Proceedings of ICTACEM 2017
International Conference on Theoretical, Applied, Computational and Experimental Mechanics
December 28-30, 2017, IIT Kharagpur, India
ICTACEM-2017/349

On the order of accuracy analysis of SDWLS method

Sourabh P. Bhat[a], S. Parameswaran[b] and J. C. Mandal[c*]
[a,b] Research Scholar, IIT Bombay, Mumbai, INDIA
[c] Professor, IIT Bombay, Mumbai, INDIA

1. INTRODUCTION & OBJECTIVE

The use of high order methods have advantages over the lower order methods [1], in terms of accuracy as well as computational efficiency. However, a lack of robustness and difficulties in implementation of high order methods on unstructured mesh has been observed. A relatively new method called the solution dependent weighted least square (SDWLS) [2, 3] tries to overcome these difficulties by using a very simple to implement methodology on arbitrary mesh. In this work, the order of accuracy analysis of the SDWLS method, using linear and quadratic reconstruction, is performed.

2. RESULTS & DISCUSSIONS

2.1 Two-Dimensional Scalar Problems

A two-dimensional scalar advection equation may be written as,

$$u_t + u_x + u_y = 0 \tag{1}$$

The exact solution of this equation is simple translation of the initial distribution with a velocity of $(1, 1)$. A standard test case [4, 5] with an initial smooth distribution given by (2) is solved by using various reconstruction methods and an order of accuracy analysis is performed on structured and unstructured mesh.

$$u = \sin\left(\frac{\pi}{2}(x + y)\right) \qquad -2 \leq x \leq 2; \quad -2 \leq y \leq 2 \tag{2}$$

2.1.1 *Smooth sinusoidal distribution (on unstructured mesh)*

The equation (1) is solved on a unstructured mesh using the Local Lax-Friedrich (LLF) Riemann solver, using two Gauss-Quadrature points and a $CFL = 0.3$. The results of the accuracy analysis are displayed in table 1. It can be observed that the L_1 error in case of SDWLS-L drops at an order of about 1.7 and in case of SDWLS-Q the error drops at an order of about 2. The SDWLS-L method does not behave very well in terms of L_∞ error as the error drops only at the rate of about 1, while SDWLS-Q maintains the order of about 2. As expected the magnitude of error in case of SDWLS-Q is smaller compared to SDWLS-L by an order of magnitude. The results from the literature [4, 5] for third order WENO and

*Further author information: (Send correspondence to J.C.M)
S.P.B.: E-mail: spbhat.mail@gmail.com, Telephone: +91-8291475328
S.P.: E-mail: sanalp@iitb.ac.in, Telephone: +91-9167096523
J.C.M.: E-mail: mandal@aero.iitb.ac.in, Telephone: +91-22-25767129, Address: Department of Aerospace Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400076, INDIA

fourth order WENO show 3.0 and 4.0 respectively as the order for fine mesh. The magnitude of error of the WENO3 on fine mesh is better compared to SDWLS-Q, however on coarser mesh SDWLS-Q behaves much better, comparing the results from the literature [6] with similar mesh sizes.

