

# Lecture 1: Elements of Radiative Transfer

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## 1 Preamble

As opposed to expectations, there is not going to be much GFD in this lecture. There will be no equations of motion. The reason is that to understand climate, it is necessary to merge understanding of the large-scale motion with the thermodynamics and specifically with radiative transfer. Most problems that have not been solved in climate science are connected to the interaction of radiative transfer with water substances.

This lecture will therefore review how radiation passing through the atmosphere interacts with its constituents, most importantly with water in its different phases. Much of the fundamentals will only be alluded to, because of the time constraint.

One example of where the interaction of radiation with water substances in the atmosphere is crucial is the Madden–Julian oscillation. This wave that moves around the globe in the tropics is not yet well understood—it is one of the phenomena for which there are more theories than theoreticians. As climate scientists are taking radiation and its interaction with water more seriously, this problem will likely be solved in the near future.

Another one of these problems is tropical cyclogenesis. This also is likely to be understood in the near future.

## 2 Useful facts and numbers

This is a loose compilation of useful facts and numbers that are fundamental controls of Earth’s climate. Much of the detail that could take up entire courses will be skipped over here.

The sun has a luminosity of about  $L_0 = 3.9 \times 10^{26} \text{ J s}^{-1}$ , which corresponds to a radiative flux of  $6.4 \times 10^7 \text{ W m}^{-2}$  at the top of the photosphere. Sunlight is received at the Earth that orbits the sun at a mean distance of about  $d = 1.5 \times 10^{11} \text{ m}$ , so that the flux density is  $S_0 = L_0/4\pi d^2 = 1370 \text{ W m}^{-2}$ . The solar flux exhibits temporal variability, especially associated with the 11-year solar cycle. The amplitude of these variations is of the order of a tenth of a percent of the mean value, and is treated as negligible in this lecture series. However, variations in the solar flux must be taken into consideration when studying climate variability and trends on decadal and multi-decadal time scales.

The incoming solar radiation must be balanced by outgoing terrestrial radiation. A blackbody of temperature  $T$  emits a spectral flux, integrated over a hemisphere, of  $\sigma T^4$ , where  $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^4$  is the Stefan–Boltzmann constant. According to this

Stefan–Boltzmann law, the emitted radiative flux depends strongly on the body’s temperature. The sun’s emission temperature as inferred from the Stefan–Boltzmann law at the top of the photosphere is about 6000 K.

The amount of solar flux absorbed by the Earth is equal to  $S_0(1 - a_p)\pi r_p^2$ , where  $a_p$  is the planetary albedo and  $r_p$  is Earth’s radius. The planetary albedo is the fraction of the solar flux that is reflected back to space. This is due to the combined effect of scattering by clouds, by reflection off ice and other surfaces, and scattering by air molecules and aerosols. In the present climate, Earth’s albedo is about  $a_p = 0.3$ .

Because the total area of Earth is  $4\pi r_p^2$ , the absorbed solar flux per unit area is  $S_0(1 - a_p)/4$ . If the atmosphere were completely transparent and the Earth emitted as a blackbody of uniform temperature  $T_e$ , then

$$\frac{S_0}{4}(1 - a_p) = \sigma T_e^4,$$

Using the numbers cited above, this gives  $T_e = 255 \text{ K} = -18^\circ\text{C}$ . This is much colder than what is observed, which is due to the fact that the atmosphere is not transparent in the infrared. In the more general case, the relation above can be taken as the definition of the effective emission temperature.

The atmosphere is not completely transparent to solar radiation. Due to the presence of water vapor ( $\text{H}_2\text{O}$ ), clouds, ozone ( $\text{O}_3$ ) in the stratosphere, and carbon dioxide ( $\text{CO}_2$ ), some solar radiation is absorbed as it passes through the atmosphere. Terrestrial radiation is absorbed even more strongly, even though it barely interacts with the atmosphere’s major constituents, nitrogen ( $\text{N}_2$ ) and oxygen ( $\text{O}_2$ ). The primary absorbers of terrestrial radiation are water in its three phases,  $\text{CO}_2$ , methane ( $\text{CH}_4$ ), and nitrous oxide.

The distribution of radiative fluxes across frequencies for a blackbody in thermodynamic equilibrium at temperature  $T$  is given by Planck’s law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2 \left( e^{\frac{h\nu}{kT}} - 1 \right)},$$

where  $\nu$  is the frequency,  $h$  is the Planck constant,  $k$  is the Boltzmann constant, and  $c$  is the speed of light. The atmosphere is in thermodynamic equilibrium except at very high altitudes, where the gas density is so low that the atmosphere is very nearly transparent. Planck’s law tells us that the distribution of radiative fluxes across frequencies, or equivalently wavelengths, depends on its temperature  $T$ . Integrating this expression over all frequencies and all angles in a hemisphere gives the Stefan–Boltzmann law:

$$\pi \int_0^\infty B_\nu(T) d\nu = \sigma T^4,$$

where the Stefan–Boltzmann constant can be written in terms of more fundamental physical constants,  $\sigma = 2\pi^5 k^4 / 15c^2 h^3$ .

