Lattice-based Commit-Transferrable Signatures and Applications to Anonymous Credentials

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Abstract. Anonymous Credentials are an important tool to protect user's privacy for proving possession of certain credentials. Although various efficient constructions have been proposed based on pre-quantum assumptions, there have been limited accomplishments in the post-quantum and especially practical settings. This research aims to derive new methods that enhance the current state of the art.

To achieve this, we make the following contributions. By distilling prior design insights, we propose a new primitive to instantiate *signature with protocols*, called commit-transferrable signature (CTS). When combined with a multi-theorem straight-line extractable non-interactive zero-knowledge proof of knowledge (NIZKPoK), CTS gives a modular approach to construct anonymous credentials. We then show efficient instantiations of CTS and the required NIZKPoK from lattices, which are believed to be post-quantum hard. Finally, we propose concrete parameters for the CTS, NIZKPoK, and the overall Anonymous Credentials, based on Module-SIS and Ring-LWE. This would serve as an important guidance for future deployment in practice.

Keywords: Anonymous Credentials; Commit-Transferrable Signature; Lattice-Based Cryptography; Post-Quantum Security

1 Introduction

In an Anonymous Credential system [15, 17, 37, 38], users interact with organizations, obtain digital credentials from them, and prove possession of these credentials anonymously and unlinkably. Anonymous Credentials are increasingly important in practice: they have been implemented by industry leaders such as IBM[6] and Microsoft[7], have found their way into industrial standards

[6] https://idemix.wordpress.com/
[7] https://www.microsoft.com/en-us/research/project/u-prove/

(such as the TCG standard), and underlined the policies that both the United States government[8] and the EU government[9] have towards balancing privacy and legitimate identification and authentication needs.

The advent of quantum computing threatens the security of all the prior anonymous credential constructions whose efficiency was suitable for use in practice, since all of them require either the RSA or the discrete logarithm assumption to hold (in fact, they need even stronger assumptions). The goal of this paper is to give more efficient Anonymous Credentials based on standard lattice assumptions, as they provide a plausible foundation against quantum attacks. Moreover, we propose a modular approach so that each building block might be improved individually for future work.

Anonymous Credentials from general-purpose crypto tools. The well-known approach [16, 39] to giving Anonymous Credentials is to provide: (1) a commitment scheme for committing to $\boldsymbol{x}$ representing a user's private input, e.g., her secret key; (2) a digital signature scheme for signing $\boldsymbol{x}$ (under the commitment); (3) an efficient and secure two-party protocol between a user and a signer to prove that the user's (private) input $\boldsymbol{x}$ is consistent with the commitment and then the protocol generates a signature of $\boldsymbol{x}$; and finally (4) a suite of efficient zero-knowledge proof systems that allow the user to prove (i) knowledge of the commitment opening and (ii) knowledge of a signature from the signer on the commitment opening.

Even though each of these general building blocks can be achieved under post-quantum assumptions, however, realizing them efficiently is still a significant and on-going research direction. Therefore, it is interesting and important to determine a new approach for more efficient constructions.

Relevant Research. We notice that the research of Anonymous Credentials has deep connections with the following two cryptographic objects: (1) Group Signatures and (2) Blind Signatures. Conceptually for all these objects, there are three roles – User, CA (certificate authority), Verifier, in the system, yet they post different privacy requirements. Particularly, Group Signatures requires privacy for User against Verifier, i.e., Verifier only knows the signature is output by some one in the group, but does not know the concreted signer. Yet group signature does not require privacy for User when getting a credential from CA. On the other hand, Blind Signatures requires privacy for User against the CA, i.e., CA signs a hidden message as the credential, but the verification step reveals the message to Verifier. Finally, Anonymous Credentials requires privacy on both sides – the User's private input (e.g., her ID or messages) is hidden from both CA and Verifier.

Our Approach. Our goal is to achieve an efficient lattice-based anonymous credential, avoiding any heavy cryptographic machinery such as general secure two-party computation and general zero-knowledge proofs in the above paradig-

[8] https://www.nist.gov/news-events/events/2011/12/meeting-privacy-enhancing-cryptography

[9] https://abc4trust.eu/

m. To achieve this goal, we first distill prior design insights, such as *signature with protocols* in [16], from prior work that realized the above general diagram, and then propose a primitive named as commit-transferable signature (CTS), with special properties that are friendly for efficient instantiating from lattices.

Briefly, the high level idea of the prior framework of *signature with protocols* is the following – for an efficient signature scheme, we would identify a commitment scheme and an efficient zero-knowledge proof of knowledge system, such that they can be elegantly combined, yielding a protocol for signing a committed value, and a zero-knowledge proof of knowledge of the signature. This work takes another perspective – by blending a signature scheme with a proper commitment scheme as one object, we are able to see better lattice insights, leading to more efficient lattice-based instantiation and thus more practical anonymous credentials.

Particularly, CTS encompasses both a non-interactive commitment algorithm Commit and a signature scheme $(\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Verify})$, with the following properties. Our first key property is that, in a CTS scheme, it is possible to compute a signature directly on the commitment value comm, where $\mathsf{comm} \leftarrow \mathsf{Commit}(\boldsymbol{x})$. This way, instead of designing a secure two-party protocol as described in (3) above, it is sufficient to simply require that the user performs a zero-knowledge proof of knowledge of the opening of comm. Once the signer verifies this proof, it can compute the signature σ on input comm. Moreover, it is possible to verify a signature σ through inputting the commitment comm rather than the value $\boldsymbol{x}$; therefore, to prove possession of a signature on the opening of comm, it is sufficient to reveal σ and prove knowledge of the opening of comm.

Our second key property is to require that, from the signature σ of the commitment $\mathsf{comm} \leftarrow \mathsf{Commit}(\boldsymbol{x})$, the user will be able to compute a new signature σ' for a new commitment $\mathsf{comm}' \leftarrow \mathsf{Commit}(\boldsymbol{x})$. That requires two additional algorithms: $\mathsf{Randomize}$ to randomize comm into comm', and $\mathsf{Transfer}$ to transform the signature σ into a new signature σ' with respect to the new commitment comm'. It is important that the resulting pair $(\mathsf{comm}', \sigma')$ is unlinkable to the original pair (comm, σ). Thus, in order to prove that the contents of comm' were signed, it is sufficient to just reveal σ'. More technical details and the formulation on CTS are deferred to Section 1.3.

From CTS to Anonymous Credentials and More. Using a CTS scheme and an appropriate (non-interactive) zero-knowledge proof of knowledge ($\mathsf{NIZKPoK}$), we can construct Anonymous Credentials and the other related object, namely, Group Signatures and Blind Signatures. We first elaborate on the case of Anonymous Credentials.

Suppose User whose secret key is $\boldsymbol{x}$ needs to obtain a credential from some CA. First, it forms a commitment $\mathsf{comm} \leftarrow \mathsf{Commit}(\boldsymbol{x})$ and proves to CA that he knows the opening to the commitment by using a $\mathsf{NIZKPoK}$. Next, CA runs the Sign algorithm on input comm, obtains the signature σ, and returns it to the user. After the signature is obtained, suppose that the user wants to prove possession of this credential, he uses the $\mathsf{Randomize}(\cdot)$ and $\mathsf{Transfer}(\cdot)$ algorithms to obtain a new commitment comm' to $\boldsymbol{x}$ and the issuer's signature σ' on comm'. Now, the user sends the resulting $(\mathsf{comm}', \sigma')$ to the verifier as a credential. As $(\mathsf{comm}', \sigma')$

is unlinkable to the original (comm, σ), we achieve the important property of unlinkability. We can further prove that it is computationally infeasible to forge a σ^* with respect to $\mathsf{comm}^* = \mathsf{Commit}(\boldsymbol{x}^*)$ that a signature of commitment of $\boldsymbol{x}^*$ has never been issued.

By instantiating a half-fledged CTS (which might allow more efficient instantiations), we can achieve *Group Signatures* and *Blind Signatures*. Particularly, for Group Signatures, User only needs to send $\boldsymbol{x}$ in the clear at the first stage, i.e., viewing $\boldsymbol{x}$ as the trivial commitment, as the privacy of User is not required in this phase. After obtaining σ, User runs $\mathsf{Randomize}(\cdot)$ and $\mathsf{Transfer}(\cdot)$ to produce a hiding comm' and a corresponding signature σ'.[10] On the other hand, for Blind Signatures, User follows the first half of the Anonymous Credential construction, yet later modify the randomized algorithm as: comm' just reveals $\boldsymbol{x}$. Again this is not an issue as the privacy of Blind Signatures is not required against Verifier. In fact, the constructions of [21,22] can be viewed as realizing the half-fledged of our notion of CTS. Besides, as CTS is essentially a non-interactive version of "signature with protocols", this notion may be useful for other privacy-preserving applications related to "signature with protocols".

For the NIZKPoK system, the recent work [21] identified a necessary property, i.e., the system needs to be multi-theorem straight-line extractable, as otherwise, the security proof of the whole system (Blind Signatures or Anonymous Credentials) would incur an exponential loss.[11] It is important to determine efficient multi-theorem straight-line extractable proof systems from lattices for the particular commitment relation in the CTS construction.

Focus of This Work. Our main goal is to construct an efficient Anonymous Credential based on some standard lattice assumptions. As discussed before, this can be achieved by determining the following questions.

> (**Main Questions**) Can we design an efficient full-fledged CTS from standard lattice assumptions? Can we construct an efficient straight-line extractable NIZK for the commitment relation of the CTS?

1.1 Our Contributions

To address the main questions, we make the following contributions.

- We formalize the notion of CTS and its security requirements. Together with a straight-line extractable NIZK, CTS gives a simple way to construct Anonymous Credentials and other useful privacy preserving tools, such as Group Signatures and Blind Signatures.
  Moreover, the CTS-based Anonymous Credentials can be extended to the attribute-based setting, by further embedding attributes to the committed message and designing proper NIZK to prove the message relation satisfying a certain policy.

[10] To be able to open the group signature scheme, we still need to add a verifiable encryption to the signature.

[11] We also notice that an exception for this requirement in the current work [33]. We will elaborate this technical difference in the following content.

– We show how to instantiate CTS from some well-studied lattices assumptions, i.e., the module learning with errors (M-LWE) and module short integer solutions (M-SIS).
– We construct an efficient lattice-based straight-line extractable NIZKPoK for our CTS commitment relation in the classical random oracle model. To achieve this, we employ the encrypt-and-prove approach in [2].
– We determine parameters for all the required components for evaluating concrete efficiency.

Below we present our concrete parameters and findings, and defer the detailed analysis on the asymptotical size parameters in Sections C and D.6.

In the following tables, we show concrete parameters of Anonymous Credentials of various security levels. Particularly, our simple yet selectively secure CTS (in Section 4) can derive selectively secure Anonymous Credentials, whose concrete parameters are presented in Table 1. By scaling up the security parameter and applying the complexity leveraging argument, we can derive adaptively secure Anonymous Credentials with concrete parameters[12] in Table 2. Alternatively, we also directly construct an adaptively secure CTS as in Section D, implying asymptotically efficient adaptively secure Anonymous Credentials with concrete parameters in Table 3. For the currently used security levels (say 128 bit-security) however, the scheme via the complexity leveraging (as Table 2) is much more efficient. We leave it as an interesting open problem to optimize such directly adaptive CTS and the derived Anonymous Credential.

1.2 Comparison with Recent Progress

Here we present a comparison between our contributions and relevant recent works, for a clear identification of our advancements over the state of the art.

Anonymous Credentials. Several earlier works [18, 26, 36, 52] have made attempts to construct lattice-based anonymous credentials, yet their approaches have various drawbacks and thus unsatisfactory. Particularly, the work [18] only achieved a weaker notion called anonymous attribute token system, where user anonymity is protected only against verifiers, but not the CA. The schemes [18,36] are not concretely efficient, and the schemes [26,52] do not achieve the important property – unlinkability. Thus, all these approaches are not suitable for scenarios that require the full-fledged anonymous credentials.

Concurrent Works. Very recently, two independent and concurrent works [12, 33] have constructed efficient lattice-based anonymous credentials. Here

[12] Here we consider 128 bits for the User ID length, and scale up the selectively secure scheme to roughly 256-bit security. This implies an adaptively secure scheme of 128 bit-security after applying the complexity leveraging argument. Particularly, our technical route is that: first construct a CTS with 256-bit selective security and 128-bit message space, then achieve 128-bit adaptive security through complexity leveraging the message space. Based on this, we just need to combine a multi-theorem straight-line extractable NIZKPoK with 128-bit security to obtain the final anonymous credentials system with 128-bit security.

	PP	PK	SK	Pseudonym	Signature	Credential	Bit-security
Params 1	25.49MB	276.5KB	10.5KB	1.25 MB	117.85KB	192.85KB	128

Table 1. Our selective Anonymous Credentials from Ring-LWE and Modulus-SIS. Here, we denote PP as public parameter, PK as public key, SK as secret key. All values in this table are computed from the example parameters of Params 1 in the Tables 11 and 10.

	PP	PK	SK	Pseudonym	Signature	Credential	Bit-security
Params 2	24.66MB	440KB	15KB	2.01MB	236.56KB	372.56KB	128

Table 2. Our adaptively secure Anonymous Credentials by applying the complexity leveraging argument to the selectively secure scheme. All values in this table are computed from the example parameters of Params 2 in the Tables 11 and 10.

	PP	PK	SK	Pseudonym	Signature	Credential	Bit-security
Params 3	321.4GB	205.4MB	1.17MB	530.96MB	16.32MB	24.77MB	128

Table 3. Our adaptive secure Anonymous Credentials from a direct construction of adaptively secure CTS. All values in this table are computed from the example parameters of the Table 16.

we undertake a comparative analysis of the findings, highlighting the unique merits of our approach despite the existence of these concurrent works.

First, the work [33] instantiates the necessary building blocks following the approach of "signature with protocols", and then derives an anonymous credential system based on the M-LWE and M-SIS assumptions. In efficiency, the credential size of their protocol is about 639 KB for 128 bit-security.

The other work [12] takes a different approach to construct non-interactive lattice-based solutions in the random oracle model. Their scheme exhibits a highly competitive level of concrete efficiency. E.g., the size of credentials is about 122 - 133 KB for 128 bit-security, or 26 - 29 KB under another new assumption. However, there are two important caveats to consider. First, their efficient scheme only achieves a very basic anonymous credential system without incorporating pseudonyms, which can be desirable for enabling some useful features, e.g., selective tracking of holders [14,39]. Second, security of all their schemes [12] depends on some new variations of ideal lattice problems.

Even though these two issues can be handled in theory, it remains challenging to derive a system with comparable efficiency under their paradigm. In particular, resolving the first issue would require additional commitments and proofs on top of their basic schemes, e.g., proving equality between committed and signed values, yet the concrete blowup needs to be re-evaluated. The second issue presents a tougher challenge, as adapting their approach to rely on more well-studied assumptions (e.g., RLWE) appears to necessitate proving knowledge of pre-images for random oracles. Unfortunately, there is currently no efficient lattice-based proof technique available to fulfill this need.

Considering the insights gained from the current post-quantum standardization process [9,19,44,51], a cautious and conservative approach would always be necessary and valuable. By building schemes under more well-studied hardness

foundations, we can mitigate the risks associated with unforeseen weaknesses in the new assumptions, ensuring better confidences in the overall security.

Besides, there are obvious differences on the unforgeability model among this paper and [12, 33]. Particularly, for the unforgeability, [33] directly proves the well-formedness of the commitment (or pseudonym) through using NIZK systems, rather than NIZKPoK. But, [12] and ours use NIZKPoK systems to prove the well-formedness of the commitment. In fact, both choices are reasonable, relying on different proof strategies or underlying assumptions.

Overall, when just using NIZK proof systems to prove the well-formedness, [33] can directly use Fiat-Shamir heuristic to obtain non-interactive protocol. However, in the case of using NIZKPoK systems to prove the well-formedness of the commitment, such as [12] and ours, multi-theorem straight-line extractability will be unavoidable for non-interactive settings.

Summary. Our work, along with the two concurrent works, possesses unique merits in different aspects. In summary, both our work and the work [33] achieve anonymous credential systems with pseudonyms under the more extensively studied assumptions (i.e., M-SIS and M-LWE). However, our work offers advantages over [33] in terms of smaller credential size.

When comparing our work to [12], we observe that their concrete parameters are smaller, yet their efficient instantiation is for a basic anonymous credential system without pseudonyms, and moreover their security relies on new and less-studied assumptions. Thus, we believe that these two works have incomparable advantages and both deserve attentions. Below we present Table 4 for comparisons between our work and these concurrent works.

	PP	PK	SK	Pseudonym	Signature	Credential	Assumption	Security
[33]	0.27MB	9.56MB	10.59MB	–	317KB	724KB	M-LWE, M-SIS	128
[12]	–	–	–	$\perp$	–	122KB	ISIS_f	128
Ours	24.66MB	440KB	15KB	2.01MB	236.56KB	372.56KB	M-LWE, M-SIS	128

Table 4. Comparison of efficiency estimates of Anonymous Credentials Systems between ours, [33] (its Table H.4) and [12]. In [12,33], some of concrete values are not explicitly listed, so we just use the symbol "-" for these columns. Besides, as the current construction of [12] does not support pseudonym application immediately, we just use the symbol $\perp$ to represent its size. Moreover, here we focus on the non-interactive version of the underlying assumptions, so we do not list the efficiency of [12] based on the interactive ISIS_f assumption.

Straight-line Extractable Lattice-based NIZKPoK. Next we present relevant works of straight-line extraction for lattice proofs. Generally, there are two main approaches to achieve this notion for lattice proofs:

1. The technique of extractable linear homomorphic commitments, e.g., [12,21].
2. The instantiation of the well-known encrypt-and-prove paradigm from lattices, e.g., [2, 10].

For practical parameters, recent works [2, 10] have focused on optimizing proof sizes in the classical random oracle model (ROM), and currently, the second approach following encrypt-and-prove paradigm achieves much better proof sizes.

This work follows the second, i.e., encrypt-and-prove approach. Particularly, we first choose a variant of Regev encryption and set the related underlying ring together with other parameters, such that all the necessary random vectors can be encrypted with about 128 bit security. Secondly, we use the LNP approach in [41] to prove the encrypted vectors satisfying certain relations together with the well-formedness of Regev encryption. More details on the encrypted vectors and the concreted relations are deferred to the following contents in Section 1.3.

It is worth noting that the first approach has an advantage in terms of extendability of security analysis to the quantum random oracle model (QROM) [21], though a significant efficiency overhead of add-ons is required under current techniques. An interesting open problem is whether we can enhance the efficiency and analysis in the QROM settings for either the first or second approach. As further research is needed to explore potential improvements, we believe that all of the aforementioned works, including our own contributions, would provide valuable guidance and serve as stepping stones towards achieving this goal.

1.3 Technical Overview

We present an overview of our techniques of how to efficiently construct the required CTS and straight-line extractable NIZKPoK from lattices. First we informally describe the notion of CTS and then present our technical insights. Next, we present the intuition of our efficient instantiation of mult-theorem straight-line extractable NIZKPoK. These two pieces naturally give Anonymous Credentials as we discussed above.

Commit-transferrable Signatures. Informally, a CTS is a combination of a re-randomizable commitment and a signature, with the following algorithms $(\mathsf{Commit}, \mathsf{Randomize}, \mathsf{Sign}, \mathsf{Transfer}, \mathsf{Verify})$. Intuitively, a user can send $\mathsf{comm} \leftarrow \mathsf{Commit}(x)$ to the signer, who will run the algorithm Sign to produce a signature σ on the commitment comm. Later on, the user can re-randomize the commitment $\mathsf{comm}' \leftarrow \mathsf{Randomize}(\mathsf{comm})$ and then derive a transferred signature $\sigma' \leftarrow \mathsf{Transfer}(\mathsf{comm}, \mathsf{comm}', \sigma)$ with respect to the randomized commitment comm'. For security, the CTS requires input privacy, signature unlinkability, and unforgeability. These properties can be roughly captured by – (1) the signer does not learn any information of x, (2) one cannot learn information about the original commitment-signature pair (comm, σ) from the re-randomized-transferred pair $(\mathsf{comm}', \sigma')$, and (3) an adversary cannot forge a valid σ' with respect to $\mathsf{comm}' \leftarrow \mathsf{Commit}(x^*)$, if any commitment of x^* has not been signed by the signer. Below we explain how to construct such a primitive from lattices.

Warm Up. To achieve selectively secure CTS, intuitively, the first step is to obtain a scheme that allows to sign on commitments, i.e., blending a commitment scheme and signature scheme in an appropriate way. This can be achieved by using ABB signature [1] and GSW commitment [32], as observed by the work [21, 22]. Briefly, the ABB scheme has public key of the form $(\mathbf{A}_0, \mathbf{B}_0, \boldsymbol{u})$ (i.e., two matrices and one vector), and the secret key is the trapdoor, i.e., $\mathbf{T}_{\mathbf{A}_0}$, of the matrix $\mathbf{A}_0$. The signature of m is a short vector $\boldsymbol{s}$, satisfying $[\mathbf{A}_0|\mathbf{B}_0+m\mathbf{G}]\cdot\boldsymbol{s} = \boldsymbol{u}$,

where $\mathbf{G}$ is the gadget matrix of [46]. The GSW commitment uses a public matrix $\mathbf{A}$. To commit to a message m, it outputs $\mathbf{A}\cdot\mathbf{R}+m\mathbf{G}$, where $\mathbf{R}$ is a short random matrix. To open, one just reveals the message and randomness. Next, we describe the idea of [22] to blend these two together.

To sign on commitment $\mathbf{C} = \mathbf{A}\cdot\mathbf{R} + m\mathbf{G}$, the signer first generates the matrix $\mathbf{F} = [\mathbf{A}_0|\mathbf{B}_0 + \mathbf{C}]$, and then generates a short vector $\sigma := \boldsymbol{s}$ that satisfies $\mathbf{F}\cdot\boldsymbol{s} = \boldsymbol{u}$. This can be achieved by using the trapdoor sampling technique of [46]. Suppose the commitment $\mathbf{C}$ does not need to be re-randomized, then the user can simply generate a transferred signature σ' by a ZK proof of knowledge that she holds a short vector with respect to the lattice $\Lambda^{\perp}_{\boldsymbol{u}}(\mathbf{F}) = \{\boldsymbol{z} : \mathbf{F}\cdot\boldsymbol{z} = \boldsymbol{u}$ and $\boldsymbol{z}$ is short$\}$. Intuitively, the zero-knowledge property guarantees that one cannot learn information about the original signature σ from the transferred one, i.e., σ'. The unforgeability follows from SIS using the ABB analysis of [1].

Handle Re-randomized Commitments. The above technique achieves a half of the goal, which means just transferring the original signature $\boldsymbol{s}$ to one (i.e., ZK proof π) with respect to the same commitment $\mathbf{C}$. To achieve the full-fledge of our goal, we need to handle how to transfer signatures with respect to a re-randomized commitment $\mathbf{C}'$.

We observe that GSW commitment can be easily re-randomized, i.e., just setting $\mathbf{C}' = \mathbf{C} + \mathbf{A}\cdot\mathbf{R}'$ for some short random matrix $\mathbf{R}'$. It is easy to show that given $(\mathbf{C}, \mathbf{C}')$, one cannot determine whether the underlying messages are related or not. Given this, we define another matrix for verification with respect to $\mathbf{C}'$ as $\mathbf{F}' = [\mathbf{A}_0|\mathbf{B}_0 + \mathbf{C}'|\mathbf{A}]$. So now our goal is to generate a short vector $\boldsymbol{s}'$ such that $\mathbf{F}'\cdot\mathbf{s}' = \boldsymbol{u}$, and then set σ' to be a ZK proof of knowing a short vector in $\Lambda^{\perp}_{\boldsymbol{u}}(\mathbf{F}')$. By the security of GSW and ZK proof of knowledge, it is easy to argue that one cannot learn information about (comm, σ) from the $(\mathsf{comm}', \sigma')$.

