http://oeis.org/A005995 - F(1,5,n)

Original work by Ata Aydin Uslu – Hamdi Goktan Ozmenekse

15.03.2012

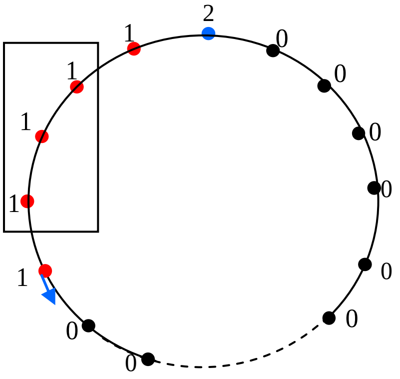
Explanation: Number of bracelets made with 1 blue, 5 identical red and n identical black beads.

Usage: Chemistry: Paraffin numbers, Maths: Circular permutations of identical objects

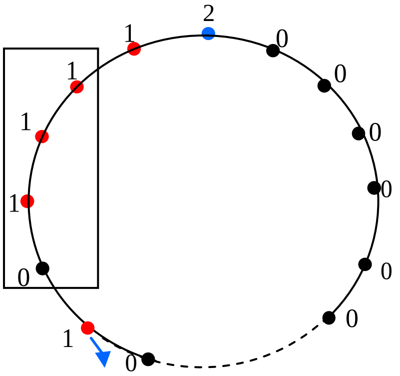
Theorem : If F(1,5,n) is the number of bracelets made with 1 blue, 5 identical red and n identical black beads

$$F(1,5,n)=\frac{(n+3).(n+2).(n+1).n}{24}+F(1,3,n)+F(1,5,n-2)$$

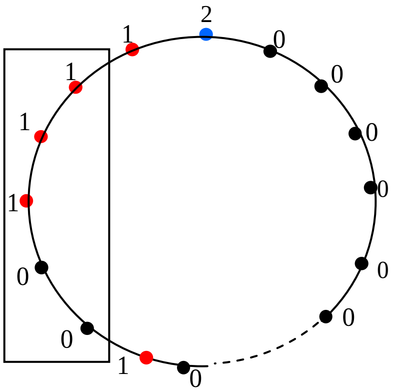
Proof :



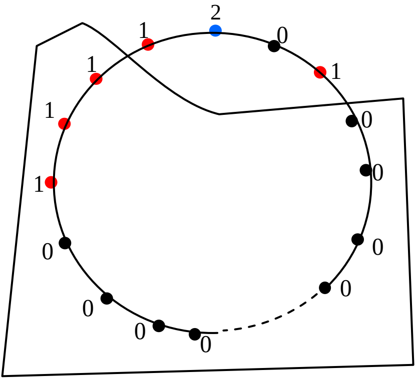
For 111 there are $\binom{3}{3}$ combinatorial states



For 1110 there are $\binom{4}{3}$ combinatorial states



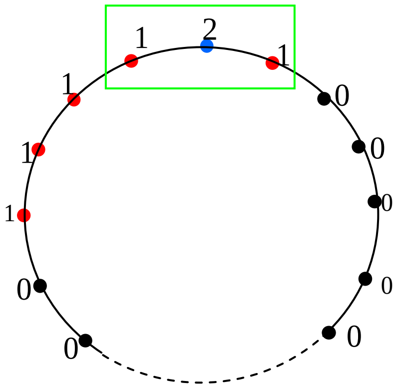
For 11100 there are $\binom{5}{3}$ combinatorial states



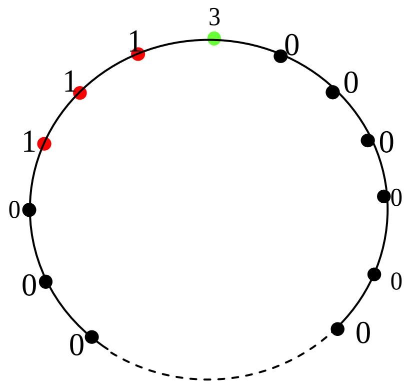
If we continue similarly, for $111\underbrace{00...0}_{n-1}$ there are $\binom{n+2}{3}$ combinatorial states