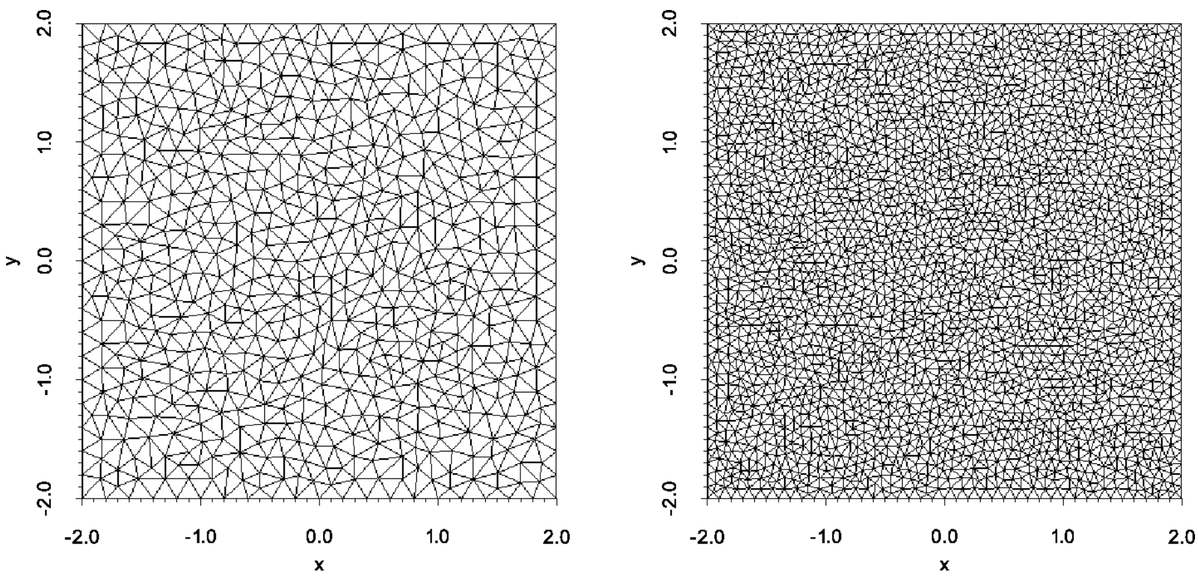

Figure 1: Mesh with 1218 and 4898 cells – for scalar advection equation

Table 1: 2D Linear advection problem – unstructured mesh, T=2, RK3

Method	Mesh Cells	L_1 Error	L_1 Order	L_∞ Error	L_∞ Order
SDWLS-L	308	1.4587×10^{-1}	–	3.4983×10^{-1}	–
	1218	6.2217×10^{-2}	1.240	1.4327×10^{-1}	1.299
	4898	1.7940×10^{-2}	1.787	5.7394×10^{-2}	1.315
	20076	4.5129×10^{-3}	1.957	2.1144×10^{-2}	1.416
	80186	1.2476×10^{-3}	1.857	1.2088×10^{-2}	0.808
	322940	4.0421×10^{-4}	1.618	6.0216×10^{-3}	1.000
SDWLS-Q	308	6.3938×10^{-2}	–	1.2679×10^{-1}	–
	1218	1.5111×10^{-2}	2.098	2.9719×10^{-2}	2.110
	4898	3.6505×10^{-3}	2.042	6.9103×10^{-3}	2.097
	20076	8.8506×10^{-4}	2.009	1.5355×10^{-3}	2.133
	80186	2.2109×10^{-4}	2.003	3.7314×10^{-4}	2.043
	322940	5.4800×10^{-5}	2.003	8.9400×10^{-5}	2.051

2.2 Two-Dimensional Euler Equations

The general form of two dimensional conservation equations of gas dynamics can be written as,

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0 \tag{3}$$

where, $\mathbf{U}$ is the vector of conservative variables and $\mathbf{F}$ and $\mathbf{G}$ are flux vectors. In case of Euler equations,

$$\mathbf{U} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho E \end{bmatrix}; \quad \mathbf{F} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (\rho E + p)\, u \end{bmatrix}; \quad \mathbf{G} = \begin{bmatrix} \rho u v \\ \rho u v \\ \rho v^2 + p \\ (\rho E + p)\, v \end{bmatrix} \tag{4}$$

where, $E = \left(e + \frac{u^2+v^2}{2}\right)$, $e = c_v T = \frac{RT}{(\gamma-1)}$, and R and γ are gas constants.

2.2.1 *Isentropic vortex (on structured mesh)*

The literature [4] and [5] can be referred to for detailed description of this problem. An isentropic vortex travels such that, after one full travel cycle, the exact solution is the initial condition itself as shown in figure 2. The table 2 shows the order of accuracy analysis performed using Cartesian structured grid using various methods with gradually refined grid size starting from a grid of 40×40 up to a grid of 640×640. The values presented are the errors in ρ (density).