To achieve this, we first express $\mathbf{F}' = [\mathbf{F}|\mathbf{0}] + [\mathbf{0}|\mathbf{A}\cdot\mathbf{R}'|\mathbf{A}] = [\mathbf{A}_0|\mathbf{B}_0 + \mathbf{C}|\mathbf{0}] + [\mathbf{0}|\mathbf{A}\cdot\mathbf{R}'|\mathbf{A}]$. Then through denoting $\boldsymbol{s} = \begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\end{bmatrix}$, we observe, $\mathbf{F}'\cdot\begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ \mathbf{0}\end{bmatrix} = [\mathbf{A}_0|\mathbf{B}_0+\mathbf{C}]\cdot\begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\end{bmatrix} + \mathbf{A}\cdot\mathbf{R}'\cdot\boldsymbol{s}_2 = \boldsymbol{u} + \mathbf{A}\cdot\mathbf{R}'\cdot\boldsymbol{s}_2$, so if we can find a short $\boldsymbol{z} = \begin{bmatrix}\boldsymbol{z}_1\\ \boldsymbol{z}_2\\ \boldsymbol{z}_3\end{bmatrix}$ such that $\mathbf{F}'\cdot\begin{bmatrix}\boldsymbol{z}_1\\ \boldsymbol{z}_2\\ \boldsymbol{z}_3\end{bmatrix} = -\mathbf{A}\cdot\mathbf{R}'\cdot\boldsymbol{s}_2$, then $\boldsymbol{s}'$ can be simply set to $\begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\boldsymbol{z}_1\\ \boldsymbol{z}_2\\ \boldsymbol{z}_3\end{bmatrix}$, fulfilling our goal. By the special structure of $\mathbf{F}'$, we can just set $\boldsymbol{z} = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\\ -\mathbf{R}'\cdot\boldsymbol{s}_2\end{bmatrix}$. Thus the overall $\boldsymbol{s}' = \begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ -\mathbf{R}'\cdot\boldsymbol{s}_2\end{bmatrix}$. It is not hard to verify all the prior steps, implying that $\mathbf{F}'\cdot\boldsymbol{s}' = \boldsymbol{u}$.

Conceptually, the user can massage the randomness $\mathbf{R}'$ (for the re-randomization of the commitment) and the signature $\boldsymbol{s}$ obtained from the signer, to derive a related witness, i.e., $\boldsymbol{s}'$ for the related lattice $\Lambda^{\perp}_{\boldsymbol{u}}(\mathbf{F}')$. Thus, a ZK proof of

knowledge can serve as the transferred signature σ' for the re-randomized $\mathbf{C}'$. We notice that lattice-based ZK proofs for general NP languages exist in the standard model [48] albeit poor efficiency. On the other hand, the particular proof system we need can be instantiated efficiently in the random oracle model [28]. The whole approach can be further optimized by using ideal lattices, i.e., Ring-SIS/LWE, as identified by the work [5, 22, 43].

We notice that we can further improve efficiency of the construction idea above by using multiple BDLOP commitments on related messages [6], similar to the work [21,22]. Thus in our main construction, we will present in the BDLOP form, and our parameters are set with respect to this more efficient version.

Straight-line Extractable Proofs. The next important piece is to construct an efficient multi-theorem straight-line extractable NIZKPoK, proving the well-formedness of the commitment in CTS. Informally, for a multi-theorem straight-line extractable proof, there exists an extractor who can extract multiple witnesses from an adversary who generates multiple valid proofs, and moreover the extraction does not need rewinding. As pointed out by [2, 10, 21], this is an important feature for non-interactive blind signatures and anonymous credentials. For our CTS, specifically we need to prove knowledge of BDLOP commitments, which we recall below. A BDLOP commitment of message m has the structure:

$$\mathsf{Commit}(m; \boldsymbol{r}) = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \boldsymbol{r} + \begin{bmatrix} \mathbf{0} \\ m \end{bmatrix} \begin{matrix} \mod q_1 \\ \mod q_2 \end{matrix} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix},$$

where $\boldsymbol{r}$ is the randomness (ring elements with small coefficients) and m is the message. There are two different moduli with some flexibility in the design. For efficiency optimizations, we can set $q_1 \ll q_2$, and in our application, we additionally require q_2 to be a large prime of a special form, e.g., congruent to 3 or 5 modulo 8.

There are various efficient lattice proofs of knowledge about $m, \boldsymbol{r}$ in the literature [5,6,13,22,25] in the random oracle model. However, the knowledge extraction of these constructions requires to rewind the random oracle, and as pointed out by [21, 34], this would incur an exponential security loss in the application of blind signatures and anonymous credentials. To achieve efficient straight-line extractable proof, as we discussed in the prior section, we take the approach of encrypt-and-prove, which is currently better optimized than the other one using extractable linear homomorphic commitments.

The general paradigm is to encrypt the witness and then prove well-formedness of the encryption and consistency of the encrypted witness (with the BDLOP commitment). In our specific case, we can just encrypt the randomness $\boldsymbol{r}$ of the above BDLOP commitment, i.e., $\mathsf{Enc}(\boldsymbol{r})$ and then prove well-formedness of the encryption, upper bound of ℓ_2 norm for $\boldsymbol{r}$, and $\mathbf{A}_1 \cdot \boldsymbol{r} = t_1$. The $\boldsymbol{r}$ can be extracted easily in a straight-line manner, by decrypting the ciphertext given the secret key of Enc. Then one can derive $m := t_2 - \mathbf{A}_2\boldsymbol{r}$, which would be consistent with what was originally committed to by the binding property of the commitment.

To instantiate this idea, one could consider the currently most optimized lattice proof (in the classical random oracle model) [2], which takes the following

high-level step. First they instantiate a RLWE-type encryption scheme $\mathsf{Enc}(\cdot)$ and then use the ABDLOP commitment to commit to $\boldsymbol{r}$ and the randomness to generate the encryption $\mathsf{Enc}(\boldsymbol{r})$, say ρ. Then they use the LNP proof technique [41] to prove (1) the randomness ρ and $\boldsymbol{r}$ are small; (2) ρ and $\boldsymbol{r}$ satisfy the linear equation as in this particular encryption algorithm, implying that the ciphertext is well-formed; and (3) $\mathbf{A}_1 \cdot \boldsymbol{r} = t_1 \pmod{q_1}$.

One More Subtlety. We identify a technical subtlety – for our anonymous construction, there still remains a gap towards the full overall security, even if one proves well-formedness of BDLOP commitments using a straight-line extractable proof, due to a possible mix-and-match attack. To tackle this, we identify a stronger form of well-formedness, where it is computationally infeasible to generate tuples (t_1, t_1', t_2) such that both (t_1, t_2) and (t_1', t_2) can be proved well-formed. This stronger property suffices for deriving secure anonymous credentials and can be realized in a simple and efficient way. We present more details in Section 2.3.

2 Preliminaries

Notations. $\mathbb{Z}$ and $\mathbb{R}$ denote the sets of integers and real numbers. Throughout this paper, we use λ to denote the security parameter, which is the implicit input for all algorithms. A function $f(\lambda) > 0$ is negligible and denoted by $\mathsf{negl}(\lambda)$ if for any $c > 0$ and sufficiently large λ, $f(\lambda) < 1/\lambda^c$. A probability is called to be overwhelming if it is $1 - \mathsf{negl}(\lambda)$. A column vector is denoted by a bold lower case letter (e.g., $\boldsymbol{x}$). A matrix is denoted by a bold upper case letter (e.g., $\mathbf{A}$). For a vector $\boldsymbol{x}$, its Euclidean norm (also known as the ℓ_2 norm) is defined to be $\|\boldsymbol{x}\| = (\sum_i x_i^2)^{1/2}$, and its infinity norm is defined to be $\|\boldsymbol{x}\|_\infty = \max_i |x_i|$. For a matrix $\mathbf{A}$, its ith column vector is denoted by $\boldsymbol{a}_i$ and its transposition is denoted by $\mathbf{A}^\top$. The Euclidean norm of a matrix is the ℓ_2 norm of its longest column: $\|\mathbf{A}\| = \max_i \|\boldsymbol{a}_i\|$. For any matrix $\mathbf{B} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_m)$, we use $\widetilde{\mathbf{B}} = (\tilde{\boldsymbol{b}}_1, \ldots, \tilde{\boldsymbol{b}}_m)$ to denote the Gram-Schmidt orthogonalization of $\mathbf{B}$. Besides, we refer to $\|\widetilde{\mathbf{B}}\|$ as the Gram-Schmidt norm of $\mathbf{B}$. Let $R = \mathbb{Z}[x]/(x^d + 1)$ be a cyclotomic ring, with d be a power of 2. And the norm of an element in R_q will be the norm of its unique representative with coefficients in $[-(q-1)/2, (q-1)/2]$. For matrix $\mathbf{A}$ in $R^{\ell \times l}$, we use $s_1(\mathbf{A}) = \mathsf{max}_{\|\boldsymbol{x}\|} \left(\frac{\|\mathbf{A}\boldsymbol{x}\|}{\|\boldsymbol{x}\|} \right)$ to denote its operator norm. For positive $\beta \in \mathbb{R}$, we use S_β to denote the set of all polynomials of infinity norm less than β, i.e., $S_\beta = \{a \in \mathcal{R} \mid \|a\|_\infty \leq \beta\}$.

For positive integers n, q, let $[n]$ denote the set $\{1, ..., n\}$ and $\mathbb{Z}_q$ denote the ring of integers modulo q. For a distribution or a set $\mathcal{X}$, we write $x \xleftarrow{\$} \mathcal{X}$ to denote the operation of sampling an uniformly random x according to $\mathcal{X}$. For two distributions $\mathcal{X}, \mathcal{Y}$, we let $\mathsf{SD}(\mathcal{X}, \mathcal{Y})$ denote their statistical distance. We write $\mathcal{X} \overset{s}{\approx} \mathcal{Y}$ to mean that they are statistically close, and $\mathcal{X} \overset{c}{\approx} \mathcal{Y}$ to say that they are computationally indistinguishable.

Due to the space limit, we defer the detailed background notations, definitions, and lemmas on lattices, rejection sampling, and algebraic structure of cyclotomic rings to Appendices A.1, A.2, and A.4, respectively.

2.1 M-LWE and M-SIS

Now we introduce the hard problems on which our schemes rely, which are denoted as M-LWE and M-SIS.

Definition 2.1 (M-SIS [35]) *The* $\mathsf{M\text{-}SIS}_{q,\ell,m,\beta}$ *problem (over an implicit ring R) is defined as follows. Given an uniformly random matrix* $\mathbf{A} \in R_q^{\ell \times m}$*, output vector* $\boldsymbol{z} \in R^m$ *such that* $\mathbf{A}\boldsymbol{z} = 0$ *and* $0 < \|\boldsymbol{z}\| \leq \beta$*.*

Definition 2.2 (M-LWE [35]) *The decision* $\mathsf{M\text{-}LWE}_{q,\ell,m,S_t}$ *problem (over an implicit ring R) is defined as follows. For* $\boldsymbol{s} \xleftarrow{\$} S_t^\ell$*, use* $A_{q,\boldsymbol{s}}$ *to denote the distribution of* $(\boldsymbol{a}, \langle \boldsymbol{a}, \boldsymbol{s} \rangle + e) \in R_q^\ell \times R_q$*, where* $\boldsymbol{a} \xleftarrow{\$} R_q^\ell$ *and* $e \xleftarrow{\$} S_t$*. The goal is to distinguish m samples from either* $A_{q,\boldsymbol{s}}$ *or* $\mathcal{U}(R_q^\ell, R_q)$*.*

Notice that for $\mathsf{M\text{-}LWE}_{q,\ell,m,S_t}$, if $\ell = 1$ and $t = 1$, it can also be called as $\mathsf{RLWE}_{q,1,m}$.

2.2 Syntax of Commitment

We give a formal definition of commitment schemes, following the presentation of [6,22]. A commitment scheme consists three algorithms ($\mathsf{CKeyGen}, \mathsf{Commit}, \mathsf{Open}$), with the security parameter 1^λ as implicit input:

$\mathsf{CKeyGen}$ is a PPT algorithm that outputs the public parameters params containing the descriptions of the message space $\mathcal{M}$ and randomness space $\mathcal{R}$.
Commit is a PPT algorithm that, on input the public parameters params and a message $x \in \mathcal{M}$, outputs the commitment c and its related randomness $r \in \mathcal{R}$.
Open is a deterministic poly-time algorithm that, on input the public parameters params, a message $x \in \mathcal{M}$ and values c and $r \in \mathcal{R}$, outputs a bit $b \in \{0, 1\}$.

A secure commitment scheme requires the two properties: hiding and binding. We defer the presentation to Appendix A.3.

2.3 BDLOP Commitment Scheme

We use as a building block the efficient lattice-based commitment scheme in [6, 22], implicitly denoted as BDLOP Commitment. Particularly, BDLOP Commitment consists of three algorithms ($\mathsf{CKeyGen}$,Commit,Open) as follows.

- $\mathsf{CKeyGen}$ (1^λ): Given the security parameter λ as input, the algorithm first sets the parameters n, k, ℓ, q_1, q_2, and ring $R = \mathbb{Z}[x]/\langle x^N + 1 \rangle$ where N is a power of 2, or other cyclotomic rings as Table 5, and then chooses random matrices $\mathbf{A}_1' \xleftarrow{\$} R_{q_1}^{n \times (k-n)}$ and $\mathbf{A}_2' \xleftarrow{\$} R_{q_2}^{\ell \times (k-n-\ell)}$. Finally, the algorithm outputs the public paramters $\mathsf{params} := \mathbf{A}_0 = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix}$ with $\mathbf{A}_1 := [\mathbf{I}_n, \mathbf{A}_1'] \in R_{q_1}^{n \times k}$, $\mathbf{A}_2 := [\mathbf{0}^{\ell \times n}, \mathbf{I}_\ell, \mathbf{A}_2'] \in R_{q_2}^{\ell \times k}$.

– $\mathsf{Commit}(\mathsf{params}, \boldsymbol{m}; \boldsymbol{r})$: In order to commit to a message $\boldsymbol{m} \in R_{q_2}^{\ell}$, the algorithm first samples a random short vector $\boldsymbol{r} \xleftarrow{\$} S_{\beta}^{k}$, and then outputs $\mathsf{comm} := \begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{m} \end{bmatrix}$.[13]
– $\mathsf{Open}(\mathsf{params}, \mathsf{comm})$: For $\mathsf{comm} := (\boldsymbol{t}_1^{\top}, \boldsymbol{t}_2^{\top})^{\top} \in R_{q_1}^{n} \times R_{q_2}^{\ell}$, there are two types of openings for slightly different commitment relations in the literature: relaxed one $(c, \bar{\boldsymbol{r}}, \boldsymbol{m})$ and exact one $(\boldsymbol{r}, \boldsymbol{m})$.
Here we will choose to use the latter one. This is because for the efficiency of our specific constructions of CTS and Anonymous Credentials system, we need to extract the exact randomness $\boldsymbol{r}$ for each commitment comm, rather than the relaxed randomness $\bar{\boldsymbol{r}}$.[14]
Particularly, The valid exact opening is with respect to the following exact relation

$$\hat{L} =: \Big\{ \mathsf{comm} : \exists (\boldsymbol{m}, \boldsymbol{r}) \text{ such that } \mathsf{comm} = \mathsf{Commit}(\mathsf{params}, \boldsymbol{m}, \boldsymbol{r}) \Big\}.$$

A valid opening of $\mathsf{comm} := (\boldsymbol{t}_1^{\top}, \boldsymbol{t}_2^{\top})^{\top} \in R_{q_1}^{n} \times R_{q_2}^{\ell}$ consists of a message $\boldsymbol{m} \in R_{q_2}^{\ell}$, and a short vector $\boldsymbol{r} = (r_1, \ldots, r_k)^{\top} \in R^{k}$, such that $\begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \boldsymbol{r} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{m} \end{bmatrix}$, where for all i, $\|r_i\|_{\infty} \leq \beta$.

Besides, there are two additional algorithms for the randomness vector in the valid commitment.

– $\mathsf{Combine}(\boldsymbol{r}, \boldsymbol{r}')$: Given two vectors $\boldsymbol{r} \in S_{\beta}^{k}$ and $\boldsymbol{r}' \in S_{\beta}^{k}$, output $\hat{\boldsymbol{r}} = \boldsymbol{r} + \boldsymbol{r}' \in S_{2\beta}^{k}$.
– $\mathsf{Randomize}(\mathsf{params}, \mathsf{comm}, \boldsymbol{r}')$: Taking as input params, $\boldsymbol{r}' \in S_{\beta}^{k}$, and a commitment comm, output $\mathsf{comm}' = \mathsf{comm} + \mathbf{A}_0 \cdot \boldsymbol{r}'$.[15]

According to [6,22], we know that BDLOP Commitment satisfies binding and hiding properties, following from $\mathsf{M\text{-}SIS}_{q_1,n,k,8\sqrt{2}\cdot\eta\cdot\kappa\cdot\beta\cdot k\cdot N}$ and $\mathsf{M\text{-}LWE}_{q_2,k-n-\ell,n+\ell}$, respectively. Here, η is the parameter for rejection sampling as in Lemma A.9, κ is the parameter for the challenge set of NIZKPoK system as in Table 5.

Well-Formedness. For our application, we need to prove the well-formedness of BDLOP commitments along with the commitment generation. This task has been studied in the original BDLOP scheme and several follow up works, e.g.,

[13] For a general parameter $\tau \geq 1$, if we choose a random short matrix $\mathbf{R} \xleftarrow{\$} S_{\beta}^{\ell \times \tau}$, then we can use such a BDLOP commitment scheme to commit to a message matrix $\mathbf{M} \in R_{q_2}^{\ell \times \tau}$.

[14] In fact, even just with the proof with respect to the relaxed relation, we can also extract the exact vector $\boldsymbol{r}$ through using the encrypt-and-prove paradigm. But, this might result in a relatively inefficient construction.

[15] Notice that, if comm is a valid commitment of m with randomness $\boldsymbol{r}$, then comm' is still a valid commitment of m, but with randomness $\hat{\boldsymbol{r}} = \mathsf{Combine}(\boldsymbol{r}, \boldsymbol{r}')$.

[6,22]. Particularly, given the public matrices $\mathbf{A}_1, \mathbf{A}_2$ and commitment $\mathsf{comm} := (\boldsymbol{t}_1^\top, \boldsymbol{t}_2^\top)^\top$, the relation can be described as: there exist vector $\boldsymbol{r}, \boldsymbol{m}$ such that

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{bmatrix},$$

where $\mathbf{0} \in R_{q_1}^{n \times \ell}$ and $\mathbf{I} \in R_{q_2}^{\ell \times \ell}$ denote the zero and identity matrices.

For the original BDLOP scheme where the message space is $R_{q_2}^{\ell}$, i.e., $\boldsymbol{m}$ can be any element in $R_{q_2}^{\ell}$, well-formedness can be proved in a relaxed way, i.e., by showing that there exists a $\boldsymbol{r}$ such that $\mathbf{A}_1 \cdot \boldsymbol{r} = f \cdot \boldsymbol{t}_1$, with respect to the relaxed relation

$$L_{\gamma_1', q_1, q_2, \bar{\mathcal{C}}} := \Big\{ (\mathbf{A}_1, \mathbf{A}_2, \boldsymbol{t}_1, \boldsymbol{t}_2) : \exists\ (\boldsymbol{r}, \boldsymbol{m}) \text{ and } f \in \bar{\mathcal{C}} \text{ such that } 0 < \|\boldsymbol{r}\| \leq \gamma_1', \text{ and } \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \boldsymbol{r} + \begin{bmatrix} \mathbf{0} \\ \boldsymbol{m} \end{bmatrix} = f \cdot \begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{bmatrix} \Big\}.$$

In fact, this is what the "proof of opening" does in several prior works [6, 22].

However, our application requires a stronger form of well-formedness, which is not implied by what we just described above. Particularly, our application needs a stronger commitment-proof binding property that for any $(\boldsymbol{t}_1, \boldsymbol{t}_2)$ that can be proved to be well-formed, it is computationally infeasible to find another $\boldsymbol{t}_1'$ such that one can prove well-formedness for $(\boldsymbol{t}_1', \boldsymbol{t}_2)$. This is an important requirement that prevents the mix-and-match attacks for our anonymous credential systems. We formalize this in Appendix A.5.

Next we argue that the original BDLOP does not satisfy this property by the following example. One first generates $\mathsf{Commit}(\boldsymbol{m}) = (\boldsymbol{t}_1, \boldsymbol{t}_2)$ honestly for an arbitrary $\boldsymbol{m}$, and then computes $\boldsymbol{t}_1' = \mathbf{A}_1 \cdot \boldsymbol{r}'$, with $\boldsymbol{r} \neq \boldsymbol{r}'$. Then we can interpret $(\boldsymbol{t}_1', \boldsymbol{t}_2)$ as $\mathsf{Commit}(\boldsymbol{m}' = \boldsymbol{t}_2 - \mathbf{A}_2 \cdot \boldsymbol{r}')$. As the message space is the full ring vector $R_{q_2}^{\ell}$, this interpretation is valid, and thus $(\boldsymbol{t}_1', \boldsymbol{t}_2)$ can still be considered to be well-formed. Thus, it is easy to generate two proofs for these two commitments, breaking the commitment-proof binding property.

To tackle this, we identify a simple property – as long as the BDLOP message space is "short", i.e., $\|\boldsymbol{m}\| \leq \gamma_2'$ for some parameter γ_2', then the stronger form of well-formedness is implied naturally!

Particularly, we notice that this stronger well-formedness can also be expressed as two *exact* linear relations:

$$\mathbf{A}_1 \cdot \boldsymbol{r} = \boldsymbol{t}_1 \text{ and } \begin{bmatrix} \mathbf{A}_2 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{m} \end{bmatrix} = \boldsymbol{t}_2.$$

So, through using the LNP proof framework, i.e., Figure 10 in [41], we can first commit to vectors $\boldsymbol{r}$ and $\boldsymbol{m}$, and then prove their exact norm bound and the liner relations. The advantage of adopting such an exact relation proof is that the overhead can be amortized with other $\mathsf{NIZKPoK}$ parts, which will reduce the full proof size significantly. Thus, we use such an exact proof approach in our instantiations for multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ in Section 5.

Except with achieving the stronger well-formedness by the above exact relation, we can also get it by the relaxed relation. Our intuition is that, if the

adversary can come up with such a tuple $(\boldsymbol{t}_1, \boldsymbol{t}_2, \boldsymbol{t}'_1)$, then there is a reduction that breaks the M-SIS problem (with proper parameters). Due to space limit, we defer more details in Appendix A.5.

2.4 Non-interactive Zero-knowledge Proof

Let's recall the notion of non-interactive zero-knowledge (NIZK) proof system.

Definition 2.3 ([23]) *Let $\mathfrak{R}$ be a relation. A non-interactive proof system for $\mathfrak{R}$ is a tuple of PPT algorithms* (Setup, Prove, Verify, SimSetup) *having the following interfaces (where 1^λ are implicit inputs to* Prove, Verify, SimSetup*):*

- $\mathsf{Setup}(1^\lambda)$: *given a security parameter λ, outputs a string* crs.
- $\mathsf{Prove}(\mathsf{crs}, x, w)$*: given a string* crs *and a statement-witness pair $(x, w) \in \mathfrak{R}$, outputs a proof π.*
- $\mathsf{Verify}(\mathsf{crs}, x, \pi)$*: given a string* crs*, a statement x, and a proof π, either accepts or rejects.*
- $\mathsf{SimSetup}(1^\lambda)$*: given a security parameter λ, outputs a simulated string $\widehat{\mathsf{crs}}$ and a trapdoor* tk.