### 3 Absorption in the atmosphere

How does radiation interact with a gas? The basis of our understanding for how photons get absorbed by gas molecules is quantum physics, which will only be alluded to here.

An isolated atom can absorb photons only at discrete frequencies, those that correspond by  $E_\nu = h\nu$  to differences in the available discrete energy levels of the atom's electron shell. Molecules have additional energy levels associated with additional degrees of freedom. These additional energy levels allow absorption of photons at a larger number of frequencies (Fig. 1).

For  $\text{N}_2$  and  $\text{O}_2$ , only a few extra degrees of freedom are available. Compared to a single atom, they have an extra vibrational mode, but they remain relatively weak absorbers.

The more complex the molecule becomes, the more degrees of freedom it has, and thus the more absorption lines are present. Carbon monoxide ( $\text{CO}$ ), for example, is asymmetric and thus has a permanent dipole, which gives it rotational modes of interaction with photons. More complex molecules allow additional rotational and vibrational degrees of freedom—and combinations of different modes of excitation. Photodissociation or ionization also leads to absorption.




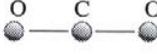

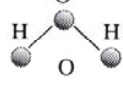
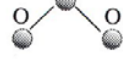
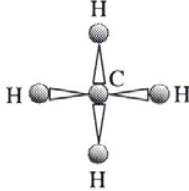
The discrete energy levels of the molecules give rise to absorption lines at discrete frequencies. But if the absorption were only present at truly discrete frequencies, no absorption at all would be possible, as the continuous spectrum of solar radiation contains only an infinitesimal amount of energy in an infinitesimal frequency window. In reality, the atmosphere absorbs a finite amount of energy due to the broadening of absorption lines by two effects (Fig. 2). First, because of the movement of the molecules in a gas, the absorption frequencies get Doppler shifted slightly. The molecules move in random directions, so averaged over all molecules, absorption can occur over a distribution around the nominal absorption frequency. Second, the collision between molecules also broadens the absorption line. This is called pressure broadening, because the collision rate depends on pressure.

Elements of the absorption properties of gases have been known since the 19th century. John Tyndall measured the absorption capabilities of the various atmospheric gases in the infrared, concluding that water vapor was the most powerful absorber.

Water is a strong absorber because of its bent triatomic structure and its permanent dipole. It has a purely rotational absorption band and bands due to rotation, vibration, and translation. Ozone, while also absorbing in the infrared, is mostly important because it is involved in photodissociation in the stratosphere that absorbs ultra-violet (UV) light emitted by the sun.  $\text{CO}_2$  has no permanent dipole, so it is not as strong an absorber as  $\text{H}_2\text{O}$ , but a dipole is present when the vibrational mode is excited, so  $\text{CO}_2$  is an important absorber in the infrared as well. Other strongly absorbing constituents are nitrogen dioxide ( $\text{N}_2\text{O}$ ) and  $\text{CH}_4$ .

It is interesting to note that the main absorbers  $\text{H}_2\text{O}$  and  $\text{CO}_2$  only make up a small fraction of the total mass of the atmosphere (Fig. 3). The bulk of the atmospheric molecules,  $\text{N}_2$  and  $\text{O}_2$ , are radiatively inert. The most abundant strong absorber is  $\text{H}_2\text{O}$ . While the  $\text{CO}_2$  concentration is to leading order uniform, the  $\text{H}_2\text{O}$  concentration is highly variable. To leading order, the  $\text{H}_2\text{O}$  concentration is controlled by temperature, through the Clausius–Clapeyron relation, but deviations from this are possible when the air is sub-saturated.

The average lifetime of a water molecule in the atmosphere is on the order of two weeks. This is very short compared to that of  $\text{CO}_2$ , which decays on two exponential time scales, one on the order of decades that is controlled by upper ocean uptake, the other on the order of millennia that is controlled by carbon cycling into the deep ocean. Carbon is also taken out of the atmosphere by weathering of the rocky land surface, but that process operates

<u>Molecule</u>	<u>Arrangement</u>	<u>Permanent Dipole Moment</u>
N <sub>2</sub>		No
O <sub>2</sub>		No
CO		Yes
CO <sub>2</sub>		No
N <sub>2</sub> O		Yes
H <sub>2</sub> O		Yes
O <sub>3</sub>		Yes
CH <sub>4</sub>		No

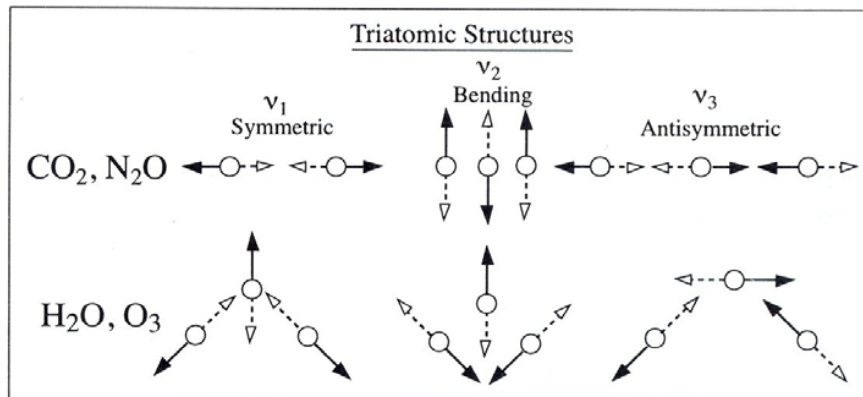
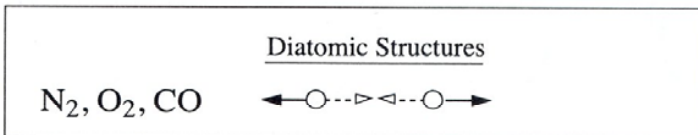