If we sum up all the states, then

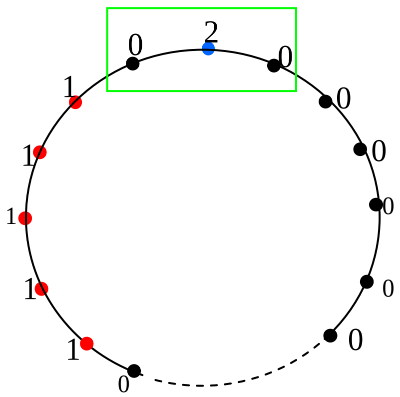
$$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+\ldots+\binom{n+2}{3}=\binom{n+3}{4}=\frac{(n+3).(n+2).(n+1).n}{24}$$



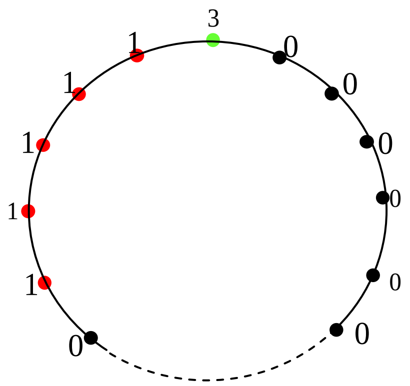
$(121)\to 3$



There are $F(1,3,n)$ states



$(121)\to 4$



There are $F(1,5,n-2)$ states

Total number of possible states:

$$F(1,5,n)=\frac{(n+3).(n+2).(n+1).n}{24}+F(1,3,n)+F(1,5,n-2)$$

Using $F(1,3,1)=2$ and $F(1,3,2)=6$,

$F(1,5,1)=3$
$F(1,5,2)=12$
$F(1,5,3)=28$

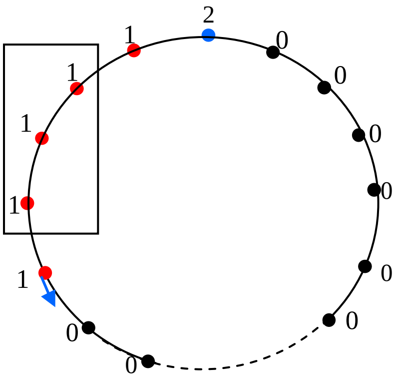
$F(1,5,4) = 66$
$F(1,5,5) = 126$
$F(1,5,6) = 236$

Turkish:

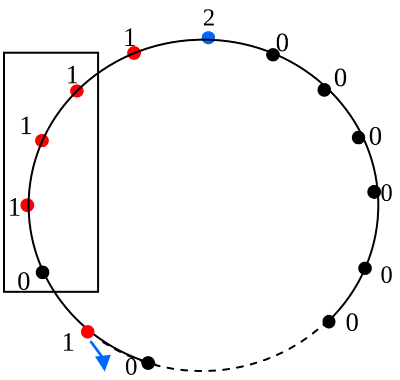
Teorem 3 : 1 tane özdeş mavi, 5 tane özdeş kırmızı ve n tane özdeş siyah boncuklar ile yapılacak bilekliklerin sayısı $F(1,5,n)$ ise

$$F(1,5,n) = \frac{(n+3).(n+2).(n+1).n}{24} + F(1,3,n) + F(1,5,n-2) \text{ dir.(........)}$$

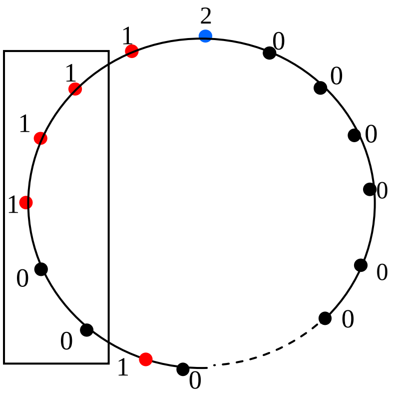
İSPAT :