A secure NIZK should have three properties: Completeness, Soundness, and Zero-knowledge. Due to space limitation, we defer the definitions in Appendix A.6. As argued by [5, 21, 25, 27], Fiat-Shamir based proof systems in the random oracle model satisfy these properties. Many recent lattice-based efficient NIZKs are Fiat-Shamir based, so they also enjoy this property. Notice that, even crs is explicitly outputted by the algorithm Setup, the above definition still cover the case of Random Oracle based NIZK, just as used in [2, 12, 21, 33]

3 Commit-Transferable Signatures

Following prior work [7], our goal is to obtain a signature scheme that can be combined with an appropriate commitment scheme and zero-knowledge proof-of-knowledge protocols to obtain an Anonymous Credential scheme.

We will first describe the key *novel* building block we need: a signature scheme whose message space consists of commitments. Our starting point is a non-interactive commitment algorithm Commit parameterized by params chosen according to the Setup algorithm, i.e., $\mathsf{params} \leftarrow \mathsf{Setup}(1^\lambda)$. The commitment scheme should admit additional algorithms that allow for randomizing commitments. Particularly, given a commitment $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$ and randomness Rand', there is an algorithm Randomize that outputs another commitment $\mathsf{comm}' = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand}'')$ to the same message m. An additional Combine operation is for combining Rand' with the randomness Rand of the commitment comm, i.e., $\mathsf{Rand}'' = \mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}')$.

The novel property of a *commit-transferable* signature is that, given a signature σ on a commitment $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$, it is possible to obtain a signature σ' on a different commitment to the same message, $\mathsf{comm}' =$

$\mathsf{Commit}(\mathsf{params}, m; \mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}'))$. The unforgeability property is defined that an adversary querying for signatures on commitments whose openings are known $m_1, \ldots, m_n$ will not be able to produce a signature on a commitment that opens to a new message $m' \neq m_i$, for $\forall\, i \in [n]$. We notice that the requirement of commitments whose openings are known can be achieved by requiring the adversary to provide an additional (non-interactive) zero-knowledge proof of knowledge in the applications, and thus our simpler form of unforgeability for CTS suffices. As discussed in the introduction, our applications need an additional property called straight-line extraction for the NIZKPoK. We discuss more details in Remark 3.5 and Section 5.

More formally: let (Setup, Commit) be a non-interactive randomizable commitment scheme that admits (Randomize, Combine) for randomizing commitments; let (KeyGen, Sign, Verify) be a signature scheme, and let Transfer be an additional algorithm with the following input-output behavior:

Setup Let λ be the security parameter. $\mathsf{Setup}(1^\lambda)$ outputs params, the parameters for the commitment scheme and the signature scheme; these parameters also define the message space $\mathcal{M}$, randomness space $\mathcal{R}$ for the commitment scheme, the randomness space $\mathcal{R}'$ for the Randomize algorithm, and the output space $\mathcal{R}''$ of the Combine algorithm.

Commit Let $m \in \mathcal{M}$, $\mathsf{Rand} \in \mathcal{R}$. $\mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$ outputs comm, a commitment to m using randomness Rand. There is no separate opening algorithm: opening can be achieved by revealing m and Rand.

Randomize **and** Combine Let $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$, with $\mathsf{Rand} \in \mathcal{R}$. $\mathsf{Randomize}(\mathsf{params}, \mathsf{comm}, \mathsf{Rand}, \mathsf{Rand}')$ returns the commitment $\mathsf{comm}' = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}'))$, where $\mathsf{Combine} : \mathcal{R} \times \mathcal{R}' \mapsto \mathcal{R}''$ is an efficiently computable operation on elements of $\mathcal{R}$ and $\mathcal{R}'$.

KeyGen Given params, $\mathsf{KeyGen}(\mathsf{params})$ outputs a secret key sk and the corresponding public key pk for commit-transferrable signature system.

Sign Let $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$. $\mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{comm})$ outputs a signature σ with respect to comm.

Transfer Let $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$, $\mathsf{comm}' = \mathsf{Randomize}(\mathsf{params}, \mathsf{comm}, \mathsf{Rand}')$, $\sigma = \mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{comm})$. On input $(\mathsf{params}, \mathsf{pk}, \sigma, m, (\mathsf{Rand}, \mathsf{Rand}'))$, the algorithm Transfer outputs a signature σ' with respect to the randomized commitment comm'.

Verify On input $(\mathsf{params}, \mathsf{pk}, \mathsf{comm}, \sigma)$, the algorithm Verify either accepts or rejects. For simplicity, our syntax does not distinguish whether the signature σ is an original or a transferred one. In the construction, we need to specify two different procedures when verifying different types of the signatures.

Definition 3.1 (Correctness) *Let* Setup, Commit, Randomize, Combine, KeyGen, Sign, Verify *and* Transfer *be efficient algorithms with input-output behavior as above. They define a* correct *randomizable commitment scheme if for all* params *that are output by* Setup, *for all* $m \in \mathcal{M}$, $\mathsf{Rand} \in \mathcal{R}$, $\mathsf{Rand}' \in \mathcal{R}'$, $\mathsf{Randomize}(\mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand}), \mathsf{Rand}') = \mathsf{Commit}(\mathsf{params}, m; \mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}'))$.

Moreover, they define a correct *commit-transferable signature scheme if for all* params *that are output by* Setup, *for all* $m \in \mathcal{M}$, $\mathsf{Rand} \in \mathcal{R}$, $\mathsf{Rand}' \in \mathcal{R}'$, $\sigma \leftarrow$

$\mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{Commit}(\mathsf{params}, m, \mathsf{Rand}))$, $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{params})$, $\sigma' \leftarrow \mathsf{Transfer}(\mathsf{params}, \mathsf{pk}, \sigma, m, (\mathsf{Rand}, \mathsf{Rand}'))$, *both* $\mathsf{Verify}(\mathsf{params}, \mathsf{pk}, \mathsf{Commit}(\mathsf{params}, m, \mathsf{Rand}), \sigma)$ *and* $\mathsf{Verify}(\mathsf{params}, \mathsf{pk}, \mathsf{Commit}(\mathsf{params}, m, \mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}')), \sigma')$ *accept.*

Additionally, we require that commit-transferrable signature schemes satisfy several properties: unlinkablity/simulatability and unforgeability. Intuitively, unlinkablity means that for any two messages m_0, m_1, it is infeasible to distinguish their honest transferred signatures σ'_0 and σ'_1 (output by the algorithm $\mathsf{Transfer}$). Simulatability means that the transferred signature σ' itself does not leak information about the input x (and also the randomness). Clearly, simulatability is much stronger property, and implies unlinkability. Thus, it is sufficient for us to just focus on simulatability.

Below we formulate the property of simulatability by the zero-knowledge paradigm, requiring that a simulator without knowing the input x and randomness can generate an indistinguishable σ' for an arbitrary number of queries.

Definition 3.2 (Simulatability) *We say that the* $\mathsf{Transfer}$ *algorithm can be* simulatable *if there exists a two-stage probabilistic polynomial time simulator* $\mathcal{S}$ *which can simulate the transfer algorithm in an indistinguishable way, without knowing the input* x *and randomness to the commitment* comm*. More formally, we define the syntax of the two-stage simulation process as follow.*

- *First,* $\mathcal{S}$ *generates* params*, together with some trapdoor information* Trap*.*
- *Second,* $\mathcal{S}$ *is given input* params *with the trapdoor* Trap*, and any arbitrary* pk*,* comm*. Then* $\mathcal{S}$ *can generate a simulated transferred signature* $\widetilde{\sigma}'$*.*

Then the simulatability requires that for $t = \mathsf{poly}(\lambda)$*, any* $\{m_i\}_{i\in[t]} \in \mathcal{M}$*, randomness* $\{\mathsf{Rand}_i, \mathsf{Rand}'_i\}_{i\in[t]}$*, no probabilistic polynomial time distinguisher* $\mathcal{D}$ *can distinguish* $(\mathsf{params}, \mathsf{pk}, \{\mathsf{comm}'_i\}_{i\in[t]}, \{\sigma'_i\}_{i\in[t]})$ *from* $(\mathsf{params}, \mathsf{pk}, \{\mathsf{comm}'_i\}_{i\in[t]}, \{\widetilde{\sigma}'_i\}_{i\in[t]})$ *with better than a negligible advantage, where*

- *in the former, the* params *and* pk *are sampled honestly, each* $\mathsf{comm}_i = \mathsf{Commit}(\mathsf{params}, m_i; \mathsf{Rand}_i)$*,* $\sigma_i \leftarrow \mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{comm}_i)$*,* $\mathsf{comm}'_i = \mathsf{Randomize}(\mathsf{params}, \mathsf{comm}_i, \mathsf{Rand}'_i)$*, and* $\sigma'_i \leftarrow \mathsf{Transfer}(\mathsf{params}, \mathsf{pk}, \sigma_i, m_i, (\mathsf{Rand}_i, \mathsf{Rand}'_i))$*;*
- *in the latter,* params *is generated by the simulator,* pk *is sampled honestly,* comm'_i *is generated as above, and* $\widetilde{\sigma}'_i$ *is generated by the simulator.*

Definition 3.3 (Unforgeability for Commitment Relation) *We say that the algorithms as above define an* unforgeable *commit-transferable signature if for all probabilistic polynomial-time adversaries* $\mathcal{A}$*, the probability that* $\mathcal{A}$ *wins the following game is negligible:*

Input generation phase: *On input* 1^λ*, the challenger generates* $\mathsf{params} \leftarrow \mathsf{Setup}(1^\lambda)$*,* $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\mathsf{params})$*.*

Query phase: *Given* $(\mathsf{params}, \mathsf{pk})$ *as input, the adversary* $\mathcal{A}$ *can access to the following oracle:* $\mathcal{A}$ *makes queries with the form of* $(\mathsf{comm}_i, m_i, \mathsf{Rand}_i)$*, and gets as responses* $\sigma_i = \mathsf{Sign}(\mathsf{params}, \mathsf{sk}, \mathsf{comm}_i)$ *if* $\mathsf{comm}_i = \mathsf{Commit}(\mathsf{params}, m_i; \mathsf{Rand}_i)$*, or* $\perp$ *else.*

Challenge phase: *Finally, the adversary* $\mathcal{A}$ *outputs* $(m^*, \mathsf{Rand}, \sigma)$. *Let* $\mathsf{comm}^* = \mathsf{Commit}(\mathsf{params}, m^*; \mathsf{Rand})$, *and* $\mathcal{A}$ *wins the game if* $\mathsf{Verify}(\mathsf{params}, \mathsf{pk}, \mathsf{comm}^*, \sigma)$ *accepts, and* m^* *has never been queried in the query phase.*

The scheme is selectively secure if the adversary needs to commit to the challenge message m^* *before the input generation phase, and is adaptively secure if this condition is not required.*

Remark 3.4 *Our notion of unforgeability requires the adversary to make queries of the form* $(\mathsf{comm}, m, \mathsf{Rand})$ *such that* $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m, \mathsf{Rand})$. *In practical applications such as anonymous credentials and blind signature, this form can be enforced by requiring the adversary to provide a zero-knowledge proof of knowledge* π, *i.e., knowing a witness* (m, Rand) *such that* $\mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m, \mathsf{Rand})$. *In this way, an adversary who makes queries of* (comm, π) *can be made equivalent to an adversary who makes queries of* $(\mathsf{comm}, m, \mathsf{Rand})$.

Remark 3.5 *As pointed out by [21], there is a subtlety about proving knowledge of the commitment in the applications to anonymous credentials and blind signatures – the knowledge extraction needs to be straight-line, as the rewinding extraction would incur an exponential security loss (in the number of queries). In Section 5, we show how to instantiate a competitively efficient straight-line extractable* NIZKPoK *required by our anonymous credential construction.*

Remark 3.6 *A weaker notion of selective security can be considered where in the above unforgeability game, the adversary needs to commit to both* (m^*, Rand) *before the input generation phase. However, this weaker notion suffers from a drawback – the upgrade to the adaptive security via the complexity leveraging would incur* $|m^*|+|\mathsf{Rand}|$ *bits of security loss, whereas the above stronger selective notion only incurs* $|m^*|$ *bits security loss. As our construction (in Section 4) can directly achieve the stronger notion as Definition 3.3, we do not consider this weaker variant in this work.*

Finally the overall security of the CTS can be defined as follow.

Definition 3.7 (Secure commit-transferable signature) *The algorithms* Setup, Commit, KeyGen, Sign, Verify *and* Transfer *constitute a secure commit-transferable signature scheme if they constitute correct, simulatable and unforgeable (for exact commitment relation) commit-secure signature scheme, i.e. satisfy Definitions 3.1, 3.2, 3.3; and the commitment scheme* $(\mathsf{Setup}, \mathsf{Commit})$ *is hiding and binding, satisfying Definitions A.10, A.11.*

4 Efficient Construction for CTS

In this section, we first present a lattice-based commit-transferrable signature scheme, and then show that it satisfies the properties of correctness, simulatability, and unforgeability as defined in Section 3. Our construction uses the following building blocks: (1) the BDLOP commitment scheme $\Gamma = \Gamma.\{\mathsf{CKeyGen}, \mathsf{Commit},$

Open, Combine, Randomize}, and (2) a NIZKPoK system $\Pi^{(1)} = \Pi^{(1)}.\{\mathsf{Setup}, \mathsf{Prove}, \mathsf{VerifyProve}, \mathsf{SimSetup}\}$ for the following language (parameterized by $\gamma', q \in \mathbb{N}$)[16]

$$L_{\gamma',q,\bar{\mathcal{C}}} = \Big\{(\mathbf{B}, \boldsymbol{u}) \in R_{N,q}^{\ell \times (\ell \cdot (2\tau+1)+\hat{\ell}+k-n)} \times R_{N,q}^{\ell} : \exists\, \boldsymbol{x} \in R_{N,q}^{\ell \cdot (2\tau+1)+\hat{\ell}+k-n} \text{ and } f \in \bar{\mathcal{C}} \text{ such that } 0 < \|\boldsymbol{x}\| \leq \gamma' \text{ and } \mathbf{B} \cdot \boldsymbol{x} = f \cdot \boldsymbol{u}\Big\}.$$

4.1 Construction

We first describe the required parameters in Table 5. Notice that in this work, we consider the cyclotomic rings $R_N = \mathbb{Z}[X]/(X^N+1)$ and $R_d = \mathbb{Z}[X]/(X^d+1)$ with N, d be powers of 2, and $t = N/d$. This type of ring is commonly used in many constructions, as it is easy to analyze to the norm bounds under ring operations, convenient to implement, and has an efficient zero-knowledge proof system.

Param.	Description
λ	Security parameter
R, N, d, t	Cyclotomic Ring for CTS and its dimensions N, d with $t = N/d$
q_1, q_2	Moduli used for BDLOP commitment scheme
n, k, ℓ	Dimensions for the underlying BDLOP commitment scheme
$\mathcal{M}, \bar{\mathcal{M}}$	$\mathcal{M} = \bar{M}^{\ell}$, $\bar{\mathcal{M}}$ is a subset of the ring R_{N,q_2} consisting of
ω, ζ	2^{ζ} non-zero binary polynomials elements with ℓ_1-norm be ω
δ, τ	$\boldsymbol{g}^{\top} = (1, \delta, \ldots, \delta^{\tau-1})$, $\delta = \lfloor q_2^{1/\tau} \rceil$
S_β	Set of all elements in R_N with ℓ_∞ norm at most β
α	Parameter used in SamplePre
$\hat{\ell}$	Public matrix $\mathbf{D} \in R_N^{\ell \times (\ell+\hat{\ell})}$
η, M	Parameters for rejection sampling algorithm
γ	ℓ_2 norm parameter used in Verify algorithm for original signature
$\mathcal{C}, \kappa$	Challenge set of the NIZKPoK system $\Pi^{(1)}$ $\mathcal{C} = \{c \in R : \|c\|_1 = \kappa, \|c\|_\infty = 1\}$
$\bar{\mathcal{C}}$	The set of differences $\mathcal{C} - \mathcal{C}$ except 0
γ'	ℓ_2 norm parameter for "short" vectors in the language of Π
δ_0	Root-Hermite Factor
Bit-sec	Bit-security in time for our construction

Table 5. Parameters of Commit-Transferrable Signature Scheme

In our Construction 4.1, we directly set the dimensions of the underlying BDLOP commitment as n, k, ℓ, following from the presentation of Definition 2.3.

Construction 4.1 (Commit-Transferrable Signature) *Our* CTS *is constructed as follow.*

[16] Under current state of art, such a system $\Pi^{(1)}$ can be efficiently instantiated from lattice-based assumptions, just as stated in Section 4.2.

– $\mathsf{Setup}(1^\lambda)$*: On input the security parameter* 1^λ*, the algorithm does:*

1. *Run* $\Gamma.\mathsf{CKeyGen}$ *to get* $\mathbf{A} := \begin{bmatrix} \mathbf{I}_n, & \mathbf{A}_1 \\ \mathbf{0}^{\ell\times n}, & \mathbf{A}_2 \end{bmatrix} \leftarrow \Gamma.\mathsf{CKeyGen}(1^\lambda)$*, where* $[\mathbf{I}_n, \mathbf{A}_1] \in R_{N,q_1}^{n\times k}$ *and* $[\mathbf{0}^{\ell\times n}, \mathbf{A}_2] \in R_{N,q_2}^{\ell\times k}$*, with* $\mathbf{A}_1 \in R_{N,q_1}^{n\times(k-n)}$ *and* $\mathbf{A}_2 = (\mathbf{I}_\ell, \mathbf{A}_2') \in R_{N,q_2}^{\ell\times(k-n)}$*. Note that the commitment scheme sets message space* $\mathcal{M} = \bar{\mathcal{M}}^\ell \subseteq R_{N,q_2}^\ell$ *with randomness space* $S_1^k \subseteq R_N^k$*, where* $\bar{\mathcal{M}}$ *is a subset of the ring* R_{N,q_2} *consisting of* 2^ζ *non-zero binary polynomials elements. More specifically, let* $\mathcal{B}$ *be the set of non-zero binary polynomials in* R_{d,q_2}*. Then, we can formally define the message space as* $\bar{\mathcal{M}} := \{m(X^t) \in R_{N,q_2} : m \in \mathcal{B} \text{ and } \|m\|_1 = \omega\}$*.*
2. *Sample a random vector* $\mathbf{D} \xleftarrow{\$} R_{N,q_2}^{\ell\times(\ell+\hat{\ell})}$*.*
3. *Set parameters* κ, γ, γ'*, and a gaussian parameter* α*;*
4. *Run* $\Pi.\mathsf{Setup}(1^\lambda)$ *to get a common reference string* crs*;*
5. *Output* $\mathsf{params} := (\mathbf{A}, \mathbf{D}, q_1, q_2, N, \kappa, \gamma, \gamma', \alpha, \mathcal{M}, \mathcal{R}, \mathsf{crs})$*.*

– $\mathsf{Commit}(\mathsf{params}, m; \mathsf{Rand})$*: On input* params*, message* $m \in \bar{\mathcal{M}}$*, and randomness* $\mathsf{Rand} := \mathbf{R} \in S_1^{k\times(\ell\cdot\tau)}$*, the algorithm does the following.*

1. *Set* $\mathbf{G} = \mathbf{I}_\ell \otimes \boldsymbol{g}^\top \in R_N^{\ell\times(\ell\cdot\tau)}$*, where* $\mathbf{I}_\ell \in R_N^{\ell\times\ell}$ *is the identity matrix and* $\boldsymbol{g}^\top = (1, \delta, \ldots, \delta^{\tau-1})$*.*
2. *Compute* $\mathsf{comm} = \Gamma.\mathsf{Commit}(\mathbf{A}, m\mathbf{G}; \mathbf{R})$ *as the commitment of* m*, i.e.,*

$$\mathsf{comm} := \mathbf{C} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \mathbf{R} + \begin{bmatrix} \mathbf{0}^{n\times(\ell\cdot\tau)} \\ m \cdot \mathbf{G} \end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}.$$

– $\mathsf{Randomize}(\mathsf{params}, \mathsf{comm}, \mathsf{Rand}')$*: On input* params*,* $\mathsf{Rand}' := \mathbf{R}' \in S_1^{k\times(\ell\cdot\tau)}$*, and* comm*, the algorithm computes and outputs* $\mathsf{comm}' = \Gamma.\mathsf{Randomize}(\mathbf{A}, \mathsf{comm}, \mathbf{R}')$*,*[17] *i.e.,*

$$\mathsf{comm}' := \mathbf{C}' = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \mathbf{R} + \begin{bmatrix} \mathbf{0}^{n\times(\ell\cdot\tau)} \\ m \cdot \mathbf{G} \end{bmatrix} + \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \mathbf{R}' \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}.$$

– $\mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}')$*: Taking as input two randomness* $\mathsf{Rand} := \mathbf{R} \in S_1^{k\times(\ell\cdot\tau)}$*, and* $\mathsf{Rand}' := \mathbf{R}' \in S_1^{k\times(\ell\cdot\tau)}$*, the algorithm computes and outputs* $\tilde{\mathbf{R}} \in S_2^{k\times(\ell\cdot\tau)}$*, where* $\tilde{\mathbf{R}} = \mathbf{R} + \mathbf{R}'$*.*

– $\mathsf{KeyGen}(\mathsf{params})$*: On input* params*, the algorithm does:*

1. *Sample* $\mathbf{T} \xleftarrow{\$} S_1^{(\ell+\hat{\ell})\times(\ell\cdot\tau)}$*, and set* $\mathbf{A}_0 = \mathbf{D} \cdot \mathbf{T} + \mathbf{G} \in R_{N,q_2}^{\ell\times(\ell\cdot\tau)}$*.*
2. *Sample* $\mathbf{B} \xleftarrow{\$} R_{N,q_2}^{\ell\times(\ell\cdot\tau)}$ *and a non-zero* $\boldsymbol{u} \xleftarrow{\$} R_{N,q_2}^\ell$*.*
3. *Output* $\mathsf{pk} := (\mathbf{A}_0, \mathbf{B}, \boldsymbol{u})$*, and* $\mathsf{sk} := \mathbf{T}$*.*

– $\mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{comm})$*: On input* params*,* pk*,* sk*, and* comm*, the algorithm does the following:*

1. *Parse* $\mathsf{comm} := \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$*, where* $\mathbf{C}_1 \in R_{N,q_2}^{n\times(\ell\cdot\tau)}$*, and* $\mathbf{C}_2 \in R_{N,q_2}^{\ell\times(\ell\cdot\tau)}$*.*

[17] Notice that, if comm is a valid commitment of $m\mathbf{G}$ with randomness $\mathbf{R}$, then comm' is still a valid commitment of m, but with randomness $\hat{\mathbf{R}} = \Gamma.\mathsf{Combine}(\mathbf{R}, \mathbf{R}')$.

2. *Set* $\mathbf{F}_{\mathsf{comm}} = \big[[\mathbf{D}|\mathbf{A}_0]\big|\mathbf{B}_{\mathsf{comm}}\big|\mathbf{A}_2\big] = \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}_2]\big|\mathbf{A}_2\big]$, *and sample* $\mathsf{Sig}_{\mathsf{comm}} := \begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3\end{bmatrix} \leftarrow \mathsf{SamplePre}(\big[[\mathbf{D}|\mathbf{A}_0]\big|\mathbf{B}_{\mathsf{comm}}\big|\mathbf{A}_2\big], \mathbf{T}, \boldsymbol{u}, \alpha)$,[18] *and output* $\mathsf{Sig}_{\mathsf{comm}}$ *as the signature of* comm, *where* $\boldsymbol{s}_1 = \begin{bmatrix}\boldsymbol{s}_{1,1}\\ \boldsymbol{s}_{1,2}\end{bmatrix}$, *and* $\boldsymbol{s}_{1,1} \in R_N^{\ell+\hat{\ell}}, \boldsymbol{s}_{1,2} \in R_N^{\ell\cdot\tau}, \boldsymbol{s}_2 \in R_N^{\ell\cdot\tau}$, $\boldsymbol{s}_3 \in R_N^{k-n}$.