Figure 1: Illustration of the structure of molecules and their modes of interaction with radiation by dipole moments, vibrational modes, and rotational modes

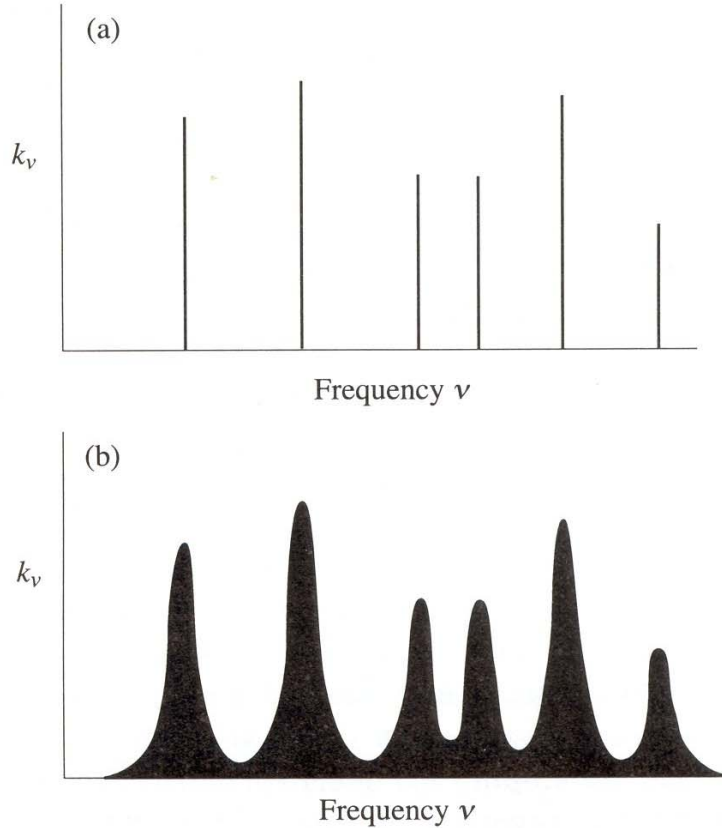


Figure 2: Illustration of the broadening of spectral lines (a) to continuous absorption bands (b) by Doppler and pressure effects

on geological time scales. The long lifetime of  $\text{CO}_2$  in the atmosphere allows it to be well mixed. The  $\text{CO}_2$  concentration is currently about 0.04 %.

Long-lived gases tend to be well-mixed. Gases with a shorter lifetime can be more highly concentrated near sources (Fig. 4). Ozone, for example, is enhanced in the stratosphere, where it is created by photodissociation. Carbon monoxide is mostly present in the troposphere, where its anthropogenic sources are. Water is also mostly present in the troposphere, but it is highly variable in space and time. It also undergoes phase transitions.

Water vapor is the most interesting of the trace gases, owing to its absorption properties and its highly variable concentration. The  $\text{H}_2\text{O}$  concentration is a function of the climate state, and the climate state is dependent on the distribution of  $\text{H}_2\text{O}$ . In fact, there are strong feedbacks, which have often been ignored. As alluded to in the preamble, recognizing the interplay of radiation and dynamics, which may lead to a self-regulation of the climate system, may allow progress in long-standing problems in atmospheric and climate science.

The  $\text{CO}_2$  concentration has been rising due to anthropogenic emissions (Fig. 5). Direct measurements of  $\text{CO}_2$  concentrations at Mauna Loa show an accelerating increase since 1959 to currently about 400ppm. Ice cores that contain air bubbles from pre-industrial times tell

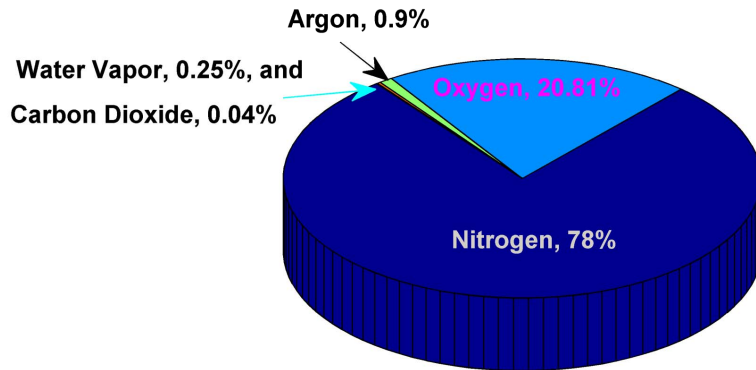


Figure 3: Atmospheric constituents by mass, showing that the strongly absorbing gases occur in small concentrations

us that the background, interglacial  $\text{CO}_2$  concentration was about 280ppm. Superposed on this increase is a small seasonal cycle due to enhanced uptake of  $\text{CO}_2$  by the deciduous forest in boreal summer that has no equivalent in the austral summer, because there is less land mass in the southern hemisphere.

