometrics* **52**(3), 324–334.

Coolen, F. P. A. and Coolen-Maturi, T. (2012), Generalizing the signature to systems with multiple types of components, *in* 'Complex Systems and Dependability', Springer.

Daneshkhah, A. and Bedford, T. (2008), Sensitivity analysis of a reliability system using gaussian processes, *in* T. Bedford, J. Quigley, L. Walls, B. Alkali, A. Daneshkhah and G. Hardman, eds, 'Advances in Mathematical Modeling for Reliability', IOS Press, chapter 2, pp. 46–62.

Samaniego, F. J. (1985), 'On closure of the IFR class under formation of coherent systems', *IEEE Transactions on Reliability* **R-34**(1), 69–72.

Tanner, M. A. and Wong, W. H. (1987), 'The calculation of posterior distributions by data augmentation', *Journal of the American Statistical Association* **82**(398), 528–540.