– $\mathsf{Transfer}(\mathsf{params}, \mathsf{pk}, \mathsf{Sig}_{\mathsf{comm}}, m, (\mathsf{Rand}, \mathsf{Rand}'))$*: On input* $\mathsf{params}, \mathsf{pk}$, *a signature* $\mathsf{Sig}_{\mathsf{comm}}$, *message* m, *randomness* $\mathsf{Rand} := \mathbf{R} \in S_1^{k\times(\ell\cdot\tau)}$ *for generating the commitment* comm *for* m, *the additional randomness* $\mathsf{Rand}' := \mathbf{R}' \in S_1^{k\times(\ell\cdot\tau)}$ *for the rerandomization of* comm, *the algorithm does the followings:*

1. *Parse* $\mathsf{Sig}_{\mathsf{comm}}$ *as vector* $\begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3\end{bmatrix}$, *where* $\boldsymbol{s}_1 \in R_N^{(1+\tau)\cdot\ell+\hat{\ell}}$, $\boldsymbol{s}_2 \in R_N^{\ell\cdot\tau}$, $\boldsymbol{s}_3 \in R_N^{k-n}$.
2. *Run* $\mathsf{Commit}(\mathsf{params}, m; \mathbf{R})$ *and obtain:* $\mathsf{comm} := \mathbf{C} = \begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$.
3. *Run* $\mathsf{Randomize}\ (\mathsf{params}, \mathsf{comm}, \mathbf{R}')$ *and obtain* $\mathsf{comm}' := \mathbf{C}' = \begin{bmatrix}\mathbf{C}'_1\\ \mathbf{C}'_2\end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$.
4. *Compute a (temporary) signature* $\mathsf{Sig}_{\mathsf{comm}'}$ *as*

$$\mathsf{Sig}_{\mathsf{comm}'} := \begin{bmatrix}\boldsymbol{s}_1\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3 - \tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2\end{bmatrix} = \begin{bmatrix}\boldsymbol{s}_{1,1}\\ \boldsymbol{s}_{1,2}\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3 - \tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2\end{bmatrix} \in R^{\ell\cdot(2\tau+1)+\hat{\ell}+k-n},$$

   where we denote $\tilde{\mathbf{R}} = \mathbf{R} + \mathbf{R}' = \begin{bmatrix}\tilde{\mathbf{R}}_1\\ \tilde{\mathbf{R}}_2\end{bmatrix} \in R_N^{k\times(\ell\cdot\tau)}$, *with* $\tilde{\mathbf{R}}_1 \in R_N^{n\times(\ell\cdot\tau)}$ *and* $\tilde{\mathbf{R}}_2 \in R_N^{(k-n)\times(\ell\cdot\tau)}$.
5. *Compute* $\mathbf{F}_{\mathsf{comm}'} := \big[[\mathbf{D}|\mathbf{A}_0]\big|\mathbf{B}_{\mathsf{comm}'}\big|\mathbf{A}_2\big] = \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}'_2]\big|\mathbf{A}_2\big]$.
6. *Run the prove algorithm and output* $\mathsf{Sig}'_{\mathsf{comm}'} := \pi \leftarrow \Pi^{(1)}.\mathsf{Prove}(\mathsf{crs}_2, (\mathbf{F}_{\mathsf{comm}'}, \boldsymbol{u}), \mathsf{Sig}_{\mathsf{comm}'})$, *proving that* $\mathsf{Sig}_{\mathsf{comm}'}$ *is a short* ℓ_2 *norm vector and satisfies* $\mathbf{F}_{\mathsf{comm}'}\cdot\mathsf{Sig}_{\mathsf{comm}'} = \boldsymbol{u}$, *through using the* NIZKPoK *system* $\Pi^{(1)}$ *with the relaxed language* $L_{\gamma',q_2,\bar{\mathcal{C}}}$.

– $\mathsf{Verify}(\mathsf{params}, \mathsf{pk}, \mathsf{comm}, \mathsf{Sig})$*: On input* $\mathsf{params}, \mathsf{pk}, \mathsf{comm}, \mathsf{Sig}$, *the algorithm does the following.*

1. *Parse* $\mathsf{comm} := \mathbf{C} = \begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$, *where* $\mathbf{C}_1 \in R_{N,q_2}^{n\times(\ell\cdot\tau)}$, *and* $\mathbf{C}_2 \in R_{N,q_2}^{\ell\times(\ell\cdot\tau)}$;
2. *Based on the type of* Sig, *the verification works as follow.*
   - *If* Sig *is a non-zero short vector within* ℓ_2 *norm* γ, *then the algorithm does*

[18] Here, we implicitly use $\mathbf{T}$ as the $\mathbf{G}$-trapdoor of the matrix $[\mathbf{D}|\mathbf{A}_0]$, which can be easily extended to get the corresponding $\mathbf{G}$-trapdoor for $\big[[\mathbf{D}|\mathbf{A}_0]\big|\mathbf{B}_{\mathsf{comm}}\big|\mathbf{A}_2\big]$.

(a) *Set matrix* $\mathbf{F}_{\mathsf{comm}} := \left[[\mathbf{D}|\mathbf{A}_0]\middle|[\mathbf{B}+\mathbf{C}_2]\middle|\mathbf{A}_2^\top\right]$.

(b) *Check whether* Sig *satisfies* $\mathbf{F}_{\mathsf{comm}} \cdot \begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{bmatrix} = \boldsymbol{u} \in R_{N,q_2}$.

- *If* Sig *is a proof of the* NIZKPoK *system* $\Pi^{(1)}$,
  (a) *Set matrix* $\mathbf{F}_{\mathsf{comm}} := \left[[\mathbf{D}|\mathbf{A}_0]\middle|[\mathbf{B}+\mathbf{C}_2]\middle|\mathbf{A}_2^\top\right]$.
  (b) *Run the verify algorithm (with respect to language* $L_{\gamma',q_2,\bar{\mathcal{C}}}$*)* $\Pi_2.\mathsf{VerifyProve}(\mathsf{crs}, (\mathbf{F}_{\mathsf{comm}}, \boldsymbol{u}), \mathsf{Sig})$ *and output its result.*

Lemma 4.2 (Correctness) *For parameters* N, q_2, α, γ*, the* NIZKPoK *system* $\Pi^{(1)}$ *for the relaxed language* $L_{\gamma',q_2,\bar{\mathcal{C}}}$*, Construction 4.1 satisfies the correctness property as defined in Definition 3.1, where*

$$\gamma = \alpha\sqrt{2\cdot\left(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\right)\cdot N}$$

$$\gamma' \geq \left(\left(\sqrt{k-n}+\sqrt{\ell\cdot\tau}\right)\cdot N\cdot\alpha\sqrt{2\cdot\ell\cdot\tau}+\alpha\sqrt{\left(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\right)\cdot N}\right)$$

The correctness directly follows the correctness of BDLOP commitment, the completeness of the NIZKPoK system Π and our parameter settings. Due to space limitation, we defer the proof in Appendix B.1.

4.2 Instantiation of NIZKPoK system $\Pi^{(1)}$ in CTS

Before presenting the NIZKPoK system $\Pi^{(1)}$, we first specify the concrete language $L_{\gamma',q_2,\bar{\mathcal{C}}}$ in the algorithms Transfer and Verify,

$$L_{\gamma',q_2,\bar{\mathcal{C}}} = \Big\{(\mathbf{F}_{\mathsf{comm}'}, \boldsymbol{u}) \in R_{N,q_2}^{\ell\times(\ell\cdot(2\tau+1)+\hat{\ell}+k-n)} \times R_{N,q_2}^{\ell} : \exists\, \boldsymbol{x} \in R^{\ell\cdot(2\tau+1)+\hat{\ell}+k-n}$$
$$\text{and } f \in \bar{\mathcal{C}} \text{ such that } 0 < \|\boldsymbol{x}\| \leq \gamma' \text{ and } \mathbf{F}_{\mathsf{comm}'}\cdot\boldsymbol{x} = f\cdot u\Big\}.$$

Then, according to [6, 22], there exists such an efficient $\Pi^{(1)}$ for $L_{\gamma',q_2,\bar{\mathcal{C}}}$. The formal theorem is presented as follows.

Theorem 4.3 ([6, 22]) *In the random oracle model, there exists a* NIZKPoK *system* $\Pi^{(1)}$ *for the relaxed language* $L_{\gamma',q_2,\bar{\mathcal{C}}}$*, with*

$$\gamma' = 2\sqrt{2\cdot\left(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\right)\cdot N}\cdot\eta\cdot\kappa$$
$$\cdot\left((\sqrt{k-n}+\sqrt{\ell\cdot\tau})\cdot N\cdot\alpha\sqrt{2\cdot\ell\cdot\tau}+\alpha\sqrt{2\cdot(\ell\cdot(2\tau+1)+\hat{\ell}+k-n)\cdot N}\right).$$

Moreover, assuming a t*-time adversary* $\mathcal{A}$ *forging a proof with probability* ε*, there exists a* $O(t/\varepsilon)$*-time extractor, who can successfully extract the witness* $\boldsymbol{x}$ *and* $c \in \bar{\mathcal{C}}$ *with probability* $\frac{1}{2}$*.*

Remark 4.4 *Notice that the concrete instantiation of* NIZKPoK *system* $\Pi^{(1)}$ *in Theorem 4.3 is essentially a Fiat-Shamir signature, which is quite practical.*

4.3 Security of CTS

In this section, we establish the simulatability and unforgeability of the above Construction 4.1.

Lemma 4.5 (Simulatability) *Suppose $\Pi^{(1)}$ is a* NIZKPoK *system, the algorithm* Transfer *in Construction 4.1 is simulatable.*

Proof. (Sketch) We show the simulatability of our construction by first constructing a two-stage PPT simulator $\mathcal{S}$, and then proving that after running any polynomial $\varrho = \mathsf{poly}(\lambda)$ times, the distribution of $\{\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by $\mathcal{S}$ are statistically close to that of $\{\mathsf{Sig}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by Transfer. Due to space limitation, we defer the full proof in Appendix B.2. □

Below, we analyse the unforgeability of Construction 4.1. Before this, we first specify the corresponding commitment relation $\hat{L}_{q_1,q_2}$ as follows.

$$\begin{aligned}\hat{L}_{q_1,q_2} := \Big\{&\mathsf{comm} : \exists(m, q_1, q_2, \mathbf{R}) \text{ such that } m \in \mathcal{M},\\ &\mathbf{R} \in S_1^{k\times(\ell\cdot\tau)} \text{ and } \mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m\cdot\mathbf{G}; \mathbf{R})\Big\}.\end{aligned}$$

Lemma 4.6 (Unforgeability) *Suppose $\Pi^{(1)}$ is a rewinding-extractable* NIZKPoK *system in the random oracle model, assume that* M-SIS$_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ *problem and* M-SIS$_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ *problem are hard with*

$$\begin{aligned}\nu =&\alpha\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha\cdot N\sqrt{2\cdot\ell\cdot\tau}+1\end{aligned}$$

$$\begin{aligned}\nu' =&\alpha'\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha'\cdot N\sqrt{2\cdot\ell\cdot\tau}+2\sqrt{\kappa},\end{aligned}$$

where $\alpha' = \gamma'/\sqrt{2\cdot\Big(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\Big)\cdot N}$. Then our above lattice-based commitment-transferrable signature scheme is partially selectively unforgeable for the exact commitment relation $\hat{L}_{q_1,q_2}$, i.e., the advantage of any PPT *adversary $\mathcal{A}$ against the partially selective unforgeability game of* CTS *is at most*

$$\mathsf{Adv}^{unforge}_{\mathcal{A}}(\lambda) \le 2\mathsf{Adv}^{\mathsf{M\text{-}LWE}}_{\mathcal{A}} + \mathsf{Adv}^{unforge^*}_{\mathcal{A}}(\lambda).$$

Due to space limitation, we defer the detailed proof and the definition of $\mathsf{Adv}^{\mathrm{unforge}^*}_{\mathcal{A}}(\lambda)$ in Appendix B.3.

5 Efficient Straight-Line Extractable NIZKPoK System

In this section, we present a multi-theorem straight-line extractable NIZKPoK system $\Pi^{(2)}$ to prove the well-formedness of commitment comm output by CTS.Commit. In a high level way, we adopt the encrypt-and-prove paradigm, as in [2, 23, 41]. Particularly, this can be achieved by first encrypting the witness, and then proving that these encrypted message under the ciphertext satisfies the corresponding relation.

For clarity of presentation, below we describe in a modular way: first we present (i) the exact commitment relation $\hat{L}_{q_1,q_2}$, and then (ii) the concrete instantiation of encryption scheme, finally (iii) prove that the encrypted witness indeed satisfies $\hat{L}_{q_1,q_2}$.

5.1 Exact Commitment Relation $\hat{L}_{q_1,q_2}$

For the above Construction 4.1, we need to prove the exact commitment relation $\hat{L}_{q_1,q_2}$ (implicitly including the commitment public parameter in the crs):

$$\hat{L}_{q_1,q_2} := \Big\{ \mathsf{comm} : \exists (m, q_1, q_2, \mathbf{R}) \text{ such that } m \in \bar{\mathcal{M}}, \mathbf{R} \in S_1^{k\times(\ell\cdot\tau)} \text{ and } \mathsf{comm} = \mathsf{Commit}(\mathsf{params}, m\cdot\mathbf{G}; \mathbf{R}) \Big\}.$$

More precisely, we need to prove the following equations over R_{N,q_1} and R_{N,q_2}:

$$\mathsf{comm} := \mathbf{C} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \cdot \mathbf{R} + \begin{bmatrix} \mathbf{0}^{n\times(\ell\cdot\tau)} \\ m\cdot\mathbf{G} \end{bmatrix} \begin{matrix} \in R_{N,q_2}^{n\times(\ell\cdot\tau)} \\ \in R_{N,q_2}^{\ell\times(\ell\cdot\tau)} \end{matrix}. \tag{1}$$

Furthermore, we can easily transfer the above Equation (1) into the following equations.

$$\mathbf{C}_1 = \mathbf{A}_1 \cdot \mathbf{R} \bmod q_1, \quad \mathbf{C}_2 = \mathbf{A}_2 \cdot \mathbf{R} + m \cdot \mathbf{G} \bmod q_2, \tag{2}$$

where $\mathbf{R} \in S_1^{k\times(\ell\cdot\tau)}$, and $m \in \bar{\mathcal{M}}$, as defined in the setup algorithm of Construction 4.1. Moreover, (mod q_1) and (mod q_2) means the computations are conducted over R_{N,q_1} and R_{N,q_2}, respectively.

5.2 Concrete instantiation of PKE for the Encrypt-and-Prove Paradigm

For the encryption scheme, we choose to use a variant of standard Regev public-key encryption scheme with $R_d = \mathbb{Z}[x]/(X^d + 1)$ as the underlying ring. For completeness, we present it as follows.

Construction 5.1 (Encryption Scheme E) *The Ring-based Regev encryption scheme is as follows.*

- $\mathsf{KeyGen}(\lambda)$*: Given a security parameter λ, the algorithm conducts the following steps:*
  1. *Choose two integers d, q_{PKE}, where d is a power of 2, and q_{PKE} is a prime;*
  2. *Set $n_{\mathsf{PKE}}, m_{\mathsf{PKE}}, k_{\mathsf{PKE}}, q_{\mathsf{PKE}}$ be integers.*

3. *For the ring* $R_d = \mathbb{Z}[X]/(X^d+1)$*, and let* $R_{d,q_{\mathsf{PKE}}} = \mathbb{Z}_{q_{\mathsf{PKE}}}[X]/(X^d+1)$*,* $\hat{S}_2$ *be the set of elements from* R_d *with* ℓ_∞*-norm* ≤ 2*.*
4. *Sample* $\mathbf{A}_{\mathsf{PKE}} \xleftarrow{\$} R_{d,q_{\mathsf{PKE}}}^{n_{\mathsf{PKE}} \times m_{\mathsf{PKE}}}$*,* $\mathbf{S} \leftarrow \hat{S}_2^{k_{\mathsf{PKE}} \times n_{\mathsf{PKE}}}, \mathbf{E} \leftarrow \hat{S}_2^{k_{\mathsf{PKE}} \times m_{\mathsf{PKE}}}$*.*
5. *Compute* $\mathbf{B}_{\mathsf{PKE}} = \mathbf{S} \cdot \mathbf{A}_{\mathsf{PKE}} + 3 \cdot \mathbf{E} (\mathrm{mod} q_{\mathsf{PKE}})$*.*
6. *Output* $\mathsf{pk} := (\mathbf{A}_{\mathsf{PKE}}, \mathbf{B}_{\mathsf{PKE}})$*,* $\mathsf{sk} := \mathbf{S}$*.*

– $\mathsf{Enc}(\mathsf{pk}, \boldsymbol{\mu})$*: Given public key* pk *and the message vector* $\boldsymbol{\mu}^\top \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}}}$*, where each coefficient of* μ_i *is from* $\{-1, 0, 1\}$*, the algorithm conducts the following steps:*

1. *Sample* $\boldsymbol{r}_{\mathsf{PKE}} \xleftarrow{\$} \hat{S}_2^{m_{\mathsf{PKE}}}$*.*
2. *Compute* $\boldsymbol{c}_0 = \mathbf{A}_{\mathsf{PKE}} \cdot \boldsymbol{r}_{\mathsf{PKE}} \in R_{d,q_{\mathsf{PKE}}}^{n_{\mathsf{PKE}}}$*,* $\boldsymbol{c}_1 = \mathbf{B}_{\mathsf{PKE}} \cdot \boldsymbol{r}_{\mathsf{PKE}} + \boldsymbol{\mu} \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}}}$*.*
3. *Output* $\mathsf{ct} = (\boldsymbol{c}_0, \boldsymbol{c}_1)$*.*

– $\mathsf{Dec}(\mathsf{pk}, \mathsf{sk}, \mathsf{ct})$*: Given public key* pk*, secret key* sk *and the ciphertext* $\mathsf{ct} = (\boldsymbol{c}_0, \boldsymbol{c}_1)$*, where* $\boldsymbol{c}_0 \in R_{d,q_{\mathsf{PKE}}}^{n_{\mathsf{PKE}}}$*,* $\boldsymbol{c}_1 \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}}}$*, the algorithm conducts the following steps:*

1. *Compute* $\boldsymbol{\mu}' = \boldsymbol{c}_1 - \mathbf{S} \cdot \boldsymbol{c}_0 \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}}}$*.*
2. *Return* $\boldsymbol{\mu}' \mod 3$*.*

Correctness of Construction 5.1. Notice that for a properly formed ciphertext $\mathsf{ct} = (\boldsymbol{c}_0, \boldsymbol{c}_1)$, the correctness of decryption holds if the ℓ_∞-norm of $\boldsymbol{\mu}' = \boldsymbol{c}_1 - \mathbf{S} \cdot \boldsymbol{c}_0 = 3\mathbf{E} \cdot \boldsymbol{r}_{\mathsf{PKE}} + \boldsymbol{\mu} \mod q_{\mathsf{PKE}}$ is smaller than $q_{\mathsf{PKE}}/2$. In this case, we can directly compute $\boldsymbol{\mu}' \mod 3$ to recover $\boldsymbol{\mu}$. For this, we need to set $q_{\mathsf{PKE}} > 2(12 \cdot m_{\mathsf{PKE}} \cdot d + 1)$. Moreover, in order to encrypt the random vectors $\boldsymbol{r}_i \in R_N$ for $i \in [4]$ and the message $m \in R_d$, we need to set $k_{\mathsf{PKE}} = \ell \cdot \tau \cdot N/d \cdot k + 1$.

Security of Construction 5.1. Clearly, the $\mathsf{IND\text{-}CPA}$ security follows from the hardness of $\mathsf{M\text{-}LWE}_{q_{\mathsf{PKE}}, n_{\mathsf{PKE}}, m_{\mathsf{PKE}}, \hat{S}_2}$. So we set q_{PKE}, d, n_{PKE} and $m_{\mathsf{PKE}} = 2 \cdot n_{\mathsf{PKE}} + k_{\mathsf{PKE}}$ to achieve sufficient security. The concrete parameter setting are presented in the following Table 11.

Overall, it is easy for us to set parameters to obtain correctness and security simultaneously.

5.3 Proof of Witness Satisfying the Relation $\hat{\boldsymbol{L}}_{\boldsymbol{q_1}, \boldsymbol{q_2}}$

Following from [2], we can instantiate the required zero-knowledge proof through using LNP proof, i.e., Figure 10 in [41]. Particularly, we need to prove the knowledge of $\boldsymbol{r}_i \in R_N \cong R_d^t$ with small norm where $t = N/d$, $m \in R_N$ is binary polynomial with certain ℓ_1-norm, such that equations in (2) are set up.

Besides, we also need to prove the well-formedness of ct. This means the existence of vector $\boldsymbol{r}_{\mathsf{PKE}} \in R_d^{m_{\mathsf{PKE}}}$ that are small and satisfy the relations of the Enc algorithm of Construction 5.1 for the message $\boldsymbol{\mu} = (\boldsymbol{r}_1 \| \ldots \| \boldsymbol{r}_{\ell \cdot \tau} \| m)$,[19] with $\mathbf{R} = (\boldsymbol{r}_1, \ldots, \boldsymbol{r}_{\ell \cdot \tau})$.

[19] Similar to [41], according to the algebraic setting of our message space, we can just express $m(X) \in R_N$ as a single binary polynomial $m \in R_d$, even $R_N \cong R_d^{N/d}$. This will clearly improve the efficiency of the used PKE scheme and the corresponding LNP proof.