Fig. 6 shows the Planck functions for a solar temperature and for typical terrestrial temperatures. The sun mostly emits in the visible range of the spectrum, while terrestrial emissions are mostly in the infrared range. This separation between solar and terrestrial radiation simplifies our conceptual understanding, as illustrated later in the lecture.

Also shown in this figure are the attenuations by absorption and scattering. The UV light is mostly absorbed by ozone in the stratosphere. The atmosphere is mostly transparent to visible light: some 90 % passes through. The absorption bands of  $\text{H}_2\text{O}$  and  $\text{CO}_2$  tend to be in the infrared range. The transmission of terrestrial infrared radiation is small, except for a window around  $10 \mu\text{m}$ .

It should be noted that this figure is a cartoonish illustration of the immensely complex absorption properties of the atmospheric gases. Very accurate numerical codes exist that compute the absorption line by line. These are computationally too expensive to be put in a climate model, so one resorts to simplified band models that are tested against the line-by-line computations. These radiation codes are quite reliable; most of the uncertainty in radiative transfer comes from scattering, especially multiple scattering in clouds.

## 4 Radiative equilibrium

Radiative equilibrium is the state of the atmosphere in which in every sample of gas, the amount of absorbed radiation equals the amount of emitted radiation. In calculating the radiative equilibrium, we make the assumption that the atmosphere is motionless, that no advective heat transfer occurs. When a system is out of radiative equilibrium, radiative transfer processes will relax it back toward the equilibrium. This relaxation will in general

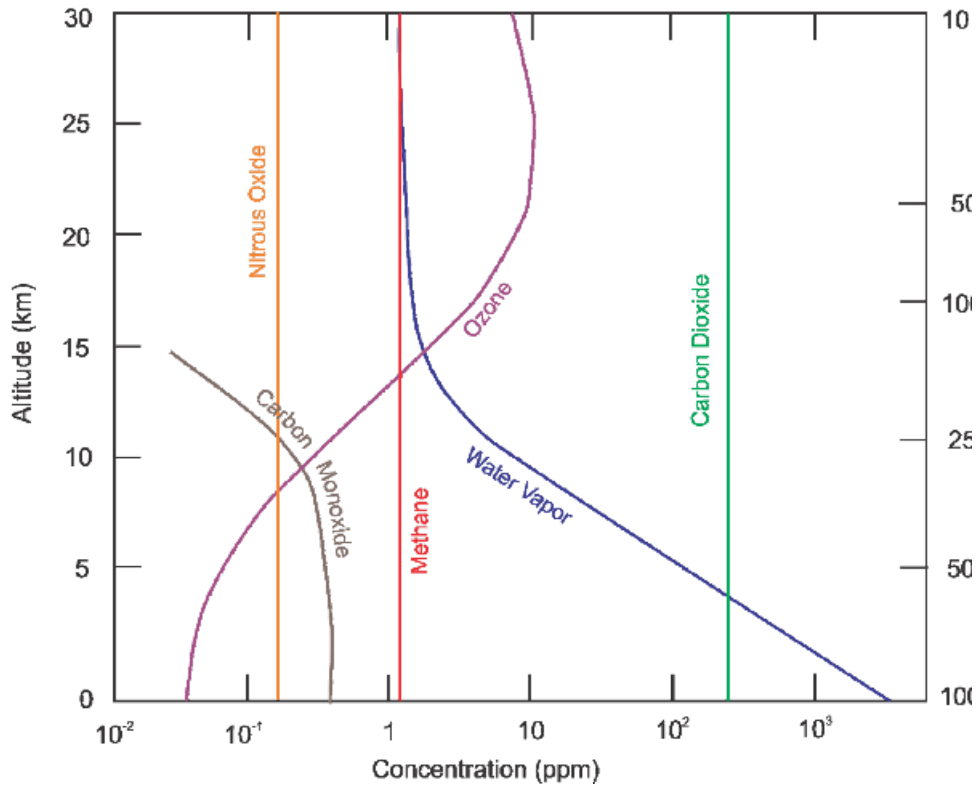


Image credit: Kerry Emanuel

Figure 4: Vertical structure of concentrations of atmospheric constituents as inferred from climatology and specified for the band model of radiative equilibrium

be more complicated than simple Newtonian relaxation, due to the nonlocality of radiative transfer. For example, a temperature perturbation at one level will have a radiative effect on the temperature of other levels, which would not occur in a Newtonian cooling model.