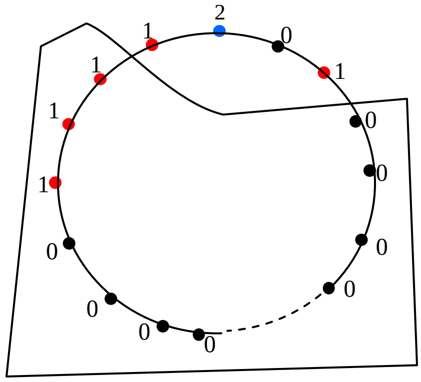
111 kendi arasındaki sıralaması $\binom{3}{3}$ tane durum vardır



1110 kendi arasındaki sıralaması $\binom{4}{3}$ tane durum vardır.



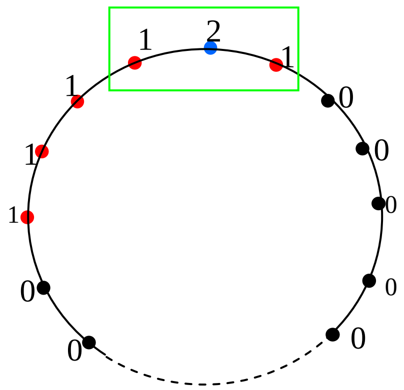
11100 kendi arasındaki sıralaması $\binom{5}{3}$ tane durum vardır.



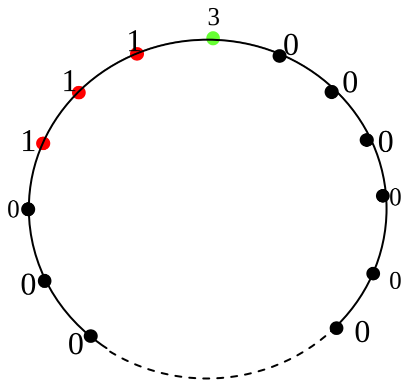
Benzer olarak devam edersek . $11\underbrace{100...0}_{n-1}$ kendi arasındaki sıralaması $\binom{n+2}{3}$ tane durum vardır.

Elde ettiğimiz bütün durumları toplarsak

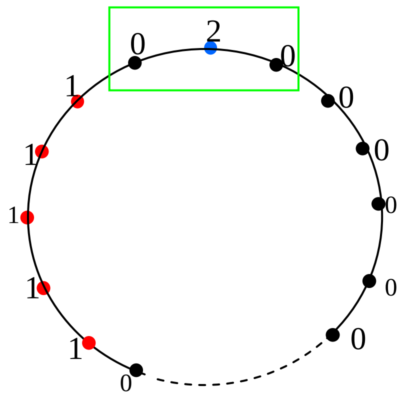
$$\binom{3}{3}+\binom{4}{3}+\binom{5}{3}+...+\binom{n+2}{3}=\binom{n+3}{4}=\frac{(n+3).(n+2).(n+1).n}{24}$$



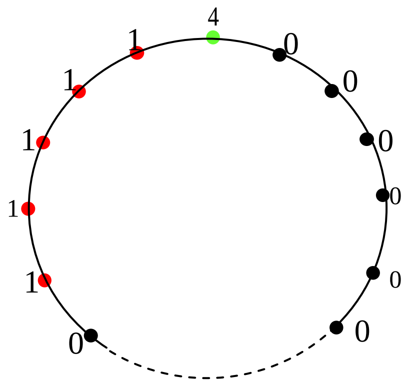
$(121)\rightarrow 3$



$F(1,3,n)$ Tane durum vardır.



$(020)\rightarrow 4$



$F(1,5,n-2)$ durum oluşur.

Oluşacak toplam durum sayısı:

$$F(1,5,n)=\frac{(n+3).(n+2).(n+1).n}{24}+F(1,3,n)+F(1,5,n-2)$$ ile ifade edebiliriz.

$F(1,3,1)=2$ ve $F(1,3,2)=6$ ve $F(1,5,1)=3$ ve $F(1,5,2)=12$

$$F(1,5,3) = 28$$
$$F(1,5,4) = 66$$
$$F(1,5,5) = 126$$
$$F(1,5,6) = 236$$