Concretely, we can first commit to the vector $(\boldsymbol{r}_{\mathsf{PKE}}\|\boldsymbol{r}_1\|\ldots\|\boldsymbol{r}_{\ell\cdot\tau}\|m)$ through using ABDLOP commitment (in the "Ajtai part") [41], and then prove several linear relations (from equations in (2) and the Enc algorithm of Construction 5.1), m are binary polynomial, and prove the following bounds:

$$(i)\ \|\boldsymbol{r}_{\mathsf{PKE}}\|_2 \leq 2\sqrt{d \cdot m_{\mathsf{PKE}}};\ (ii)\ \|\boldsymbol{r}_i\|_2 \leq \sqrt{kN};\ (iii)\ \|m\|_1 = \omega.$$

variable	description	instantiation
ρ	# of equations to prove	$t_0 = \ell \cdot \tau \cdot N/d \cdot (n+\ell)$
ρ_{eval}	# of evaluations with const. coeff. zero	1
v_e	# of exact norm proofs	$\ell \cdot \tau + 1$
v_d	# non-exact norm proofs	1
k_{bin}	length of the binary vector to prove	1
$\boldsymbol{s}_1$	committed message in the Ajtai part	$(\boldsymbol{r}_{\mathsf{PKE}}, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_{\ell\cdot\tau}, m)$
$\boldsymbol{m}$	committed message in the BDLOP part	$\emptyset$ (no message)
$f_1, \ldots, f_{t_0}$	equations to prove	Equations (2)
F_1	evaluation to prove const coeff. zero	$\sigma_{-1}(\sum_{i=0}^{d-1} X^i) \cdot m - \omega$
$\mathbf{E}_1$	public matrix for proving $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$[\mathbf{I}_{m_{\mathsf{PKE}}}\ \mathbf{0}\ \ldots\ \mathbf{0}\ 0]$
$\boldsymbol{v}_1$	public vector for proving $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$\mathbf{0}$
$\beta_1^{(e)}$	upper-bound on $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$2\sqrt{d \cdot m_{\mathsf{PKE}}}$
$\mathbf{E}_2$	public matrix for proving $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$[\mathbf{0}\ \mathbf{I}_{k\cdot N/d}\ \ldots\ \mathbf{0}\ 0]$
$\boldsymbol{v}_2$	public vector for proving $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$\mathbf{0}$
$\beta_2^{(e)}$	upper-bound on $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$\sqrt{N \cdot k}$
$\vdots$	$\vdots$	$\vdots$
$\mathbf{E}_i$	public matrix for proving $\|\mathbf{E}_i\boldsymbol{s} - \boldsymbol{v}_i\| \leq \beta_i^{(e)}$	$[\mathbf{0}\ \ldots\ \mathbf{I}_{k\cdot N/d}\ \ldots\ \mathbf{0}\ 0]$
$\boldsymbol{v}_i$	public vector for proving $\|\mathbf{E}_i\boldsymbol{s} - \boldsymbol{v}_i\| \leq \beta_i^{(e)}$	$\mathbf{0}$
$\beta_i^{(e)}$	upper-bound on $\|\mathbf{E}_i\boldsymbol{s} - \boldsymbol{v}_i\| \leq \beta_i^{(e)}$	$\sqrt{N \cdot k}$
$\vdots$	$\vdots$	$\vdots$
$\mathbf{E}_{\ell\cdot\tau}$	public matrix for proving $\|\mathbf{E}_{\ell\cdot\tau}\boldsymbol{s} - \boldsymbol{v}_{\ell\cdot\tau}\| \leq \beta_{\ell\cdot\tau}^{(e)}$	$[\mathbf{0}\ \mathbf{0}\ \ldots\ \mathbf{I}_{k\cdot N/d}\ 0]$
$\boldsymbol{v}_{\ell\cdot\tau}$	public vector for proving $\|\mathbf{E}_{\ell\cdot\tau}\boldsymbol{s} - \boldsymbol{v}_{\ell\cdot\tau}\| \leq \beta_{\ell\cdot\tau}^{(e)}$	$\mathbf{0}$
$\beta_{\ell\cdot\tau}^{(e)}$	upper-bound on $\|\mathbf{E}_{\ell\cdot\tau}\boldsymbol{s} - \boldsymbol{v}_{\ell\cdot\tau}\| \leq \beta_{\ell\cdot\tau}^{(e)}$	$\sqrt{N \cdot k}$
$\mathbf{D}_1$	public matrix for proving $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$q_{\mathsf{PKE}}^{-1} \cdot \begin{bmatrix} \mathbf{A}_{\mathsf{PKE}}, \mathbf{0} \\ \mathbf{B}_{\mathsf{PKE}}, \mathbf{I}_{k_{\mathsf{PKE}}} \end{bmatrix}$
$\boldsymbol{u}_1$	public vector for proving $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$q_{\mathsf{PKE}}^{-1} \cdot \begin{bmatrix} \boldsymbol{t}_0 \\ \boldsymbol{t}_1 \end{bmatrix}$
$\beta_1^{(d)}$	upper-bound on $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$(d \cdot m_{\mathsf{PKE}} + 1)\sqrt{(n_{\mathsf{PKE}} + 1) \cdot d}$
$\mathbf{E}_{\mathsf{bin}}$	matrix for proving binary	$[\mathbf{0}\ \ldots\ \mathbf{0}\ 1]$
$\boldsymbol{v}_{\mathsf{bin}}$	vector for proving binary	$\mathbf{0}$

Table 6. Instantiation of Figure 10 of [41] for multi-theorem straight-line extractable $\mathsf{NIZKPoK}$.

Similar to the concrete instantiations in [2,41], we just explain how to instantiate the protocol in Figure 10 of [41], instead of presenting the detailed steps of

LNP proof. Particularly, we show in Table 6 how to instantiate LNP proof, i.e., the protocol in Figure 10 of [41].

Overall, in order to help the readers to understand this process of instantiating LNP proof, we try to use the same notations as in [41] to indicate the corresponding variables. However, due to notation abusing, we use the additional superscript star to distinguish symbols of LNP instantiation from these of our CTS. Particularly, we list the concrete parameter selection in Table 11[20], among which we instantiate the variables $d^*, \kappa^*, l^*, \nu^*, \gamma_1, \gamma_2, \gamma_d$, and γ_e as in the Section 6.4 of [41] for group signature. This means our LNP instantiation has the same underlying ring structure for ABDLOP and the related proof, and the same expected repetition times $M^* = 7$. Notice that, for the case of $d \geq 512$, we set $\kappa^* = 1$ and η^* such that $\binom{d/2}{\eta^*} \times 2^{\kappa} \geq 2^{128}$, rather than through the experimental evaluation as in [41]. This is because conditioned on $d \geq 512$, the challenge space $\mathcal{C}$ for LNP proof is sufficiently large, i.e., $|\mathcal{C}| \geq 2^{128}$, even with the ℓ_∞-norm $\kappa^* = 1$.

We set $m_1 = (m_{\mathsf{PKE}} + \ell \cdot \tau \cdot k) \cdot N/d + 1$ and $v_e = \ell \cdot \tau + 1$ to commit to $(\boldsymbol{r}_{\mathsf{PKE}}, \boldsymbol{r}_1, \ldots, \boldsymbol{r}_{\ell \cdot \tau}, m)$, where v_e indicates that the number of exact ℓ_2-norm bounds we need to prove. Then, the parameters n, m_2, γ, D are chosen to make the underlying M-SIS and M-LWE problems for LNP proof have sufficient hardness, and improve the proof size, through using the compression technique as in [41].

Below, we elaborate how to choose suitable modulus q_{LNP} for our instantiation of LNP proof. Generally, our proof involves three types of equations: (i) the left side of Equations (2) with modulus q_1; (ii) the right side of Equations (2) with modulus q_2; (iii) the encryption equation of Construction 5.1 with modulus q_{PKE}. For each one, we have two approaches to complete the proof: (a) multiply another prime to both sides of one equation, and then prove the linear relations on the committed vectors over the multiplication of these two primes; and (b) directly express the equation modulo a larger modulus that is co-prime to the original modulus, and then prove the approximate ℓ_2-norm bound on linear computation of the committed vectors. Here, in order to simplify the relations to be proven, we set q_{LNP} as a multiplication of q_1 and q_2, with which we directly prove Equations (2) through using the above mentioned approach (a). With such q_{LNP}, we can prove the encryption equation of Construction 5.1 through using the above mentioned approach (b). Of course, it is possible for us to choose other much smaller q_{LNP}. For example, we can prove the left side of Equations (2), through choosing to use the above mentioned approach (b), rather than the approach (a). But this will not significantly affect our final efficiency parameters.

6 Application to Anonymous Credentials

In this section, we present how to construct Anonymous Credentials from CTS and NIZKPoK. Particularly, we first recall the definition and security requirement

[20] Due to space limitation, we present this table in Section C.

of the basic Anonymous Credentials in [37], and then describe the construction. Then we describe how to extend the basic scheme into one that supports some attribute settings.

6.1 Definition and Security of Anonymous Credentials

We use the formulation of Anonymous Credentials by Lysyanskaya [37]. A basic credential system has *users*, *organizations*, and *verifiers* as types of players. Users are entities that receive credentials. Organizations are entities that issue the credentials of the users. Finally, verifiers are entities that verify credentials of the users. Specifically, the system is defined as follows:

- AC.Setup: System parameters params are generated, users generate their secret key usk, and organizations generate their public and secret keys $(\mathsf{pk}_O, \mathsf{sk}_O)$;
- AC.Registration: A user generates a pseudonym nym, and sends it to an organization. The user's private input is usk. the organization does not have any private input.
- AC.Issue: As a result of this protocol, a user obtains a credential from an organization without revealing his private input, just based on his pseudonym nym. The user's private input to the protocol is his usk. The organization's private input is its secret key sk_O. And the user's private output is the credential Cred;
- AC.Prove: The user who is known to one organization O_1 under nym_1, and to a verifier under nym_2, and a credential Cred from O_1, proves to the verifier that he has a credential from O_1. The user's private input to this protocol consists of $(\mathsf{usk}, \mathsf{nym}_1, \mathsf{Cred})$, while the values nym_2 and pk_{O_1} are public;
- AC.Verify: The verifier verifies if the user possesses a credential Cred with respect to nym_2 from O_1 or not.

We follow the security formulation of [7] – an anonymous credential should satisfy unforgeability, anonymity, and unlikability. Intuitively, unforgeability requires that an adversary cannot provide a valid proof of credential Cred^* with respect to a pseudonym nym^* of some usk^* that he has never received a credential from an organization.

Anonymity, informally, requires two different privacy properties: (1) privacy against an organization: the organization cannot distinguish any two different users with two different private inputs in the registration process, and (2) privacy against a verifier: the proof of credential leaks no information other than the validity of owning a credential with respect to the pseudonym.

Unlinkability requires that the adversary cannot distinguish whether (nym_1, π_1) and (nym_2, π_2) are from the same user or not, where π_1, π_2 are two proofs of credentials with respect to nym_1 and nym_2, respectively.

6.2 Anonymous Credentials from CTS

Now we show how to construct an anonymous credential system from a secure CTS and a zero-knowledge proof of knowledge of commitment opening.

Building blocks. Suppose we are given a secure commit-transferable signature scheme $(\mathsf{CTS.Setup}, \mathsf{CTS.Commit}, \mathsf{CTS.Randomize}, \mathsf{CTS.KeyGen}, \mathsf{CTS.Sign}, \mathsf{CTS.Transfer}, \mathsf{CTS.Verify})$ as in Construction 4.1, and an efficient multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ $\Pi = (\mathsf{NIZKSetup(params)}, \mathsf{NIZKProve}, \mathsf{NIZKVerify}, \mathsf{SimSetup})$ for the following commitment relation $\hat{L}$ as Definition 3.3.

$$\hat{L} =: \Big\{\mathsf{comm} : \exists(m, \mathsf{Rand}) \text{ such that } \mathsf{comm} = \mathsf{Commit}(\mathsf{params}, x, \mathsf{Rand})\Big\}.$$

Then we can construct an anonymous credential system as follows:

Construction 6.1 (Anonymous Credential) *The anonymous credential scheme can be constructed in the following way.*

- $\mathsf{AC.Setup}$*: System runs* $\mathsf{CTS.Setup}$ *to obtain* $\mathsf{CTS.params}$*, and runs* $\mathsf{NIZKSetup(params)}$ *to obtain* $\mathsf{NIZKpara}$*. An honest user* U *generates her secret key* usk *by sampling* $\mathcal{M}_{\mathsf{params}}$*. An honest organization* O *generates its keys as follows:* $(\mathsf{sk}_O, \mathsf{pk}_O) \leftarrow \mathsf{CTS.KeyGen(params)}$*;*
- $\mathsf{AC.Registration}$*: The user* U *first samples* $\mathsf{Rand} \leftarrow \mathcal{R}_{\mathsf{params}}$ *and generates a commitment* $\mathsf{comm} = \mathsf{CTS.Commit(params, usk, Rand)}$*. Then* U *generates an* NIZK *proof* π *by running* $\mathsf{NIZKProve(params, comm, usk, Rand)}$*. Furthermore,* U *sends* $\mathsf{nym} = (\mathsf{comm}, \pi)$ *as the pseudonym to the organization* O*. Finally,* O *would run* $\mathsf{NIZKVerify}$ *to check whether the pseudonym (commitment) is properly formed;*
- $\mathsf{AC.Issue}$*: Suppose that a user* U *is known to organization* O *under pseudonym* $\mathsf{nym} = (\mathsf{comm}, \pi)$[21]*.* O *computes* $\sigma \leftarrow \mathsf{CTS.Sign(params, sk}_O, \mathsf{comm})$ *and gives* $\mathsf{Cred} := \sigma$ *to* U*;*
- $\mathsf{AC.Prove}$*: User* U *samples* Rand'*, and runs* $\mathsf{comm}' \leftarrow \mathsf{CTS.Randomize(comm, Rand, Rand}')$*. Then she computes* $\sigma' = \mathsf{Transfer(params, pk}_{O_1}, \mathsf{usk, Rand, Rand}', \sigma)$*, which (by correctness of the* CTS*) is a signature under* pk_{O_1} *on the commitment* comm'*. Next, she gives the verifier the values* σ' *and* $\mathsf{nym}' = (\mathsf{comm}', \pi')$*, where* π' *is an* NIZK *proof that* comm' *is properly formed as well;*
- $\mathsf{AC.Verify}$*: The verifier runs* $\mathsf{Verify(params, pk}_{O_1}, \mathsf{comm}', \sigma')$ *and the NIZK verifier of* π' *on input* $(\sigma', \mathsf{nym}' = (\mathsf{comm}', \pi'))$ *to verify* U*'s credential on the new pseudonym* nym'*.*

Security of the anonymous credential system follows from the security of CTS and $\mathsf{NIZKPoK}$ Π with respect to the commitment relation $\hat{L}$.

Theorem 6.2 *Assuming that* CTS *is secure for the exact commit relation, and* Π *is a secure multi-theorem straight-line extractable* $\mathsf{NIZKPoK}$ *system for* $\hat{L}$*, Construction 4.1 is a secure anonymous credential system.*

Proof. (Sktech) Intuitively, the anonymity against the organization follows from the security of $\mathsf{NIZKPoK}$ and hiding of the commitment scheme, and that against the verifier follows from the simulatability of the CTS, as the transferred signature does not leak information beyond the validity. The unlinkability follows by

[21] Here, we implicitly assume this π has been successfully verified, otherwise this nym will be invalid.

the hiding property of the re-randomized commitments and the simulatability of the CTS, so that any user cannot relate two pairs of pseudonym-proofs.

To prove unforgeability, we rely on the $\mathsf{NIZKPoK}$ extractor (of the commitment relation) and the unforgeability of CTS. Assuming that there exists an adversary $\mathcal{A}$ that forges a valid proof of the anonymous credential, then we can construct a reduction $\mathcal{B}$ that breaks CTS unforgeability in the following way. $\mathcal{B}$ first simulates the $\mathsf{NIZKPoK}$ and extracts $\mathcal{A}$'s (m, Rand) in the commitment of the registration queries, from the ZKPoK proof he provides. Then when $\mathcal{A}$ makes an issue query, $\mathcal{B}$ makes a signing query to the CTS challenger. As $\mathcal{B}$ has extracted the witness from the commitment, $\mathcal{B}$ can make a valid CTS signing query. It is easy to verify that as long as $\mathcal{A}$ can forge a valid proof, $\mathcal{B}$ can break the CTS unforgeability. We note that if the $\mathsf{NIZKPoK}$ is with respect to the exact commitment relation, then $\mathcal{B}$ breaks CTS unforgeability with respect to the exact relation. If the proof system is with respect to the relaxed commitment relation, then $\mathcal{B}$ breaks CTS unforgeability with respect to the relaxed relation. □

6.3 Extension to Attribute-based Settings

In the above basic anonymous credential system, the user's secret value usk is her id or some secret key. In a more general setting of attribute-based credentials, the user's secret value can include additional attributes, denoted as $\mathbf{att} = (\mathsf{att}_1, \ldots, \mathsf{att}_\ell)$ where each att_i is some small integer (or a short bit string). The user might wish to reveal some subset of the attributes to any party, e.g., an organization or a verifier, while keep the other attributes and the secret key/id private. This property of *chosen disclosure of attributes* has been identified useful in the literature [12, 18, 29, 33]. We observe that our system can easily be extended to support such an extension. Below we elaborate.

In the basic scheme, the user sets the message as the secret value, i.e., $m = \mathsf{usk}$, and generates a BDLOP commitment $\mathsf{comm} = \mathsf{Commit}(m)$ for the CTS as a pseudonym. Then the user proves well-formedness of the commitment and then the organizations would sign on the commitment. To generalize to the attribute setting, we can use m to encode usk and the attributes $\mathbf{att}$, simultaneously. For example, for $m = \sum_{i=0}^{N-1} m_i X^i \in \mathbb{Z}_{q_2}[X]/\langle X^N + 1\rangle$, we can use the coefficients to encode $(\mathsf{usk} \| \mathbf{att})$ (for simplicity we assume N to be the bit-length of $(\mathsf{usk} \| \mathbf{att})$, i.e., $N = |\mathsf{usk}| + |\mathbf{att}|$). Then the user generates $\mathsf{comm} = \mathsf{Commit}(m)$ as before, yet with m under such an encoding.

To disclose some subset of attribute, say $\mathbf{att}_\mathcal{I} = \{\mathsf{att}_i\}_{i \in \mathcal{I}}$ for $\mathcal{I} \subseteq [N]$, the user can prove well-formedness of the commitment and additionally that the coefficients of m corresponding to these attributes are consistent with $\mathbf{att}_\mathcal{I}$. To achieve this, we observe that it suffices to use the following protocol $\Pi_{\mathsf{Disclosure}}$ in Table 17, which proves well-formedness of a BDLOP commitment $\mathsf{Commit}(m)$ and as well consistency that a certain subset of coefficients in m are the same as those were disclosed. To achieve this, we present the interactive protocol

adapted from the ENS and LNP proof [25, 41] in Table 17[22], which can be made non-interactive easily using the Fiat-Shamir Transform.

We notice that the above approach supports the case when we can embed the usk and the attribute into one single ring element. A noticeable advantage is that the proof size is essentially independent of the cardinality of $\mathcal{I}$, i.e., the number of disclosed attributes. However, on the other hand, an oblivious disadvantage is that the total bit-sizes of the embedded usk and attribute will be strictly restricted, such as just 128 bits under our second parameter setting in Tables 10 and 2 for efficient implementation, if we adopt the embedding approach as in the first step of the Setup algorithm of Construction 4.1. Fortunately, we can easily enlarge this space through modifying embedding approach. More specifically, let $\mathcal{B}$ be the set of non-zero binary polynomials in R_{d,q_2}. Then, we can formally define the message space as $\bar{\mathcal{M}} := \{m = (m_1 \| \dots \| m_t) \in R_{N,q_2} : m_i \in \mathcal{B} \text{ and } \|m\|_1 = \omega\}$ with $t = N/d$. Here, we denote the part of m_1 as usk, and $(m_2, \dots, m_t)$ as attributes[23]. Then, through using LNP proof, we can prove the well-formedness of such type of message, with almost the same efficiency as original one in Construction 4.1. Technically, we use $\|m\|_1 = \omega$ to restrict the size of the identity space, which is important to obtain efficient adaptive construction from complexity leveraging. And $(m_2, \dots, m_t)$ allows us to encode sufficiently large attributes for each user.

In our particular parameter selection for selective (or adaptive) construction, the ring dimension d for LNP is 512 (or 1024), and ring dimension N for CTS is 2048 (or 4096). This allows that the final anonymous credentials system holds sufficiently large number of different users, and each user has sufficient attributes.

[22] Due to space limitation, we present this table in Section E.

[23] Notice that if d is sufficiently large, we encode many attributes into one m_i.

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Roadmap of the Appendix

Here we present a roadmap so that the readers can find relevant texts more easily. In Section A, we present additional preliminaries. In Section B, we present the security analysis of our selectively secure CTS. Going a step further, Section C provides parameter setting details for CTS construction in Section 4.1 and $\mathsf{NIZKPoK}$ system in Section 5. In Section E, we present the interactive protocol to disclose certain coefficients of a committed polynomial, which can extend the basic anonymous credentials system to attribute-based on supporting *chosen disclosure of attributes*.

Notice that the CTS in Section 4 is just proven to be selectively secure. This means we need to use the approach of complexity leveraging to achieve adaptive security, which will induce security loss related to the size of message space. As a technical supplement, in Section D, we present how to construct an adaptively secure CTS without the complexity leveraging argument, and determine concrete parameters.

A Supplementary Material for Section 2

A.1 Lattices with Algebra Structure

Below, we use R to denote a polynomial ring of the form $\mathbb{Z}[X]/(\Phi_m(X))$, where $\Phi_m(X)$ is the m^{th} cyclotomic polynomial, and denote $N = \varphi(m)$. For an integer $q \in \mathbb{Z}$, we also consider the quotient ring $R_q = R/qR$. Any element in R can be considered as a vector of its coefficients. Namely, an element $a = \sum_{i\in[N]} a_i x^i \in R$ can be seen as the vector $\boldsymbol{a} = (a_0, ..., a_{N-1})$. We call this map as coefficient embedding (denoted as $\mathsf{Coeffs}(\cdot)$). Furthermore, we can also represent a ring element $a \in R$ as a matrix in $\mathbb{Z}^{N\times N}$ by the following map $\mathsf{Rot} : R \to \mathbb{Z}^{N\times N}$:

$$\mathsf{Rot}(a) := \begin{bmatrix} \mathsf{Coeffs}(a)^\top \\ \mathsf{Coeffs}(xa \bmod \Phi(x))^\top \\ \vdots \\ \mathsf{Coeffs}(x^{N-1}a \bmod \Phi(x)\ )^\top \end{bmatrix}.$$

Furthermore, we extend this map to ring vectors and matrices by applying it entry-wise, i.e., for a vector $\boldsymbol{a}^\top = (a_1, \ldots, a_\ell) \in R^\ell$, we define $\mathsf{Rot}(\boldsymbol{a}^\top) = [\mathsf{Rot}(a_1)|\ldots|\mathsf{Rot}(a_\ell)] \in \mathbb{Z}^{n\times n\ell}$, and the map for matrices can be defined similarly. In the case of power of 2 cyclotomic rings, i.e., $\Phi(x) = x^N + 1$ for n being some power of 2, the above rotation matrix $\mathsf{Rot}(a)$ is the anti-cyclic matrix.

If $\mathcal{I}$ is an ideal in the polynomial ring R, then it is also an additive subgroup of $\mathbb{Z}^N$, and therefore a N-dimensional lattice. Such lattices are therefore sometimes referred to as *ideal lattices*. Similarly, we can also define the *module lattices* $M \subseteq (\mathbb{Q}[X]/(\Phi_m(X)))^\ell$ as a ℓN-dimensional lattice. We simply denote *ideal lattices* or *module lattices* as Λ.

Discrete Gaussian distribution. We now define the Gaussian distribution used in our schemes.

Definition A.1 *The discrete Gaussian distribution on $\Lambda \subseteq R^\ell$ centered around $\boldsymbol{v} \in R^\ell$ with standard deviation $s > 0$ is given by* $D_{\Lambda,\boldsymbol{v},s}(\boldsymbol{x}) = \frac{e^{-\|\boldsymbol{x}-\boldsymbol{v}\|^2/2s^2}}{\sum_{\boldsymbol{z}\in\Lambda} e^{-\|\boldsymbol{z}-\boldsymbol{v}\|^2/2s^2}}$. *When it is centered around $\mathbf{0}$, we denote $D_{\Lambda,s}$ for short.*

Specifically, for ring vector $\boldsymbol{x}$, we write $\boldsymbol{x} \leftarrow D_{\Lambda,s}$ to mean that $\boldsymbol{x} \in \Lambda \subseteq R^\ell$ and every coefficient of each component $x_i \in R$ is distributed according to $D_{\mathbb{Z},s}$. Then, we have the following properties.

Lemma A.2 ([40]) *Let D_s is a discrete Gaussian distribution over the ring R. Then for $\boldsymbol{x} \leftarrow D_s^\ell$, it holds* $\Pr\left[\|\boldsymbol{x}\| > t \cdot s\sqrt{\ell N}\right] \leq \left(te^{\frac{1-t^2}{2}}\right)^{\ell N}$

For positive integers δ and $k = \lceil \log_\delta(q) \rceil$, let $\boldsymbol{g}_\delta^\top = [1|\delta|\delta^2|...|\delta^{k-1}] \in R^k$ be the gadget matrix. Then we have the following lemmas.

Lemma A.3 ([46]) *There exists an efficient algorithm that on input ring vector $\boldsymbol{a} \in R_q^\ell$ such that $\mathsf{Rot}(\boldsymbol{a}^\top) \in \mathbb{Z}^{N\times N\ell}$ is full-rank, elements $x \in R_q^*, u \in R_q$ and matrix $\mathbf{R} \in R_q^{\ell\times k}$, outputs a random sample $\boldsymbol{r} \in R^{\ell+k}$ from a distribution that is statistically close to $D_{\Lambda_q^u[\boldsymbol{a}^\top|\boldsymbol{a}^\top\mathbf{R}+x\cdot\boldsymbol{g}_\delta^\top],\sigma}(\boldsymbol{x})$, where $\sigma \geq 2\sqrt{\delta^2+1}(s_1(\mathbf{R})+1)$.*

Lemma A.4 ([46]) *For $\boldsymbol{g}_\delta^\top = [1|\delta|\delta^2|...|\delta^{k-1}] \in R^k$, there exists a deterministic polynomial time algorithm $\mathbf{G}^{-1}$ which takes input $\boldsymbol{u} \in R_q^k$, and outputs $\mathbf{R} \leftarrow \mathbf{G}^{-1}(\boldsymbol{u}^\top)$ such that $\boldsymbol{g}_\delta \cdot \mathbf{R} = \boldsymbol{u}^\top$, such that $s_1(\mathbf{R}) \leq kN\delta$.*

We here recall the definition of smoothing parameter of a lattice and its upper bound as follow.