As the simplest model of radiative equilibrium, consider a one-layer atmosphere (Fig. 7). The model consists of a single layer of atmosphere at temperature  $T_a$  and a surface at temperature  $T_s$ . The atmosphere is assumed to be completely transparent to solar radiation and completely opaque to terrestrial radiation. The solar flux received is then  $S_0(1-a_p)/4 = \sigma T_e^4$ , which should be taken as a definition of the emission temperature. The surface emits black-body radiation according to the Stefan–Boltzmann law,  $\sigma T_s^4$ . The atmosphere similarly emits as a black body,  $\sigma T_a^4$ , both upward and downward. Remember that the Stefan–Boltzmann law gives the radiative flux integrated over a hemisphere.

In radiative equilibrium, the incoming solar flux must balance the outgoing terrestrial flux, which trivially gives the atmospheric temperature  $T_a = T_e$ . At the surface, the balance is between the sum of the incoming flux from the sun and the atmosphere and the emitted black-body radiation:

$$\sigma T_e^4 + \sigma T_a^4 = \sigma T_s^4.$$

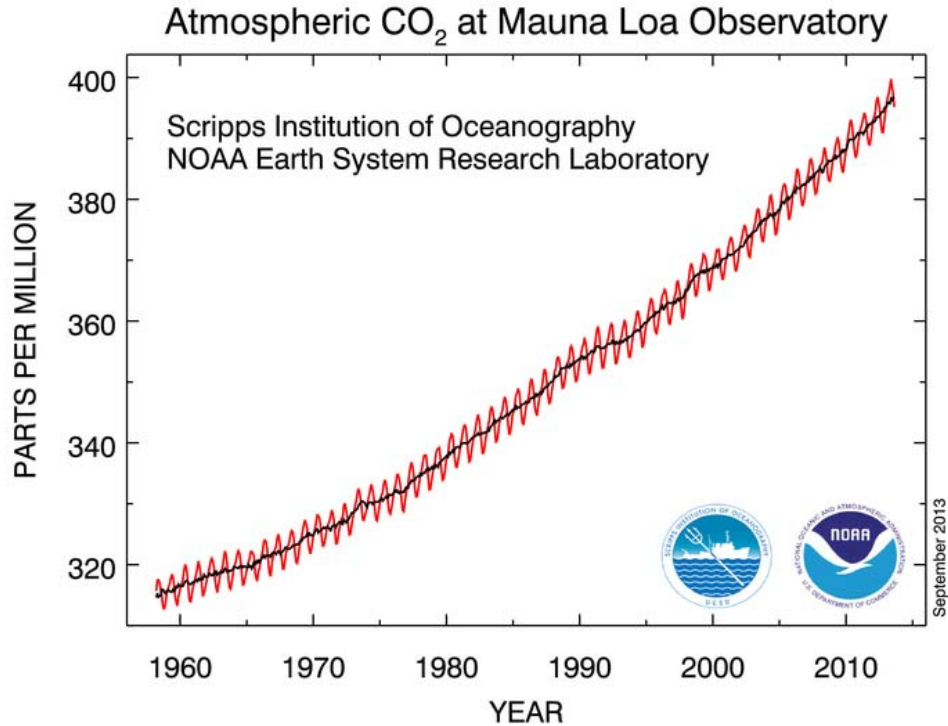


Figure 5: CO<sub>2</sub> concentration as measured at Mauna Loa Observatory (red) and running mean (black)

With  $T_a = T_e$ , this gives the surface temperature  $T_s = 2^{1/4}T_e = 303\text{K}$ . The surface temperature is higher than the emission temperature. This is the greenhouse effect: the surface receives radiation from both the sun and the atmosphere. The resulting surface temperature is higher than the observed mean surface temperature on Earth, but not too far off, considering how simple a model this is. More importantly, the simple model captures the essence of the problem, allowing conceptual understanding. It can be considered the simplest climate model.

Note that in this one-layer model, the surface receives the same amount of flux from the sun as it does from the atmosphere. In reality, it turns out that the surface receives roughly twice as much radiation from the atmosphere as it does from the sun. This provides a puzzle, as our model surface receives less radiation from the atmosphere than the true earth surface, and yet our model surface temperature still comes out too hot. We will address this issue later in the lectures.

The model can be modified by adding extra layers. The case of two atmospheric layers that are completely opaque is depicted in Fig. 8. The extra layer increases the surface temperature to  $T_s = 3^{1/4}T_e$ . Generalizing to  $n$  layers results in a surface temperature  $T_s = (n + 1)^{1/4}T_e$ . Every extra layer increases the surface temperature. It should be noted that the real atmosphere is not the continuous limit of this layered model, because as layers become thinner, they become less opaque, so in a continuous limit, the emissivity of the layers must go to zero.