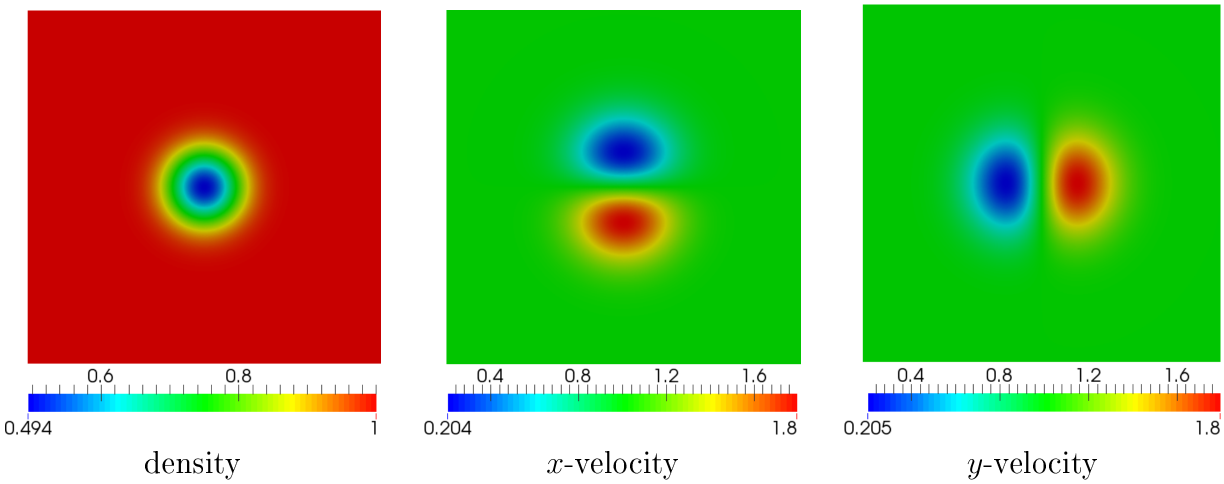

Figure 2: Solution of isentropic vortex problem

Table 2: Isentropic vortex problem – structured mesh, CFL=0.3, T=10, RK3

Method	Grid Size	L_1 Error	L_1 Order	L_∞ Error	L_∞ Order
SDWLS-L	40x40	1.6162×10^{-2}	–	3.2021×10^{-1}	–
	80x80	4.7500×10^{-3}	1.766	9.7100×10^{-2}	1.721
	160x160	1.3529×10^{-3}	1.812	5.3834×10^{-2}	0.851
	320x320	3.8434×10^{-4}	1.816	2.7490×10^{-2}	0.97
	640x640	9.2878×10^{-5}	2.049	9.4313×10^{-3}	1.543
SDWLS-Q	40x40	7.2217×10^{-3}	–	1.5153×10^{-1}	–
	80x80	1.4289×10^{-3}	2.337	2.9510×10^{-2}	2.36
	160x160	2.5447×10^{-4}	2.489	5.6347×10^{-3}	2.389
	320x320	4.3264×10^{-5}	2.556	1.0280×10^{-3}	2.454
	640x640	8.9168×10^{-6}	2.279	2.0525×10^{-4}	2.324

2.2.2 *Isentropic vortex (on unstructured mesh)*

The size of finite volume cells is maintained similar to [4] and [5] so that the results can be compared with WENO methods. The flux is integrated over the cell interfaces using two quadrature points. The table 3 shows the results obtained for order of accuracy for SDWLS-L and SDWLS-Q. It is seen that the order of accuracy of SDWLS methods reduces slightly on unstructured mesh compared to structured mesh. This can be attributed to the first order boundary conditions applied at the boundaries, due to shortage of neighboring cells.

Table 3: Isentropic vortex problem – unstructured mesh, CFL=0.3, T=10, RK3

Method	Mesh Cells	L_1 Error	L_1 Order	L_∞ Error	L_∞ Order
SDWLS-L	300	2.7387×10^{-2}	–	4.4705×10^{-1}	–
	1212	2.3037×10^{-2}	0.248	4.1500×10^{-1}	0.107
	4960	1.3521×10^{-2}	0.756	2.5222×10^{-1}	0.707
	20076	5.1255×10^{-3}	1.388	1.2212×10^{-1}	1.038
	80578	1.9941×10^{-3}	1.359	4.7943×10^{-2}	1.346
	323802	8.7036×10^{-4}	1.192	2.3380×10^{-2}	1.033
SDWLS-Q	300	2.3812×10^{-2}	–	3.9192×10^{-1}	–
	1212	8.4004×10^{-3}	1.492	1.7874×10^{-1}	1.125
	4960	1.8300×10^{-3}	2.163	4.0530×10^{-2}	2.106
	20076	3.8147×10^{-4}	2.243	8.4089×10^{-3}	2.25
	80578	7.9387×10^{-5}	2.259	1.8051×10^{-3}	2.214
	323802	1.8002×10^{-5}	2.134	3.8676×10^{-4}	2.215

REFERENCES

[1] Z. J. Wang, K. Fidkowski, R. Abgrall, F. Bassi, D. Caraeni, A. Cary, H. Deconinck, R. Hartmann, K. Hillewaert, H. T. Huynh, N. Kroll, G. May, P. O. Persson, B. van Leer, and M. Visbal, "High-order CFD methods: Current status and perspective," *International Journal for Numerical Methods in Fluids*, vol. 72, pp. 811–845, July 2013.

[2] J. Mandal and J. Subramanian, "On the link between weighted least-squares and limiters used in higher-order reconstructions for finite volume computations of hyperbolic equations," *Applied Numerical Mathematics*, vol. 58, pp. 705–725, May 2008.

[3] J. C. Mandal and S. P. Rao, "High resolution finite volume computations on unstructured grids using solution dependent weighted least squares gradients," *Computers and Fluids*, vol. 44, pp. 23–31, May 2011.

[4] C. Hu and C.-W. Shu, "Weighted Essentially Non-Oscillatory Schemes on Triangular Meshes," *Journal of Computational Physics*, vol. 150, pp. 97–127, 1999.

[5] Y. Liu and Y. T. Zhang, "A Robust Reconstruction for Unstructured WENO Schemes," *Journal of Scientific Computing*, vol. 54, no. 2-3, pp. 603–621, 2013.

[6] C.-w. Shu, "Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conservation laws," tech. rep., NASA Langley Research Center, 1997.