Definition A.5 ([47]) *For any n-dimensional lattice Λ and positive real $\epsilon_s > 0$, the smoothing parameter $\eta_{\epsilon_s}(\Lambda)$ is the smallest real $s > 0$ such that $\rho_{1/s}(\Lambda^*\backslash\{0\}) \leq \epsilon_s$, where Λ^* is the dual lattice of Λ.*

Lemma A.6 (Generalization of Lemma 2.6 in [45] to ring setting) *For any primitive matrix $\mathbf{P} \in R^{\ell\times k}$, positive reals $\alpha, \sigma > 0$, and negligible ε, if $\mathbf{P}\cdot\mathbf{P}^\top = \alpha^2 \cdot \mathbf{I}$ and $\eta_\varepsilon(\ker(\mathbf{P})) \leq \sigma$, then $\mathbf{P} \cdot D_\sigma^{kN} \overset{s}{\approx} D_{\alpha\sigma}^{\ell N}$.*

From Lemmas A.3 and A.6, we have the following lemma.

Lemma A.7 *There exists an efficient algorithm that on input ring vectors $\boldsymbol{a}_1 \in R_q^{\ell_1}, \boldsymbol{a}_2 \in R_q^{\ell_2}$ such that $\mathsf{Rot}([\boldsymbol{a}_1^\top|\boldsymbol{a}_2^\top]) \in \mathbb{Z}^{N\times N(\ell_1+\ell_2)}$ is full-rank, elements $x, c \in R_q^*, u \in R_q$ with $\|c\|_2 \leq \tau$ and matrices $\mathbf{R}_1 \in R_q^{\ell_1\times k}, \mathbf{R}_2 \in R_q^{\ell_2\times k}$, outputs a random sample $\boldsymbol{r} \in R^{\ell_1+\ell_2+k}$ from a distribution that is statistically close to $D_\sigma(\Lambda_q^u[\boldsymbol{a}_1^\top|\boldsymbol{a}_1^\top\mathbf{R}_1 + \boldsymbol{a}_2^\top\mathbf{R}_2 + x\cdot\boldsymbol{g}_\delta^\top|\boldsymbol{a}_2^\top])$, where $\sigma \geq 2\sqrt{\delta^2+1}(s_1(\begin{bmatrix}\mathbf{R}_1\\ \mathbf{R}_2\end{bmatrix})+1)$.*

Proof. Given the vector $[\boldsymbol{a}_1^\top|\boldsymbol{a}_1^\top\mathbf{R}_1 + \boldsymbol{a}_2^\top\mathbf{R}_2 + x\cdot\boldsymbol{g}_\delta^\top|\boldsymbol{a}_2^\top] \in R_q^{\ell_1+\ell_2+k}$, consider matrix

$$\mathbf{P} = \begin{pmatrix} \mathbf{I}_{\ell_1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_k \\ \mathbf{0} & \mathbf{I}_{\ell_2} & \mathbf{0} \end{pmatrix} \in R_q^{(\ell_1+\ell_2+k)\times(\ell_1+\ell_2+k)},$$

we have $[\boldsymbol{a}_1^\top|\boldsymbol{a}_2^\top|\boldsymbol{a}_1^\top\mathbf{R}_1+\boldsymbol{a}_2^\top\mathbf{R}_2+x\cdot\boldsymbol{g}_\delta^\top]=[\boldsymbol{a}_1^\top|\boldsymbol{a}_1^\top\mathbf{R}_1+\boldsymbol{a}_2^\top\mathbf{R}_2+x\cdot\boldsymbol{g}_\delta^\top|\boldsymbol{a}_2^\top]\cdot\mathbf{P}$.

Let $\ell=\ell_1+\ell_2$, $\boldsymbol{a}^\top=[\boldsymbol{a}_1^\top|\boldsymbol{a}_2^\top]\in R_q^\ell$, and $\mathbf{R}=\begin{bmatrix}-\mathbf{R}_1\\-\mathbf{R}_2\end{bmatrix}$. Clearly, we have $[\boldsymbol{a}^\top|\boldsymbol{a}_1^\top\mathbf{R}_1+\boldsymbol{a}_2^\top\mathbf{R}_2+x\cdot\boldsymbol{g}_\delta^\top]\cdot\begin{bmatrix}\mathbf{R}\\1\end{bmatrix}=x\cdot\boldsymbol{g}_\delta^\top$, where x and 1 are invertible ring elements. Hence, we can view this matrix $\mathbf{R}$ as the $\mathbf{G}$-trapdoor.

Therefore, by Lemma A.3, we can sample vector $\boldsymbol{r}\in R^{\ell+k}$ such that $[\boldsymbol{a}^\top|\boldsymbol{a}^\top\mathbf{R}+x\cdot\boldsymbol{g}_\delta^\top]\cdot\boldsymbol{r}=u\ (\bmod q)$, and the distribution of $\boldsymbol{r}$ is statistically close to $\mathcal{D}_\sigma(\Lambda_q^u[\boldsymbol{a}^\top|\boldsymbol{a}^\top\mathbf{R}+x\cdot\boldsymbol{g}_\delta^\top])$, where $\sigma\geq 2\sqrt{\delta^2+1}(s_1(\mathbf{R})+1)$. As a result, $[\boldsymbol{a}_1^\top|\boldsymbol{a}_1^\top\mathbf{R}_1+\boldsymbol{a}_2^\top\mathbf{R}_2+x\cdot\boldsymbol{g}_\delta^\top|\boldsymbol{a}_2^\top]\cdot\mathbf{P}\cdot\boldsymbol{r}=u\ (\mathrm{mod}q)$. Furthermore, by Lemma A.6, the distribution of $\mathbf{P}\cdot\boldsymbol{r}$ is statistically close to $D_\sigma(\Lambda_q^u[\boldsymbol{a}_1^\top|\boldsymbol{a}_1^\top\mathbf{R}_1+\boldsymbol{a}_2^\top\mathbf{R}_2+x\cdot\boldsymbol{g}_\delta^\top|\boldsymbol{a}_2^\top])$ (it's easy to see $\mathbf{P}\cdot\mathbf{P}^\top=\mathbf{I}$ and $\eta_\varepsilon\leq\sigma$). This completes the proof. □

In this paper, we use the following sampling algorithm. The following lemma have been established in a sequence of works.

Lemma A.8 ([1, 31]) *Given integers $n\geq 1, q\geq 2$ there exists some $m=m(n,q)=O(n\log q)$, there exists a sampling algorithm $\mathsf{SamplePre}(\mathbf{A},\mathbf{T_A},\boldsymbol{u},s)$, that takes as input: (1) a rank-n matrix $\mathbf{A}\in\mathbb{Z}_q^{n\times m}$, (2) a "short" basis $\mathbf{T_A}$ for lattice $\Lambda_q^\perp(\mathbf{A})$, a vector $\boldsymbol{u}\in\mathbb{Z}_q^n$, (3) a Gaussian parameter $s>\|\widetilde{\mathbf{T_A}}\|\cdot\omega(\sqrt{\log m})$; then outputs a vector $\boldsymbol{r}\in\mathbb{Z}^m$ distributed statistically close to $D_{\Lambda_q^{\boldsymbol{u}}(\mathbf{A}),s}$.*

We note that when $\mathbf{A}\in R_q^{\ell\times k}$ is a ring matrix, and $\mathbf{T_A}$ is the trapdoor for $\mathbf{A}$, the $\mathsf{SamplePre}$ algorithm also works by taking $\mathbf{A}$ as a matrix in $\mathbb{Z}_q^{\ell N\times kN}$, which is the coefficient embedding of $\mathbf{A}$.

A.2 Rejection Sampling

Lemma A.9 (Rejection Sampling) *Let V be a subset of $\mathbb{R}^m$ in which all elements have norms less than T, and $h:V\to[0,1]$ be a probability distribution. Let $\sigma=\eta T$ for $\eta=O(\sqrt{\lambda})$ and*

$$M=\exp\left(\sqrt{\frac{2(\lambda+1)}{\log e}}\cdot\frac{1}{\eta}+\frac{1}{2\eta^2}\right)=O(1).$$

Now, sample $\boldsymbol{v}\xleftarrow{\$}h$ and $\boldsymbol{y}\xleftarrow{\$}D_\sigma^m$, set $\boldsymbol{z}=\boldsymbol{y}+\boldsymbol{v}$, and run $b\leftarrow\mathsf{Rej}(\boldsymbol{z},\boldsymbol{v},\sigma)$ in Table 7. Then, the probability that $b=0$ is at least $\frac{1-2^{-\lambda}}{M}$. And conditioned on $b=0$, the distribution of $(\boldsymbol{v},\boldsymbol{z})$ is within statistical distance of $\frac{2^{-\lambda}}{M}$ of the product distribution $h\times D_\sigma^m$.

A.3 Security of Commitment

A secure commitment scheme requires the two properties: hiding and binding.

$\underline{\mathsf{Rej}(\boldsymbol{z}, \boldsymbol{v}, \sigma)}$
01 $u \xleftarrow{\$} [0,1)$
02 If $u > \frac{1}{M} \cdot \exp(\frac{-2\langle \boldsymbol{z}, \boldsymbol{v} \rangle + \|\boldsymbol{v}\|^2}{2\sigma^2})$
03 return 0 (i.e. abort)
04 Else
05 return 1 (i.e. non-abort)

Table 7. Rejection Sampling.

Definition A.10 (Hiding, [6]) *We say that a commitment scheme* (CKeyGen, Commit, Open) *with message space* $\mathcal{M}$ *and randomness space* $\mathcal{R}$ *is hiding, if for all adversaries* $\mathcal{A}$*, the probability (over the randomness of* CKeyGen,Commit*, and* $\mathcal{A}$*) that* $b' = b$ *in the following experiment is negligible:*

Parameter setup *The challenger sets up* $\mathsf{params} \leftarrow \mathsf{CKeyGen}(1^\lambda)$*, and send* params *to* $\mathcal{A}$*.*

Message selection $\mathcal{A}(\mathsf{params})$ *selects two messages* $m_0, m_1 \in \mathcal{M}$*, and then sends them to* $\mathcal{C}$*.*

Commitments *The challenger computes* $\mathsf{comm}_b = \mathsf{Commit}(\mathsf{params}, m_b; r)$*, where* $b \xleftarrow{\$} \{0,1\}$*,* $r \xleftarrow{\$} \mathcal{R}$*, and sends* comm_b *to* $\mathcal{A}$*.*

Output $\mathcal{A}$ *outputs a bit* b'*.*

If $\mathcal{A}$ *are restricted to polynomial-time algorithms, then the scheme is called computationally hiding. If there is no restriction on the running time of such algorithms, then the scheme is statistically hiding.*

Definition A.11 (Binding, [6]) *We say that a commitment scheme* (CKeyGen, Commit, Open) *with message space* $\mathcal{M}$ *and randomness space* $\mathcal{R}$ *is binding, if for all adversaries* $\mathcal{A}$*, the probability*

$$\Pr\left[\begin{array}{l} \mathsf{params} \leftarrow \mathsf{CKeyGen}(1^\lambda), \\ (m, m', r, r', \mathsf{comm}) \leftarrow \mathcal{A}(\mathsf{params}) \\ s.t.\ m \neq m' \wedge \mathsf{Open}(\mathsf{params}, m, \mathsf{comm}, r) = \\ \mathsf{Open}(\mathsf{params}, m', \mathsf{comm}, r') = 1 \end{array}\right] \leq \mathsf{negl}(\lambda),$$

where the probability is taken over the randomness of CKeyGen *and* $\mathcal{A}$*.*

Similarly, if $\mathcal{A}$ *are restricted to polynomial-time algorithms, then the scheme is called computationally binding. If there is no restriction on the running time of such algorithms, then the scheme is statistically binding.*

A.4 Algebraic Structure of Cyclotomic Rings

In this section, we first recall some necessary algebraic background, and then introduce the related and necessary lemmas for our constructions.

We focus mainly on the algebraic structure of cyclotomic rings of integers $R_N = \mathbb{Z}[X]/\langle X^N + 1\rangle$ and $R_d = \mathbb{Z}[X]/\langle X^d + 1\rangle$ with dimension N and d,

respectively. Given certain prime q, we can define $R_{N,q} = \mathbb{Z}_q[X]/\langle X^N + 1\rangle$ (or $R_d = \mathbb{Z}_q[X]/\langle X^d + 1\rangle$), where each coefficient of $R_{N,q}$ (or $R_{d,q}$) are in $\mathbb{Z}_q$.

Additionally, if $d|N$ and $t = N/d$, R_d can be viewed as a subring of R_N, i.e., $R_N \cong R_d^t$. According to [41, 42], there is an efficiently computable ring isomorphism between R_N and R_d, for an appropriately defined vector multiplication in R, which preserves norms. Therefore, any relations over $R_{N,q}$ can be expressed as the corresponding relations over $R_{d,q}$.

We consider the algebraic structure of m-th cyclotomic field $K = \mathbb{Q}[X]/\langle \Phi_m(X)\rangle$ of degree $d = \phi(m)$ with the ring of integers $R = \mathbb{Z}[X]/\langle \Phi_m(X)\rangle$. Here, K is a d-degree Galois extension of $\mathbb{Q}$. Then, we use $G = \mathrm{Gal}(K/\mathbb{Q})$ to denote the Galois group of K, which consists of all automorphisms of K and is computed under composition. Clearly, all these automorphisms fix the rational numbers $\mathbb{Q}$, i.e., for any $\sigma \in G$ and any $x \in \mathbb{Q}$, it holds $\sigma(x) = x$. Conversely, cyclotomic field are Galois over $\mathbb{Q}$ meaning that only the elements of $\mathbb{Q}$ are fixed by all automorphisms in G.

Moreover, the Galois group G of K is isomorphic to $\mathbb{Z}_m^\times$, where the isomorphism $j \mapsto \sigma_j : \mathbb{Z}_m^\times \mapsto \mathrm{Gal}(K/\mathbb{Q})$ is defined by $\sigma_j(X) = X^j$. In general, the degree of a Galois extension of a field is always equal to the order of its Galois group. The main theorem of Galois theory says that there is one-by-one correspondence between the subgroups of G and the subfields of K. For example, let H to be a subgroup of G, i.e., $H < G$. Then H is corresponded to a subfield L of K, i.e., $L < K$. And H is the Galois group of K over L, i.e., $H = \mathrm{Gal}(K/L)$ consists of the automorphisms of K that fix the elements in L. Conversely, as the subfield of K, L consists precisely of all the elements that are fixed by all automorphisms in H, and thus L is called as the fixed field of H. This implies that the extension K/L is again Galois.

Furthermore, by restricting the automorphisms of K to the cyclotomic ring $R \subset K$, we get ring automorphisms of R. And the property that certain subset $S \subset R$ is fixed under automorphisms is still set up. More formally, we have the following lemma from [22].

Lemma A.12 (Theorem 3.1 in [22]) *Let K be a cyclotomic number field with the ring of integers R, and let L be a subfield of K with the ring of integers $\mathcal{S}$. Let G denote the Galois group of K, and H denote a subgroup that consists of all these automorphisms fixing L. Let q is a prime number that is inert in the subfield L, $\mu \in R_q$ be an element that is fixed modulo q by all Galois automorphisms $\sigma \in H$; that is, $\sigma(\mu) \equiv \mu (\mathrm{mod} qR)$ for all $\sigma \in H$. Then, μ is contained in the subfield $\mathcal{S}_q$ of R_q.*

Moreover, for the special case of power-of-two cyclotomic rings, given a power of 2 integer n, we denote $R = \mathbb{Z}[X]/\langle X^d+1\rangle$ as the related cyclotomic ring, since X^d+1 is the $2d$-th cyclotomic polynomial. Similarly, we denote $K = \mathbb{Q}[X]/\langle X^d+1\rangle$ as the related cyclotomic field, which is a d-degree Galois extension of $\mathbb{Q}$. Here, the Galois group G of K is isomorphic to $\mathbb{Z}_{2d}^\times$, which has the structure $\mathbb{Z}_2 \times \mathbb{Z}_{d/2}$. Notice that the cyclic subgroup $\mathbb{Z}_2$ and $\mathbb{Z}_{d/2}$ are generated by σ_{-1} and σ_5, respectively.

Given a prime q and the integer ring $R = \mathbb{Z}[X]/\langle X^d + 1\rangle$, we need to ensure the message space $\mathcal{M} \subseteq R_q$ is a subfield of $K = \mathbb{Q}[X]/\langle X^d + 1\rangle$. According to the above mentioned Galois group structure of general cyclotomic rings, the necessary and sufficient conditions for $\mathcal{M}$ to be a subfield is:

1. Its elements are fixed by a subgroup of G. This means that the message is contained in $\mathcal{S}_q = \mathcal{S}/q\mathcal{S}$ where $\mathcal{S} \subseteq R$ is the ring of integers of a subfield of K.
2. Prime number q stay inert in $\mathcal{S}$ such that $\mathcal{S}_q$ is a field.

With respect to the above two conditions, we have the following two formal lemmas from [22].

Lemma A.13 (Theorem 3.2 in [22]) *Let $d > k \geq 1$ be powers of 2. The subgroup $H = \langle \sigma_{-1}, \sigma_5^k \rangle$ of the Galois group $G = Gal(K/\mathbb{Q})$ has index k. Its fixed field L is generated by $\alpha = X^{d-\frac{d}{2k}} - X^{\frac{d}{2k}}$ over Q inside K, $L = \mathbb{Q}[\alpha] \subset K$.*

Lemma A.14 (Theorem 3.3 in [22]) *The prime numbers that are inert in the fixed field L of $\langle \sigma_{-1}, \sigma_5^k \rangle$ with $1 < k < d$ be power of two, are precisely the primes that are congruent to 3 or 5 modulo 8. They split into two prime ideals in K.*

A.5 Well-formedness of BDLOP commitment

In this section, we present how to achieve the well-formedness of BDLOP commitment, through using relaxed relation. Particularly, we first present the related relaxed relation with respect to the well-formedness of BDLOP commitment, and then present the concrete protocol. Finally, we analyze the efficiency of this concrete protocol, and compare it with that of the previous opening proof in [6,22].

We first consider the following language for the *relaxed* relation of BDLOP:

$$L_{\gamma_1',\gamma_2',q_1,q_2,\bar{\mathcal{C}}} := \Big\{ (\mathbf{A}_1, \mathbf{A}_2, \boldsymbol{t}_1, \boldsymbol{t}_2) : \exists\ (\boldsymbol{r}, \boldsymbol{m}) \text{ and } f \in \bar{\mathcal{C}} \text{ such that } 0 < \|\boldsymbol{r}\| \leq \gamma_1', \\ 0 < \|\boldsymbol{m}\| \leq \gamma_2', \text{ and } \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{m} \end{bmatrix} = f \cdot \begin{bmatrix} \boldsymbol{t}_1 \\ \boldsymbol{t}_2 \end{bmatrix} \Big\}.$$

By using the technique of [6,22], we can construct a NIZKPoK (with rewinding-type extractions) as following. For the BDLOP scheme with small non-zero message, i.e., $\|m\| \leq \gamma_2'$, the stronger well-formedness can be achieved.

Interactive Proof Protocol

The interactive version is presented in Table 8.

Proof of Theorem A.15

Theorem A.15 *In the random oracle model, for a secure BDLOP commitment scheme, there exists a NIZKPoK system Π for the relaxed language $L_{\gamma_1',\gamma_2',q_1,q_2,\bar{\mathcal{C}}}$, with $\gamma_1' = 2\sqrt{2}\eta \cdot \sqrt{\kappa} \cdot \beta \cdot kN$ and $\gamma_2' = 2\sqrt{2}\eta \cdot \sqrt{\kappa} \cdot \beta \cdot \ell N$, where η is the parameter for rejection sampling as in Lemma A.9.*

Prover $\mathcal{P}$		Verifier $\mathcal{V}$
Inputs:		
$\mathbf{A}_1 \in R_{q_1}^{n\times k}, \mathbf{A}_2 \in R_{q_2}^{\ell\times k}$		$\mathbf{A}_1, \mathbf{A}_2$
$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} \\ \mathbf{A}_2 & \mathbf{I} \end{bmatrix}$		$\mathbf{A}$,$\boldsymbol{t}_1, \boldsymbol{t}_2$
$\boldsymbol{t}_1 \in R_{q_1}^n, \boldsymbol{t}_2 \in R_{q_2}^k$		$B_1 \geq \sigma_1 \cdot \sqrt{2N \cdot k}$
$\boldsymbol{r} \in S_\beta^n, \boldsymbol{m} \in S_{\beta'}^\ell$		$B_2 \geq \sigma_2 \cdot \sqrt{2N \cdot \ell}$
$\boldsymbol{y}_1 \leftarrow \mathcal{D}_{\sigma_1}^k$		
$\boldsymbol{y}_2 \leftarrow \mathcal{D}_{\sigma_2}^\ell$		
$\boldsymbol{w}_1 = \mathbf{A}_1 \cdot \boldsymbol{y}_1$		
$\boldsymbol{w}_2 = \mathbf{A}_2 \cdot \boldsymbol{y}_1 + \boldsymbol{y}_2$		
	$\xrightarrow{\boldsymbol{w}_1, \boldsymbol{w}_2}$	
	$\xleftarrow{c}$	$c \xleftarrow{\$} \mathcal{C}$
$\boldsymbol{z}_1 = \boldsymbol{y}_1 + c \cdot \boldsymbol{r}$		
$\boldsymbol{z}_2 = \boldsymbol{y}_2 + c \cdot \boldsymbol{m}$		
If $\mathsf{Rej}(\boldsymbol{z}_1, c \cdot \boldsymbol{r}, \sigma_1) = 1$		
or $\mathsf{Rej}(\boldsymbol{z}_2, c \cdot \boldsymbol{m}, \sigma_2) = 1$, abort	$\xrightarrow{\boldsymbol{z}_1, \boldsymbol{z}_2}$	
		Check:
		$\|\boldsymbol{z}_1\| \leq B_1$, $\|\boldsymbol{z}_2\| \leq B_2$
		$\mathbf{A}_1 \cdot \boldsymbol{z}_1 = \boldsymbol{w}_1 + c \cdot \boldsymbol{t}_1$
		$\mathbf{A}_2 \cdot \boldsymbol{z}_1 + \boldsymbol{z}_2 = \boldsymbol{w}_2 + c \cdot \boldsymbol{t}_2$

Table 8. Well-formedness proof of BDLOP commitment.

Proof. Essentially, this proof consists of two steps: the first is that of proving the protocol in Table 8 is complete, statistical honest verifier zero-knowledge and computational sound under the M-SIS assumption; the second is that of making it non-interactive with the help of the standard Fiat-Shamir technique. As the second one is natural, it suffices for us to just focus on the first one. Details are given as follows.

Completeness. The vectors $\boldsymbol{z}_1, \boldsymbol{z}_2$ sent by $\mathcal{P}$ are independent and their distributions have statistical distance at most $2^{-\lambda}$ from $\mathcal{D}_{\sigma_1}^n$ and $\mathcal{D}_{\sigma_2}^\ell$ respectively, by Lemma A.9 on rejection sampling. Furthermore, Lemma A.2 implies that the bounds $\|\boldsymbol{z}_i\|_2 \leq B_i$ holds with overwhelming probability. Besides, it is easy to verify that all of the other verification equations are always true for the messages sent by $\mathcal{P}$.