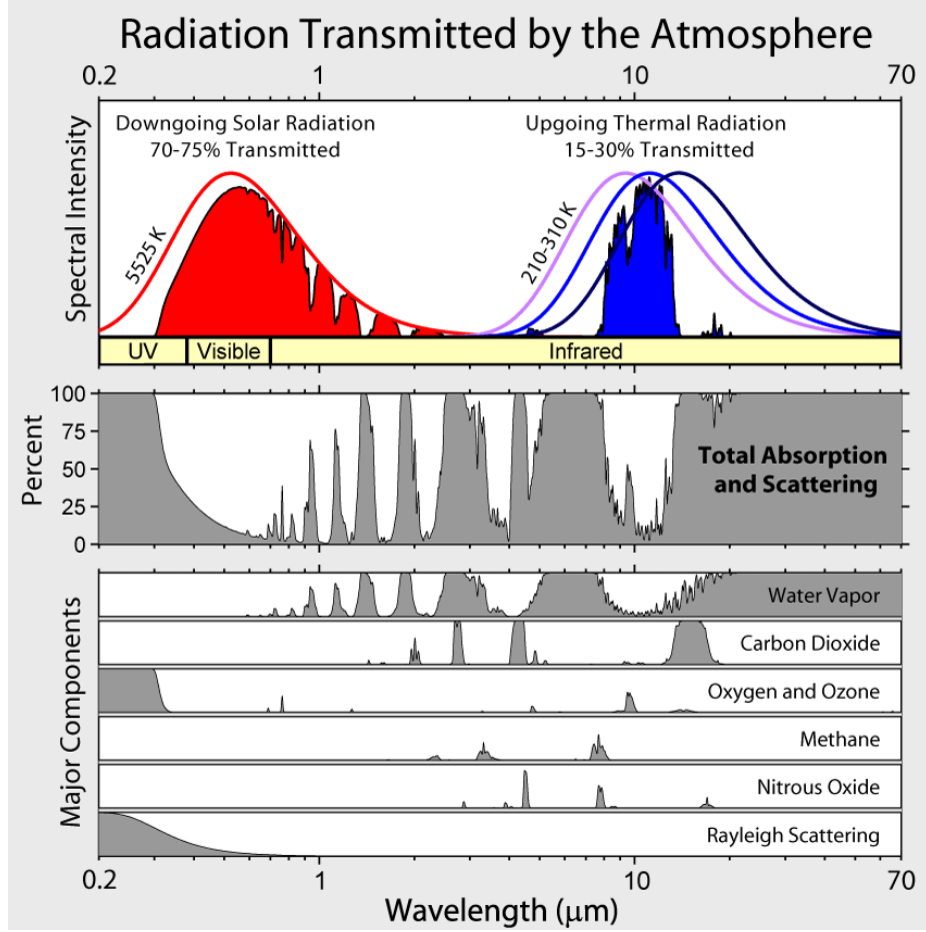


Figure 6: Planck curves of solar and terrestrial radiation as well as scattering and absorption by the atmospheric constituents

Instead of the continuous limit, let us consider two wrinkles on the two-layer model to illustrate two important aspects of radiative equilibrium.

First, consider a thin layer of gas at temperature  $T_t$  just above the layer 2. Let its emissivity be  $\varepsilon_t$  and let that emissivity tend to zero. This layer then does not affect any of the other layers, so we can consider the radiative balance of this layer independently. From

$$\varepsilon_t \sigma T_2^4 = 2\varepsilon_t \sigma T_t^4$$

we find  $T_t = 2^{-1/4} T_2 = 2^{-1/4} T_e$ . This illustrates that the radiative equilibrium temperature of an atmospheric layer can be lower than the emission temperature  $T_e$ .

Second, consider adding such a layer just above the surface. Let its temperature be  $T_a$  and its emissivity  $\varepsilon_a$  tend to zero. The balance of this layer is then

$$\varepsilon_a \sigma T_s^4 + \varepsilon_a \sigma T_1^4 = 2\varepsilon_a \sigma T_a^4,$$

so  $T_a^4 = (T_s^4 + T_1^4)/2$ . This layer therefore does not have the same temperature as the surface. This result is independent of  $\varepsilon_a$ , so long as it is sufficiently small, and illustrates that a

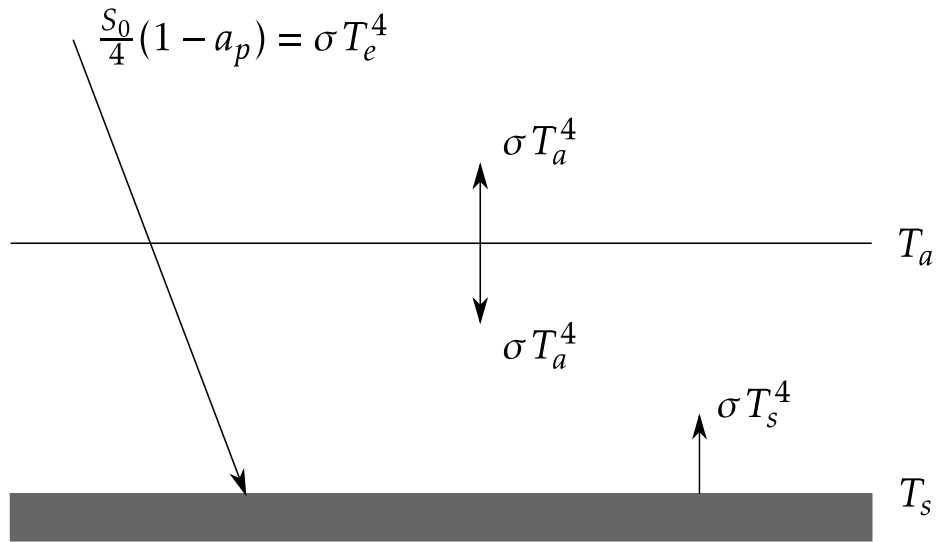


Figure 7: Radiative fluxes in a model with an atmosphere that consists of a single layer that is completely transparent to solar radiation and completely opaque to terrestrial radiation

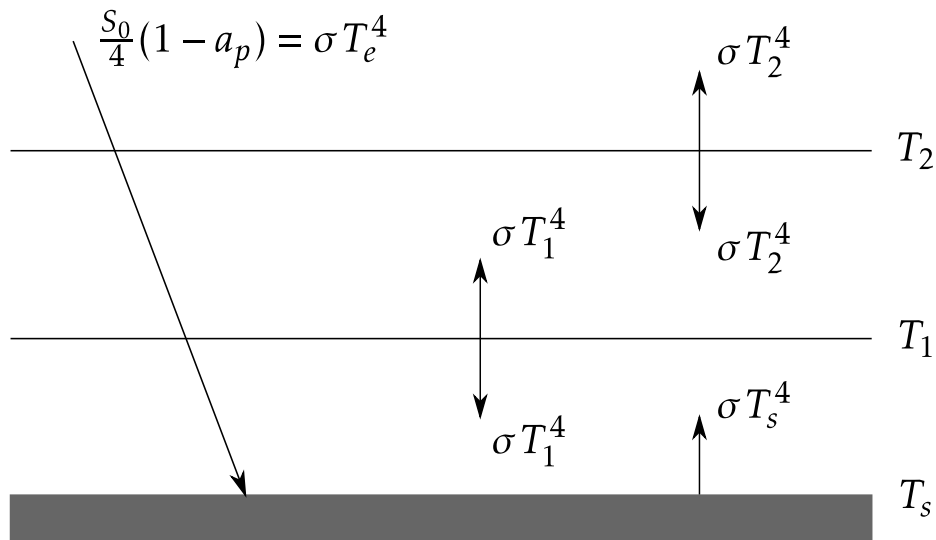


Figure 8: Radiative fluxes in a model with an atmosphere that consists of two layers that are completely transparent to solar radiation and completely opaque to terrestrial radiation

discontinuous emissivity entails a discontinuity in temperature. In radiative equilibrium, the surface atmospheric temperature is generally different from the temperature of the surface. Radiation drives the system into thermodynamic disequilibrium, which in reality is counteracted by heat diffusion or fluid motion. But this effect can be experienced in a desert during the day, where the sand is typically much hotter than the air at 2 m above ground.

We now discuss the radiative equilibrium in a much more realistic model. This model consists of a large number of layers for which the emissivity is calculated for wavelength bands from the concentrations of the atmospheric constituents. The concentrations are specified, with the exception of water vapor, for which a temperature-dependent relative humidity is prescribed.

There are a number of interesting features of the resulting radiative equilibrium (Fig. 9). There is a discontinuity of temperature at the surface—the surface temperature is different from the temperature of the atmosphere at the surface. Furthermore, the troposphere is very cold, much colder than is observed. Part of the reason is that there is a positive water vapor feedback: as the atmosphere gets cold, the amount of water vapor decreases for a fixed relative humidity, so the greenhouse effect is diminished. But the ultimate reason why the tropospheric temperature is unrealistic is that we disregard convection, which will be discussed in the next part.

The obtained temperature of the stratosphere is roughly consistent with the observed stratospheric temperatures, which indicates that the stratosphere is in a state not very far away from radiative equilibrium. There is an increase in temperature in the stratosphere, which is due to the absorption of UV light by ozone in the upper stratosphere. If we increased the  $\text{CO}_2$  concentration, the stratosphere would cool. This counterintuitive effect can be explained by realizing that the stratospheric gas becomes a better emitter for a higher  $\text{CO}_2$  concentration, so its temperature must be reduced to have the emitted radiation match the absorbed solar radiation. To understand this effect fully, a radiative-equilibrium model with two partially transparent layers would presumably be useful. In such a model, an increase in  $\text{CO}_2$  concentration could be modeled by an increase in emissivity of the upper layer.

## 5 Convection

The reason for the unrealistically cold troposphere with a very steep temperature gradient in the lower troposphere is the omission of convection. The time scale of the radiative relaxation to equilibrium is on the order of tens of days. But convection operates on a few hours, much faster than radiation. Convection drives the system toward a state of convective neutrality. The separation of time scales means that where convection occurs, the system is very near this state of convective neutrality. The state resulting from the combination of radiation and convection is called the radiative–convective equilibrium. It should be noted, however, that such an equilibrium is a state of statistical equilibrium, which is highly turbulent in nature. We begin by calculating what this equilibrium state is if phase changes of water are neglected. Later, we will add such moist processes.