Statistical honest verifier zero-knowledge. Here, we just need to prove that the protocol is zero-knowledge when $\mathcal{P}$ does not abort prior to sending $\boldsymbol{z}_i$. This is because after converting into non-interactive proofs via Fiat-Shamir transform, $\mathcal{V}$ never sees the aborting transcripts. We can prove this zero-knowledge properties by designing a PPT simulator $\mathcal{S}$ whose outputs are statistically close to the transcript of real protocol. Particularly, given matrices $\mathbf{A}_1 \in R_{q_1}^{n\times k}$, $\mathbf{A}_2 \in R_{q_2}^{\ell\times k}$, and commitment vectors $\boldsymbol{t}_1 \in R_{q_1}^n$, $\boldsymbol{t}_2 \in R_{q_2}^\ell$, $\mathcal{S}$ conducts the followings

Prover $\mathcal{P}$		Verifier $\mathcal{V}$
Inputs:		
For $i \in [\tau]$:		
$\mathbf{A}_{1,i} \in R_{q_1}^{n_i \times k_i}$		$\mathbf{A}_{i,1}, \mathbf{A}_{i,2}$
$\mathbf{A}_{2,i} \in R_{q_2}^{\ell_i \times k_i}$		$\mathbf{B}_i$
$\mathbf{B}_i \in R_{q_2}^{x \times \ell_i}$		
$\mathbf{A}_i = \begin{bmatrix} \mathbf{A}_{i,1} & \mathbf{0}_i \\ \mathbf{A}_{i,2} & \mathbf{I}_i \end{bmatrix}$		$\mathbf{A}_i, \boldsymbol{t}_{i,1}, \boldsymbol{t}_{i,2}$
with zero matrix:		
$\mathbf{0}_i \in R^{n_i \times \ell_i}$		
identity matrix:		
$\mathbf{I}_i \in R^{\ell_i \times \ell_i}$		
$\boldsymbol{t}_{i,1} \in R_{q_1}^{n_i}, \boldsymbol{t}_{i,2} \in R_{q_2}^{k_i}$		$B_{i,1} \geq \sigma_{i,1} \cdot \sqrt{N \cdot k_i}$
$\boldsymbol{r}_i \in S_{\beta}^{n_i}, \boldsymbol{m}_i \in S_{\beta'}^{\ell_i}$		$B_{i,2} \geq \sigma_{i,2} \cdot \sqrt{N \cdot \ell_i}$
s.t.		
$\begin{bmatrix} \boldsymbol{t}_{i,1} \\ \boldsymbol{t}_{i,2} \end{bmatrix} = \mathbf{A}_i \begin{bmatrix} \boldsymbol{r}_{i,1} \\ \boldsymbol{m}_{i,2} \end{bmatrix}$		
$\sum \mathbf{B}_i \boldsymbol{m}_i = \mathbf{0} \in R_{q_2}^x$		
For $\forall i \in [\tau]$		
$\boldsymbol{y}_{i,1} \leftarrow \mathcal{D}_{\sigma_{i,1}}^{k_i}$		
$\boldsymbol{y}_{i,2} \leftarrow \mathcal{D}_{\sigma_{i,2}}^{\ell_i}$		
$\boldsymbol{w}_{i,1} = \mathbf{A}_{i,1} \cdot \boldsymbol{y}_{i,1}$		
$\boldsymbol{w}_{i,2} = \mathbf{A}_{i,2} \cdot \boldsymbol{y}_{i,1} + \boldsymbol{y}_{i,2}$		
$\boldsymbol{w}_2 = \sum_i \mathbf{B}_i \mathbf{A}_{i,2} \boldsymbol{y}_{i,1}$		
	$\boldsymbol{w}_2$ $\boldsymbol{w}_{i,1}$ $\boldsymbol{w}_{i,2}$ $\longrightarrow$	
	$\longleftarrow c$	$c \xleftarrow{\$} \mathcal{C}$
$\boldsymbol{z}_{i,1} = \boldsymbol{y}_{i,1} + c \cdot \boldsymbol{r}_i$		
$\boldsymbol{z}_{i,2} = \boldsymbol{y}_{i,2} + c \cdot \boldsymbol{m}_i$		
For $\forall i \in [\tau]$, if $\mathsf{Rej}(\boldsymbol{z}_{i,1}, c \cdot \boldsymbol{r}_i, \sigma_{i,1}) = 1$ or $\mathsf{Rej}(\boldsymbol{z}_{i,2}, c \cdot \boldsymbol{m}_i, \sigma_{i,2}) = 1$, abort	$\boldsymbol{z}_1, \boldsymbol{z}_2 \longrightarrow$	
		Check: for $\forall i \in [\tau]$
		$\lVert \boldsymbol{z}_{i,1} \rVert \leq B_{i,1}$
		$\lVert \boldsymbol{z}_{i,2} \rVert \leq B_{i,2}$
		$\mathbf{A}_{i,1} \cdot \boldsymbol{z}_{i,1} = \boldsymbol{w}_{i,1} + c \cdot \boldsymbol{t}_{i,1}$
		$\mathbf{A}_{i,2} \cdot \boldsymbol{z}_{i,1} + \boldsymbol{z}_{i,2} = \boldsymbol{w}_{i,2} + c \cdot \boldsymbol{t}_{i,2}$
		$\sum \mathbf{B}_i \mathbf{A}_{i,2} \boldsymbol{z}_{i,1} = c \sum \mathbf{B}_i \boldsymbol{t}_{i,2} + \boldsymbol{w}_2$

Table 9. Linear-relationship Proof of BDLOP commitment.

– Sample $c \xleftarrow{\$} \mathcal{C}$;
– Sample $\boldsymbol{z}_1 \leftarrow \mathcal{D}^k_{\sigma_1}$, and $\boldsymbol{z}_2 \leftarrow \mathcal{D}^\ell_{\sigma_2}$;
– Set $\boldsymbol{w}_1 = \mathbf{A}_1 \cdot \boldsymbol{z}_1 - c \cdot \boldsymbol{t}_1$, $\boldsymbol{w}_2 = \mathbf{A}_2 \cdot \boldsymbol{z}_1 + \boldsymbol{z}_2 - c \cdot \boldsymbol{t}_2$;
– Output $(\boldsymbol{w}_1, \boldsymbol{w}_2, c, \boldsymbol{z}_1, \boldsymbol{z}_2)$.

Clearly, the vectors $\boldsymbol{z}_1, \boldsymbol{z}_2$ output by $\mathcal{S}$ will be accepted with overwhelming probability. Besides, the distribution of $\boldsymbol{z}_i$ output in the real protocol is within a negligible statistical distance of $\mathcal{D}^k_{\sigma_1}$ or $\mathcal{D}^\ell_{\sigma_2}$. Since $\boldsymbol{w}_1, \boldsymbol{w}_2$ are completely determined by $\mathbf{A}_1, \mathbf{A}_2, \boldsymbol{t}_1, \boldsymbol{t}_2, c, \boldsymbol{z}_1, \boldsymbol{z}_2$, the output distribution of $\mathcal{S}$ is within a negligible statistical distance of these random variables in the actual protocol.

Special soundness. Suppose there exists an adversary $\mathcal{A}$ who can produce a valid proof $\pi := (\boldsymbol{w}_1, \boldsymbol{w}_2, c, \boldsymbol{z}_1, \boldsymbol{z}_2)$ for two vectors $(\boldsymbol{t}_1, \boldsymbol{t}_2) \notin L_{\gamma_1', \gamma_2', q_1, q_2, \bar{\mathcal{C}}}$. Then, we can rewind $\mathcal{A}$ to obtain another adversary $\pi := (\boldsymbol{w}_1, \boldsymbol{w}_2, c', \boldsymbol{z}_1', \boldsymbol{z}_2')$ with $c \neq c'$ and $\bar{c} = c - c' \in \bar{C}$ is invertible. Then, we can compute $f = (c - c') \in \bar{\mathcal{C}}$, and set $\bar{\boldsymbol{r}} = \boldsymbol{z}_1 - \boldsymbol{z}_1'$, $\bar{\boldsymbol{m}} = \boldsymbol{z}_2 - \boldsymbol{z}_2'$ such that $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}} = f \cdot \boldsymbol{t}_1$, and $\mathbf{A}_2 \cdot \bar{\boldsymbol{r}} + \bar{\boldsymbol{m}} = f \cdot \boldsymbol{t}_2$.

Below, we compute the ℓ_2-norm of the extracted vectors $\bar{\boldsymbol{r}}, \bar{\boldsymbol{m}}$. According to the rejection sampling in Lemma A.9 and $\|\boldsymbol{r}\|_\infty \leq \beta$, $\|\boldsymbol{m}\|_\infty \leq \beta'$, we need to set $\sigma_1 = \eta \cdot \sqrt{\kappa} \cdot \beta \cdot \sqrt{kN}$ and $\sigma_2 = \eta \cdot \sqrt{\kappa} \cdot \beta' \cdot \sqrt{\ell N}$. And thus, we get $\|\bar{\boldsymbol{r}}\|_2 \leq 2\sqrt{2} \cdot \sigma_1 \cdot \sqrt{kN} = 2\sqrt{2}\eta \cdot \sqrt{\kappa} \cdot \beta \cdot kN = \gamma_1'$ and $\|\bar{\boldsymbol{m}}\|_2 \leq 2\sqrt{2} \cdot \sigma_2 \cdot \sqrt{\ell N} = 2\sqrt{2}\eta \cdot \sqrt{\kappa} \cdot \beta' \cdot \ell N = \gamma_2'$, where η is the parameter for rejection sampling as in Lemma A.9. This implies we can view $(\bar{\boldsymbol{r}}, \bar{\boldsymbol{m}})$ as a witness for $(\boldsymbol{t}_1, \boldsymbol{t}_2) \in L_{\gamma_1', \gamma_2', q_1, q_2, \bar{\mathcal{C}}}$. However, this is clearly contradictive with the previous assumption that $(\boldsymbol{t}_1, \boldsymbol{t}_2)$ is not in the language $L_{\gamma_1', \gamma_2', q_1, q_2, \bar{\mathcal{C}}}$, which implies the protocol is computationally special soundness. Notice that this special soundness implicitly implies the properties of computational soundness. □

Proof Size of Non-Interactive Protocol

In this section, we analyze the efficiency of the non-interactive protocol of Table 8. This means that the challenge $c \in \mathcal{C}$ is computed by $\mathcal{P}$ via hashing all previous messages and public information. And the hash function is modeled as a random oracle. In order to shorten the length of the proof, we can adopt a standard technique that is not to directly send the input to the hash function, but rather send its output (i.e. the challenge). In this case, given the transmitted vector $\boldsymbol{z}_i$, the verifier can recompute the input, through using the verification equation, and then check that the hash of these computed input terms is indeed the transmitted challenge c. As a result, the proof size of the non-interactive protocol consists of that of vectors $\boldsymbol{z}_i$ and the challenge c, i.e.,

$$k \cdot N \cdot \lceil \log(12\sigma_1) \rceil + \ell \cdot N \cdot \lceil \log(12\sigma_2) \rceil + 256,$$

where the output size of random oracle is supposed to be 256 bits.

Notice that for our parameter setting on Construction 4.1 (i.e., $n = \ell = 1, k = 3$), if we choose message polynomial $\boldsymbol{m}$ with coefficients in $\{-1, 0, 1\}$, this proof of well-formedness is just larger than the previous opening proof in [6, 22] by one third times. Clearly, this overhead is mild.

Additional Properties of the Protocol in Table 8

In this section, we first present a proof of knowledge on linear relationship as in [22]. Particularly, given a set of commitments $\boldsymbol{t}_i = \begin{bmatrix} \boldsymbol{t}_{i,1} \\ \boldsymbol{t}_{i,2} \end{bmatrix}$, we prove that their openings $\boldsymbol{m}_i$ satisfying $\sum \mathbf{B}_i \boldsymbol{m}_i = 0$ for any fixed $\mathbf{B}_i$. The detailed protocol is presented in Table 9. Here, due to the similarity with [22], we omit the detailed proof of completeness, honest-verifier zero-knowledge, and special soundness for simplity.

Moreover, just as mentioned in Section 2.3, our new well-formedness proof in Table 8 can prevent the mix-and-match attacks for our anonymous credential systems. In order to specify this more clearly, below we first introduce what the mix-and-match attack is, and then argue this will induce a solution for M-SIS problem.

Definition A.16 (Mix-and-Match attack) *Given a pair of* BDLOP *public matrices* $\mathbf{A}_1, \mathbf{A}_2$ *and two vectors* $\boldsymbol{t}_{1,1}, \boldsymbol{t}_{1,2}$*, together with an opening* NIZKPoK *proof* π *showing that* $\begin{bmatrix} \boldsymbol{t}_{1,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$ *is a valid commitment with respect to* $\mathbf{A}_1, \mathbf{A}_2$*, if the adversary can find out a new vector* $\boldsymbol{t}_{2,1}$ *together with a new opening* NIZKPoK *proof* π' *showing that* $\begin{bmatrix} \boldsymbol{t}_{2,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$ *is a valid commitment with respect to* $\mathbf{A}_1, \mathbf{A}_2$*.*

Definition A.17 (Commitment-Proof Binding Property) *We say the* BDLOP *commitment scheme and its* NIZKPoK *proof system* Π *satisfy the commitment-proof binding property, if they can resist mix-and-match attacks, i.e., it is negligible for any* PPT *adversary to find a vector* $\boldsymbol{t}_{2,1}$ *and a proof* π' *to conduct a successful mix-and-match attack.*

It is easy to verify that for BDLOP commitment scheme, the previous opening proof systems as in [6,22] can not satisfy the commitment-proof binding property. Particularly, suppose $\boldsymbol{t}_1$ is a valid commitment of $\boldsymbol{m}$, with respect to the public matrices $\mathbf{A}_1$, $\mathbf{A}_2$. This means $\boldsymbol{t}_1 = \begin{bmatrix} \boldsymbol{t}_{1,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \boldsymbol{r}_1 + \begin{bmatrix} 0 \\ \boldsymbol{m} \end{bmatrix}$. According to the opening proof for BDLOP commitment in [6, 22], given their opening proofs π_1, the corresponding extracted openings are $(\boldsymbol{m}, \bar{\boldsymbol{r}}_1, f_1)$ such that $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}}_1 = f_1 \cdot \boldsymbol{t}_{1,1}$ and $\boldsymbol{m} = \boldsymbol{t}_{1,2} - f_1^{-1} \cdot \mathbf{A}_2 \bar{\boldsymbol{r}}_1$.

In this case, through computing $\boldsymbol{t}_{2,1} = \mathbf{A}_1 \boldsymbol{r}_2$, the adversary can obtain a modified commitment $\boldsymbol{t}_1' = \begin{bmatrix} \boldsymbol{t}_{2,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$. Furthermore, the adversary can directly use $\boldsymbol{r}_2$ to generate opening proof π' for $\boldsymbol{t}_1'$, and the corresponding extracted openings are $(\boldsymbol{m}', \bar{\boldsymbol{r}}_2, f_2)$, such that $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}}_2 = f_2 \cdot \boldsymbol{t}_{2,1}$ and $\boldsymbol{m}' = \boldsymbol{t}_{1,2} - f_2^{-1} \cdot \mathbf{A}_2 \bar{\boldsymbol{r}}_2$. Clearly, this is a successful mix-and-match attack.

Fortunately, for our new proof in Table 8, this attack can be prevented.

Claim A.18 *When using the protocol in Table 8 as the opening proof, the* BDLOP *commitment satisfies the commitment-proof binding property.*

Proof. Generally, we give a reduction that if the adversary can conduct the mix-and-match attacks successfully, then we can construct a new algorithm to solve the M-SIS problem.

Particularly, suppose $\boldsymbol{t}_1 = \begin{bmatrix} \boldsymbol{t}_{1,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$ is a valid commitment of $\boldsymbol{m}$, with respect to the public matrices $\mathbf{A}_1$, $\mathbf{A}_2$. This means $\begin{bmatrix} \mathbf{A}_1 \ \mathbf{0} \\ \mathbf{A}_2 \ \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{r} \\ \boldsymbol{m} \end{bmatrix} = \begin{bmatrix} \boldsymbol{t}_{1,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$. Through using the new proof in Table 8, one can extract witness $(\bar{\boldsymbol{r}}, \bar{\boldsymbol{m}}, f)$ such that $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}} = f \cdot \boldsymbol{t}_{1,1}$ and $\mathbf{A}_2 \cdot \bar{\boldsymbol{r}} + \bar{\boldsymbol{m}} = f \cdot \boldsymbol{t}_{1,2}$, which implies

$$[\mathbf{A}_2, \mathbf{I}] \begin{bmatrix} \bar{\boldsymbol{r}} \\ \bar{\boldsymbol{m}} \end{bmatrix} = f \cdot \boldsymbol{t}_{1,2}. \tag{3}$$

Here, assume the adversary can compute a vector $\boldsymbol{t}_{2,1} \neq \boldsymbol{t}_{1,1}$ such that the modified commitment $\boldsymbol{t}'_1 = \begin{bmatrix} \boldsymbol{t}_{2,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$ is valid. This means $\begin{bmatrix} \mathbf{A}_1 \ \mathbf{0} \\ \mathbf{A}_2 \ \mathbf{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{r}' \\ \boldsymbol{m}' \end{bmatrix} = \begin{bmatrix} \boldsymbol{t}_{2,1} \\ \boldsymbol{t}_{1,2} \end{bmatrix}$ for certain vectors $\boldsymbol{r}', \boldsymbol{m}'$. Furthermore, through using the new proof in Table 8, the extracted witness is $(\bar{\boldsymbol{r}}', \bar{\boldsymbol{m}}', f')$ such that $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}}' = f' \cdot \boldsymbol{t}_{2,1}$ and $\mathbf{A}_2 \cdot \bar{\boldsymbol{r}}' + \bar{\boldsymbol{m}}' = f' \cdot \boldsymbol{t}_{1,2}$, which implies

$$[\mathbf{A}_2, \mathbf{I}] \begin{bmatrix} \bar{\boldsymbol{r}}' \\ \bar{\boldsymbol{m}}' \end{bmatrix} = f' \cdot \boldsymbol{t}_{1,2}. \tag{4}$$

Through multiplying f' and f into Equations (3) and (4), we can get

$$[\mathbf{A}_2, \mathbf{I}] \begin{bmatrix} f' \cdot \bar{\boldsymbol{r}} \\ f' \cdot \bar{\boldsymbol{m}} \end{bmatrix} = f' \cdot f \cdot \boldsymbol{t}_{1,2} \tag{5}$$

and

$$[\mathbf{A}_2, \mathbf{I}] \begin{bmatrix} f \cdot \bar{\boldsymbol{r}}' \\ f \cdot \bar{\boldsymbol{m}}' \end{bmatrix} = f \cdot f' \cdot \boldsymbol{t}_{1,2}. \tag{6}$$

And through subtracting (6) from (5), we can get

$$[\mathbf{A}_2, \mathbf{I}] \begin{bmatrix} f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}' \\ f' \cdot \bar{\boldsymbol{m}} - f \cdot \bar{\boldsymbol{m}}' \end{bmatrix} = 0. \tag{7}$$

Below, we just need to prove that $\begin{bmatrix} f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}' \\ f' \cdot \bar{\boldsymbol{m}} - f \cdot \bar{\boldsymbol{m}}' \end{bmatrix}$ is a non-zero short vector. First, as both f, f' are small, and the ℓ_2 norms of all vectors $\bar{\boldsymbol{r}}, \bar{\boldsymbol{r}}', \bar{\boldsymbol{m}}, \bar{\boldsymbol{m}}'$ are small, the ℓ_2 norm of $\begin{bmatrix} f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}' \\ f' \cdot \bar{\boldsymbol{m}} - f \cdot \bar{\boldsymbol{m}}' \end{bmatrix}$ should be bounded by a small value too.

Second, from $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}} = f \cdot \boldsymbol{t}_{1,1}$ and $\mathbf{A}_1 \cdot \bar{\boldsymbol{r}}' = f \cdot \boldsymbol{t}_{2,1}$, we know that

$$\mathbf{A}_1 \cdot (f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}') = f' f \cdot \boldsymbol{t}_{1,1} - f \cdot f' \cdot \boldsymbol{t}_{2,1}.$$

Then, by the assumption that $\boldsymbol{t}_{2,1} \neq \boldsymbol{t}_{1,1}$, we know the above equation is non-zero, which implies $(f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}') \neq 0$. Finally, this implies the vector $\begin{bmatrix} f' \cdot \bar{\boldsymbol{r}} - f \cdot \bar{\boldsymbol{r}}' \\ f' \cdot \bar{\boldsymbol{m}} - f \cdot \bar{\boldsymbol{m}}' \end{bmatrix}$ is non-zero.

Overall, this implies if there exists the adversary successfully conducting mix-and-match attacks, then we can construct another reduction algorithm to solve M-SIS problem with respect to $[\mathbf{A}_2, \mathbf{I}]$. □

A.6 Security for NIZK

Let's recall the notion of non-interactive zero-knowledge (NIZK) proof system.

Definition A.19 ([23]) *Let $\mathfrak{R}$ be a relation. A non-interactive proof system Π for $\mathfrak{R}$ is a tuple of PPT algorithms* (Setup, Prove, Verify, SimSetup) *having the following interfaces (where 1^λ are implicit inputs to* Prove, Verify, SimSetup*):*

- $\mathsf{Setup}(1^\lambda)$: *given a security parameter λ, outputs a string* crs.
- $\mathsf{Prove}(\mathsf{crs}, x, w)$*: given a string* crs *and a statement-witness pair $(x, w) \in \mathfrak{R}$, outputs a proof π.*
- $\mathsf{Verify}(\mathsf{crs}, x, \pi)$*: given a string* crs*, a statement x, and a proof π, either accepts or rejects.*
- $\mathsf{SimSetup}(1^\lambda)$*: given a security parameter λ, outputs a simulated string* $\widehat{\mathsf{crs}}$ *and a trapdoor* tk*.*

A secure NIZK system Π should have three properties: Completeness, Soundness, and Zero-knowledge. As argued by [5, 21, 25, 27], Fiat-Shamir based proof systems in the random oracle model satisfy these properties. Many recent lattice-based efficient NIZKs are Fiat-Shamir based, so they also enjoy this property. We require that the following three properties hold:

- *Completeness*: for every $(x, w) \in \mathfrak{R}$ and every λ, $\mathsf{Verify}(\mathsf{crs}, x, \pi)$ accepts with probability 1, over the choice of $\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda)$ and $\pi \leftarrow \mathsf{Prove}(\mathsf{crs}, x, w)$.
- *Soundness* : let $L_{\mathfrak{R}}$ be the language defined by relation $\mathfrak{R}$. For any PPT adversary $\mathcal{A}$,

  $$\mathsf{Pr}_{\mathsf{crs}\leftarrow\mathsf{Setup}(1^\lambda)}\left[\exists x \ s.t. \pi^* \leftarrow \mathcal{A}(\mathsf{crs}, x) : \mathsf{Verify}(\mathsf{crs}, x, \pi^*) \text{ accepts} \wedge x \notin L_{\mathfrak{R}}\right] \leq \mathsf{negl}(\lambda).$$

- *Zero-Knowledge* : There exists one PPT algorithm SimProve, such that, for any PPT adversary $\mathcal{A}$ we have $|\mathsf{Pr}[\mathcal{A} \ wins] - \frac{1}{2}| \leq \mathsf{negl}(\lambda)$ in the following game:
  1. The challenger samples $(\widehat{\mathsf{crs}}, \mathsf{tk}) \leftarrow \mathsf{SimSetup}(1^\lambda)$ such that $\widehat{\mathsf{crs}}$ is indistinguishable from crs output by Setup, and gives the simulated $\widehat{\mathsf{crs}}$ to $\mathcal{A}$.
  2. The adversary $\mathcal{A}$ chooses $(x, w) \in \mathfrak{R}$ and gives these to the challenger.
  3. The challenger samples $\pi_0 \leftarrow \mathsf{Prove}(\mathsf{crs}, x, w), \pi_1 \leftarrow \mathsf{SimProve}(\widehat{\mathsf{crs}}, x, \mathsf{tk}), b \leftarrow \{0, 1\}$ and gives π_b to $\mathcal{A}$.
  4. The adversary $\mathcal{A}$ outputs a bit b' and wins if $b' = b$.