Historically, radiative–convective equilibrium was first analyzed in the 1960s. Since then, climate scientists have rapidly moved on to three-dimensional modeling, building

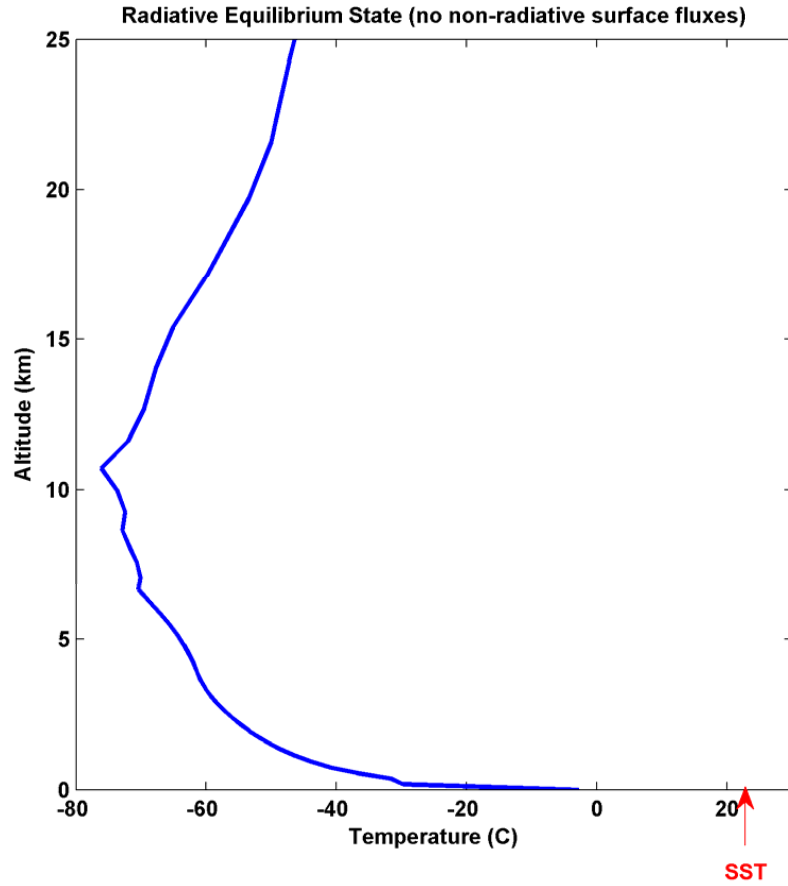


Figure 9: Radiative equilibrium of band model with concentrations of greenhouse gases specified and relative humidity prescribed

ever more complex models. Moving on so quickly might have been a historical mistake as much fundamental insight can still be gained from this very idealized radiative–convective equilibrium.

Let’s consider the force balance of a cube of fluid of density  $\rho_b$  under the effect of gravity  $g$  (Fig. 10). The forces in the vertical on the box are then the sum of the box’s weight and the pressure forces on the horizontal faces:

$$-\rho_b g \Delta x \Delta y \Delta z + p(z) \Delta x \Delta y - p(z + \Delta z) \Delta x \Delta y = 0.$$

Dividing through by the volume of the box and taking the limit  $\Delta z \rightarrow 0$  gives

$$\frac{\partial p}{\partial z} = -\rho_b g,$$

which is called the hydrostatic balance.

Allowing a vertical acceleration modifies the balance to

$$-\rho_b g \Delta x \Delta y \Delta z + p(z) \Delta x \Delta y - p(z + \Delta z) \Delta x \Delta y = \rho_b \frac{dw}{dt} \Delta x \Delta y \Delta z,$$

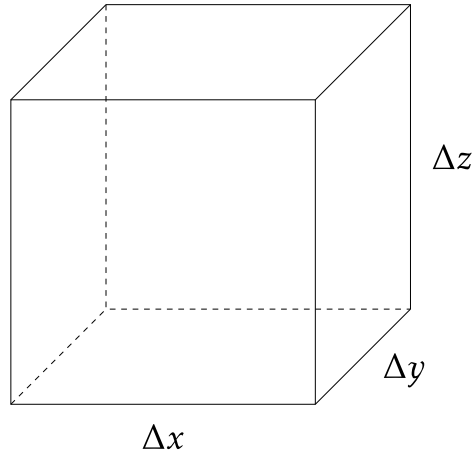


Figure 10: Box of air for which the vertical force balance is considered

where  $w$  is the vertical velocity. The vertical acceleration is then

$$\frac{dw}{dt} = -\frac{1}{\rho_b} \frac{\partial p}{\partial z} - g.$$

If the environment is in hydrostatic balance and the pressure is the same in the box and the environment, then

$$\frac{dw}{dt} = g \left( \frac{\rho_e}{\rho_b} - 1 \right) = g \frac{\rho_e - \rho_b}{\rho_b} = B,$$

which is called buoyancy. The environmental density was here denoted by  $\rho_e$ .

Next we will assess whether a vertical profile of temperature is stable by displacing a parcel and calculating whether it gets pushed back to its original position, in which case we call it stable, or whether it gets accelerated away from its original position, in which case we call it unstable. This is not a trivial calculation, because the parcel's density changes as it is displaced, because its pressure changes. Throughout, we will neglect viscous stresses.