Notice that in the above zero-knowledge game, if we allow the adversary $\mathcal{A}$ to choose any polynomial numbers of (x_i, w_i), and all the resulting $\{\pi_{i,0}\}$ and $\{\pi_{i,1}\}$ are still indistinguishable, we say that Π is a multi-theorem NIZK system.

We define proof of knowledge which is a stronger property than soundness. Generally, a NIZK system is called NIZKPoK if we can efficiently recover the

witness w from the valid proof output by the adversary. More formally, we say a non-interactive system is a proof of knowledge, if there exists a pair of PPT algorithms $(\mathsf{SimSetup}, \mathsf{Ext})$, such that $\mathsf{SimSetup}$ outputs a correctly generated $\widehat{\mathsf{crs}}$ together with an extraction key tk, and Ext can use tk to extract a valid witness from a proof.

Moreover, we consider two flavors for proof of knowledge: single-proof extractability and multi-theorem straight-line extractability.

Definition A.20 (Single-Theorem Extractability in [21]) *An* NIZK *proof system is single-proof extractable if there exists a* PPT *extractor* Ext*, constant* c_1*,* c_2*,* e *and a non-negligible polynomial* $p(\lambda)$ *such that for any* crs*, any* $x \in L_{\mathfrak{R}}$*, any* $Q = \mathsf{poly}(\lambda)$*, and* PPT *adversary* $\mathcal{A}$ *that makes at most* Q *random oracle queries with*

$$\Pr\left[\pi \xleftarrow{\$} \mathcal{A}(\mathsf{crs}, x) : \mathsf{Verify}(\mathsf{crs}, x, \pi) = 1\right] \geq \mu(\lambda),$$

then we have,

$$\Pr\left[w \xleftarrow{\$} \mathsf{Ext}^{\mathcal{A}}(\mathsf{crs}, x) : (x, w) \in \mathfrak{R}\right] \geq \frac{1}{p(\lambda \cdot Q^e)} \cdot \mu(\lambda)^{c_1} - \mathsf{negl}(\lambda),$$

where the runtime of Ext *is upper bounded by* $c_2 \cdot \mathsf{Time}(\mathcal{A})$ *and we assume one oracle access to* $\mathcal{A}$ *takes* $\mathsf{Time}(\mathcal{A})$*.*

Particularly, if we compile a sigma protocol with the Fiat-Shamir transform, then we have $(c_1, c_2, e) = (2, 2, 1)$ and $p(\lambda) = 1$ via rewinding the prover and the forking lemma [8, 50]. Additionally, we need to use a stronger extractability, i.e., multi-theorem straight-line extractability, where we can directly extract witnesses from multiple pairs of statement and proof output by the adversary. Moreover, for such multiple-theorem extractability, we allow the adversary to choose the queried statements adaptively.

Definition A.21 (Multi-Theorem Extractability in [21]) *An* NIZK *system is multi-theorem straight-line extractable, if there exists a* PPT *oracle simulator* $\mathsf{SimSetup}$ *and a* PPT *extractor* Ext *with the following properties:*

CRS indistinguishability. *For any* PPT *adversary* $\mathcal{A}$*, we have*

$$\begin{aligned}\mathsf{Adv}(\mathcal{A}) :=& \Big| \Pr[\mathsf{crs} \leftarrow \mathsf{Setup}(1^\lambda) : \mathcal{A}(\mathsf{crs}) = 1] \\ &- \Pr[(\widehat{\mathsf{crs}}, \mathsf{tk}) \leftarrow \mathsf{SimSetup}(1^\lambda) : \mathcal{A}(\widehat{\mathsf{crs}}) = 1]\Big| \leq \mathsf{negl}(\lambda).\end{aligned}$$

Straight-Line Extractability. *There exists constants* c, e_1, e_2 *and polynomial* $p(\lambda)$ *such that for any* $Q = \mathsf{poly}(\lambda)$ *and* PPT *adversary* $\mathcal{A}$ *that makes at most* Q *random oracle queries with*

$$\begin{aligned}\Pr\Big[(\widehat{\mathsf{crs}}, \mathsf{tk}) \leftarrow \mathsf{SimSetup}(1^\lambda), \{(x_i, \pi_i)\}_{i \in [Q_s]} \leftarrow \mathcal{A}(\widehat{\mathsf{crs}}) :& \\ \forall i \in [Q_s], \mathsf{Verify}(\widehat{\mathsf{crs}}, x_i, \pi_i) = 1\Big] \geq \mu(\lambda),&\end{aligned}$$

we have

$$\begin{aligned}\Pr\Big[&(\widehat{\mathsf{crs}},\mathsf{tk})\leftarrow\mathsf{SimSetup}(1^\lambda),\{(x_i,\pi_i)\}_{i\in[Q_s]}\leftarrow\mathcal{A}(\widehat{\mathsf{crs}}),\\&\{w_i\leftarrow\mathsf{Ext}(1^\lambda,Q_H,Q_s,1/\mu,\mathsf{tk},x_i,\pi_i)\}_{i\in[Q_s]}:\\&\forall i\in[Q_s],(x_i,\pi_i)\in\mathfrak{R}\wedge\mathsf{Verify}(\widehat{\mathsf{crs}},x_i,\pi_i)=1\Big]\\&\geq\frac{1}{2}\cdot\mu(\lambda)-\mathsf{negl}(\lambda).\end{aligned}$$

Moreover, the running time of Ext *is upper bounded by* $Q_H^{e_1}\cdot Q_s^{e_2}\cdot\frac{1}{\mu^c}\cdot p(\lambda)$.

B Supplementary Material for Section 4

B.1 Correctness Proof for Construction 4.1

Lemma B.1 (Restatement of Lemma 4.2) *For parameters* N,q_2,α,γ, *the* NIZKPoK *system* $\Pi^{(1)}$ *for the relaxed language* $L_{\gamma',q_2,\bar{c}}$, *Construction 4.1 satisfies the correctness property as defined in Definition 3.1, where*

$$\gamma=\alpha\sqrt{2\cdot\Big(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\Big)\cdot N}$$

$$\gamma'\geq\left(\Big(\sqrt{k-n}+\sqrt{\ell\cdot\tau}\Big)\cdot N\cdot\alpha\sqrt{2\cdot\ell\cdot\tau}+\alpha\sqrt{\Big(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\right)$$

Proof. The correctness according to Definition 3.1 requires to prove the following three statements: (1) four algorithms (Setup,Commit,Randomize, Combine) define a correct randomizable commitment scheme; (2) the signature by algorithm Sign passes the verification algorithm, i.e., Verify; and (3) the transferred signature (with respect to the randomized commitment) from Transfer also passes Verify.

Notice that, statement (1) follows naturally from the used BDLOP commitment scheme Γ. And statement (2) simply follows from the fact that SamplePre outputs a short vector of lattice $\Lambda_u^\perp(\mathbf{F}_{\mathsf{comm}})$ with an overwhelming probability, and thus the verification would pass. To show statement (3), it suffices to show that $\mathbf{F}_{\mathsf{comm}'}\cdot\mathsf{Sig}_{\mathsf{comm}'}=\boldsymbol{u}$ (as defined in the algorithm Transfer) $\mathsf{Sig}_{\mathsf{comm}'}$ is within ℓ_2 norm $((\sqrt{k-n}+\sqrt{\ell\cdot\tau})\cdot N\cdot\alpha\sqrt{2\cdot\ell\cdot\tau}+\alpha\sqrt{\ell\cdot(2\tau+1)+\hat{\ell}+k-n)\cdot N})$, as the rest of the proof simply follows from the completeness of the NIZKPoK systems Π.

Particularly, for all $m\in\mathcal{M}\subseteq R_{q_2}$, $(\mathsf{sk},\mathsf{pk})$ output by KeyGen, and signature $\mathsf{Sig}_{\mathsf{comm}}=(\boldsymbol{s}_1^T,\boldsymbol{s}_2^T,\boldsymbol{s}_3^T)=((\boldsymbol{s}_{1,1}^T,\boldsymbol{s}_{1,2}^T),\boldsymbol{s}_2^T,\boldsymbol{s}_3^T)$ output by Sign, it holds

$$\mathbf{F}_{\mathsf{comm}}\cdot\begin{bmatrix}\boldsymbol{s}_1\\\boldsymbol{s}_2\\\boldsymbol{s}_3\end{bmatrix}=u\in\mathcal{R}_{q_2},$$

where $\mathbf{F}_{\mathsf{comm}} := \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}_2]\big|\mathbf{A}_2^\top\big]$. And the ℓ_2 norm of the vector $(\boldsymbol{s}_{1,1}^T, \boldsymbol{s}_{1,2}^T, \boldsymbol{s}_2^T, \boldsymbol{s}_3^T)$ is less than $\alpha\sqrt{2\cdot(\ell\cdot(2\tau+1)+\hat{\ell}+k-n)\cdot N}$. This implies

$$\mathbf{D}\cdot\boldsymbol{s}_{1,1}+\mathbf{A}_0\cdot\boldsymbol{s}_{1,2}+(\mathbf{B}+\mathbf{C}_2)\cdot\boldsymbol{s}_2+\mathbf{A}_2\cdot\boldsymbol{s}_3=\boldsymbol{u}\in\mathcal{R}_{q_2}^{\ell}.$$

We notice that the above equation is equivalent to

$$\begin{aligned}\boldsymbol{u}=&\mathbf{D}\cdot\boldsymbol{s}_{1,1}+\mathbf{A}_0\cdot\boldsymbol{s}_{1,2}+(\mathbf{B}+\mathbf{C}_2)\cdot\boldsymbol{s}_2+\mathbf{A}_2\cdot(\tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2)-\mathbf{A}_2\cdot(\tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2)+\mathbf{A}_2\cdot\boldsymbol{s}_3\\ =&\mathbf{D}\cdot\boldsymbol{s}_{1,1}+\mathbf{A}_0\cdot\boldsymbol{s}_{1,2}+(\mathbf{B}+\mathbf{C}_2+\mathbf{A}_2\cdot\tilde{\mathbf{R}}_2)\cdot\boldsymbol{s}_2+\mathbf{A}_2\cdot(\boldsymbol{s}_3-\tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2),\end{aligned}$$

which can be rewritten as

$$\Big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}_2]\big|\mathbf{A}_2^\top\Big]\cdot\begin{bmatrix}\boldsymbol{s}_{1,1}\\ \boldsymbol{s}_{1,2}\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3-\tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2\end{bmatrix}=\boldsymbol{u}.$$

Here we denote $\tilde{\mathbf{R}}=\mathbf{R}+\mathbf{R}'=\begin{bmatrix}\tilde{\mathbf{R}}_1\\ \tilde{\mathbf{R}}_2\end{bmatrix}\in R_N^{k\times(\ell\cdot\tau)}$, with $\tilde{\mathbf{R}}_1\in R_N^{n\times(\ell\cdot\tau)}$ and $\tilde{\mathbf{R}}_2\in R_N^{(k-n)\times(\ell\cdot\tau)}$.

Then we observe that

$$\mathbf{F}_{\mathsf{comm}'}:=\big[[\mathbf{D}|\mathbf{A}_0]\big|\mathbf{B}_{\mathsf{comm}'}\big|\mathbf{A}_2\big]=\big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}_2']\big|\mathbf{A}_2\big],$$

and $\mathsf{Sig}_{\mathsf{comm}'}:=\begin{bmatrix}\boldsymbol{s}_{1,1}\\ \boldsymbol{s}_{1,2}\\ \boldsymbol{s}_2\\ \boldsymbol{s}_3-\tilde{\mathbf{R}}_2\cdot\boldsymbol{s}_2\end{bmatrix}$. Now, it is easy to verify that the ℓ_2 norm of $\mathsf{Sig}_{\mathsf{comm}'}$ is within $((\sqrt{k-n}+\sqrt{\ell\cdot\tau})\cdot N\cdot\alpha\sqrt{2\cdot\ell\cdot\tau}+\alpha\sqrt{2\cdot(\ell\cdot(2\tau+1)+\hat{\ell}+k-n)\cdot N})$ and $\mathbf{F}_{\mathsf{comm}'}\cdot\mathsf{Sig}_{\mathsf{comm}'}=\boldsymbol{u}$, since for such a matrix $\widetilde{\mathbf{R}}_2\in S_1^{(k-n)\times(\ell\cdot\tau)}$, its singular value $s_1(\widetilde{\mathbf{R}}_2)$ is bounded by $(\sqrt{k-n}+\sqrt{\ell\cdot\tau})\sqrt{N}$. This completes the proof. □

B.2 Simulatability Proof for Construction 4.1

Lemma B.2 (Restatement of Lemma 4.5) *The algorithm* Transfer *in Construction 4.1 is simulatable.*

Proof. According to Definition 3.2, we need to first construct a two-stage PPT simulator $\mathcal{S}$, and then prove that after running any polynomial $t=\mathsf{poly}(\lambda)$ times, the distribution of $\{\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'_i}\}_{i\in[t]}$ output by $\mathcal{S}$ are statistically close to that of $\{\mathsf{Sig}'_{\mathsf{comm}'_i}\}_{i\in[t]}$ output by Transfer.

Particularly, the two-stage PPT simulator $\mathcal{S}$ can be constructed in the following way:

– First Stage: $\mathcal{S}$ conducts the following steps:
  1. Generate and output $(\mathbf{A},\mathbf{D},q_1,q_2,N,\kappa,\gamma,\gamma',\alpha,\mathcal{M},\mathcal{R},\mathsf{crs})$.

– Second Stage: given params, and valid pk, comm', $\mathcal{S}$ conducts the following steps:
  1. Recognize pk as $(\mathbf{A}_0, \mathbf{B}, \boldsymbol{u})$.
  2. Parse $\mathsf{comm}' := \mathbf{C}' = \begin{bmatrix} \mathbf{C}'_1 \\ \mathbf{C}'_2 \end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$;
  3. Set matrix
$$\mathbf{F}_{\mathsf{comm}'} := \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{B}+\mathbf{C}'_2]\big|\mathbf{A}_2\big],$$
  4. With respect to the $\mathsf{NIZKPoK}$ system Π for the relaxed language $L_{\gamma',q_2,\bar{\mathcal{C}}}$,
$$L_{\gamma',q_2,\bar{\mathcal{C}}} = \Big\{(\mathbf{F}_{\mathsf{comm}'}, \boldsymbol{u}) \in R_{N,q_2}^{\ell\times(\ell\cdot(2\tau+1)+\hat{\ell}+k-n)} \times R_{N,q_2}^{\ell} : \exists\, \boldsymbol{x} \in R^{\ell\cdot(2\tau+1)+\hat{\ell}+k-n} \text{ and } f \in \bar{\mathcal{C}} \text{ such that } 0 < \|\boldsymbol{x}\| \le \gamma' \text{ and } \mathbf{F}_{\mathsf{comm}'}\cdot\boldsymbol{x} = f\cdot u\Big\}.$$
     we can run the corresponding simulation algorithm to generate a simulated proof π', whose distribution is statistically indistinguishable from that of the real proof π.
  5. Output $\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'} := \pi'$.

According to the zero knowledge property of the used $\mathsf{NIZKPoK}$ system $\Pi^{(1)}$, it is clear that after running any polynomial $\varrho = \mathsf{poly}(\lambda)$ times, the distribution of $\{\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by $\mathcal{S}$ are statistically close to that of $\{\mathsf{Sig}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by $\mathsf{Transfer}$. □

B.3 Unforgeability Proof for Construction 4.1

Lemma B.3 (Restatement of Lemma 4.6) *Assume that* $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ *problem and* $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ *problem are hard with*

$$\begin{aligned}\nu =& \alpha\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N} \\ &+ \Big(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\Big)\cdot\alpha\cdot N\sqrt{2\cdot\ell\cdot\tau}+1\end{aligned}$$

$$\begin{aligned}\nu' =& \alpha'\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N} \\ &+ \Big(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\Big)\cdot\alpha'\cdot N\sqrt{2\cdot\ell\cdot\tau}+2\sqrt{\kappa},\end{aligned}$$

where $\alpha' = \gamma'/\sqrt{2\cdot\Big(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\Big)\cdot N}$. *Then our above lattice-based commitment-transferrable signature scheme is partially selectively unforgeable for the exact commitment relation* $\hat{L}_{q_1,q_2}$, *i.e., the advantage of any* PPT *adversary* $\mathcal{A}$ *against the partially selective unforgeability game of* CTS *is at most*

$$\mathsf{Adv}_{\mathcal{A}}^{unforge}(\lambda) \le 2\mathsf{Adv}_{\mathcal{A}}^{\mathsf{M\text{-}LWE}} + \mathsf{Adv}_{\mathcal{A}}^{unforge^*}(\lambda).$$

Proof. We argue the unforgeability using the series of hybrids.

$\mathsf{H_0}$: The challenger $\mathcal{B}$ runs the CTS honestly. He gives to the adversary $\mathcal{A}$ the public key pk and signatures with respect to the queried commitments comm_i. In this hybrid, we say $\mathcal{A}$ has advantage $\varepsilon = \mathsf{Adv}_{\mathcal{A}}^{\text{unforge}}(\lambda)$ in the unforgeability game. Then, it holds

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_0}(\lambda) = \mathsf{Adv}_{\mathcal{A}}^{\text{unforge}}(\lambda).$$

$\mathsf{H_1}$: The challenger $\mathcal{B}$ runs the identical procedures as H_0, except that he samples $\mathbf{R}_0 \xleftarrow{\$} S_1^{(\ell+\hat{\ell})\times(\ell\cdot\tau)}$, and set $\mathbf{B} = \mathbf{D}\cdot\mathbf{R}_0 - m\mathbf{G} \in R_{q_2}^{\ell\times(\ell\cdot\tau)}$. Here, we use m^* to denote the committed message in the challenge commitment comm^*. According to the $\mathsf{M\text{-}LWE}_{q_2,\hat{\ell},\ell,S_1}$ assumption, we know that H_0 and H_1 are computationally indistinguishable. Then, it holds

$$|\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_0}(\lambda) - \mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_1}(\lambda)| \leq \mathsf{Adv}_{\mathcal{A}}^{\mathsf{M\text{-}LWE}}(\lambda).$$

$\mathsf{H_2}$: The challenger $\mathcal{B}$ runs the identical procedures as H_1, except that he samples $\mathbf{A}_0 \xleftarrow{\$} R^{\ell\times(\ell\cdot\tau)}$, and $\mathcal{B}$ answers the signature queries through using Lemma A.7, rather than Lemma A.8. According to the $\mathsf{M\text{-}LWE}$ assumption, we know that H_1 and H_2 are computational indistinguishability. Then, it holds

$$|\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_1}(\lambda) - \mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_2}(\lambda)| \leq \mathsf{Adv}_{\mathcal{A}}^{\mathsf{M\text{-}LWE}}(\lambda).$$

Besides, we denote the challenger in H_2 as $\mathcal{B}^*$. Thus, we have

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_2}(\lambda) = \mathsf{Adv}_{\mathcal{A}}^{\text{unforge}^*}(\lambda).$$

Lemma B.4 *Let $\mathcal{A}$ be a* PPT *adversary with advantage ε in the selective unforgeability game with respect to $\mathcal{B}^*$ for the exact commitment relation $\hat{L}_{q_1,q_2}$, i.e., $\mathsf{Adv}_{\mathcal{A}}^{unforge^*}(\lambda) = \varepsilon$. Let h be a bound on the number of random oracle queries made by $\mathcal{A}$. Let*

$$\begin{aligned}\nu =& \alpha\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha\cdot N\sqrt{2\cdot\ell\cdot\tau}+1\end{aligned}$$

$$\begin{aligned}\nu' =& \alpha'\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha'\cdot N\sqrt{2\cdot\ell\cdot\tau}+2\sqrt{\kappa},\end{aligned}$$

where $\alpha' = \gamma'/\sqrt{2\cdot\Big(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\Big)\cdot N}$. Then there exists a reduction algorithm $\mathcal{R}$ for $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ or $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ such that

$$\mathsf{Adv}_{\mathcal{R}}^{\mathsf{M\text{-}SIS}}(\lambda) \geq \varepsilon\left(\frac{\varepsilon}{h} - 2^{-\lambda}\right).$$

Proof. According to our construction, the verifier needs to consider two cases: original signature and transferred signature. Thus, we need to prove the unforgeability for both cases. Overall, both of them have the similar proof process, and are based on the hardness of $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ and $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ problems, respectively. Below, we present the details for both cases in an unified form, and just separate in their different points.

Particularly, we prove that if the adversary $\mathcal{A}$ can forge a valid original/transferred signature in the selective way, then we can construct an efficient reduction algorithm $\mathcal{B}$ to solve the $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$/ $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ problem. In particular, $\mathcal{B}$ is given an uniformly random matrix $\mathbf{X} = \left[\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+k-n+1}\right] \in R_{q_2}^{\ell\times(\ell(\tau+1)+\hat{\ell}+k-n+1)}$, and need to output a vector $\boldsymbol{y}$ such that $\mathbf{X}\cdot\boldsymbol{y} = 0 \bmod q_2$ and

$$\begin{aligned}\|\boldsymbol{y}\| \leq \nu =& \alpha\sqrt{2\cdot\left(\ell(\tau+1)+\hat{\ell}+k-n\right)\cdot N} \\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha\cdot N\sqrt{2\cdot\ell\cdot\tau}+1\end{aligned}$$

or

$$\begin{aligned}\|\boldsymbol{y}\| \leq \nu' =& \alpha'\sqrt{2\cdot\left(\ell(\tau+1)+\hat{\ell}+k-n\right)\cdot N} \\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha'\cdot N\sqrt{2\cdot\ell\cdot\tau}+2\sqrt{\kappa},\end{aligned}$$

with $\alpha' = \gamma'/\sqrt{2\cdot\left(\ell\cdot(2\tau+1)+\hat{\ell}+k-n\right)\cdot N}$. Similar to the consideration in [22], we choose to use

$$\mathbf{X} = \left[\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1}, \mathbf{I}_\ell, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+2}, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1+k-n-\ell}\right],$$

which is a permute of the hermite norm form of the original matrix.

In this case, $\mathcal{B}$ conducts the following steps:

1. Choose $\mathbf{A}_1' \xleftarrow{\$} R_{N,q_1}^{n\times(k-n)}$ and set $\mathbf{A}_1 = [\mathbf{I}_n, \mathbf{A}_1'] \in R_{N,q_1}^{n\times k}$.
2. Set $\mathbf{A}_2 = (\mathbf{I}_\ell, \mathbf{A}_2') \in R_{q_2}^{\ell\times(k-n)}$, where
$$\mathbf{A}_2' = \left[\boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+2}, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1+k-n-\ell}\right].$$
3. Set $\mathbf{A} = \begin{bmatrix}\mathbf{A}_1 \\ 0, \mathbf{A}_2\end{bmatrix}$ and send it to $\mathcal{A}$.

Clearly, $\mathbf{A}$ is a valid public parameter output by $\Gamma.\mathsf{CKeyGen}$.

Next, we need to argue that $\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully for the exact commitment relation $\hat{L}_{q_1,q_2}$. In particular, we use the following Claim B.5 to specify the case.

Claim B.5 *$\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully in the unforgeability game with respect to the exact commitment relation $\hat{L}_{q_1,q_2}$.*

Proof. With this $\mathbf{A}$, according to Remark 3.6 of Definition 3.3, $\mathcal{A}$ can commit to the challenge message m^* at the beginning of unforgeability game.

Then $\mathcal{B}$ can set the public parameters in the following way:

1. Set $\mathbf{D} = (\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\ell+\hat{\ell}}) \in R_{q_2}^{\ell \times 2\ell}$, $\mathbf{A}_0 = (\boldsymbol{x}_{\ell+\hat{\ell}+1}, \ldots, \boldsymbol{x}_{\ell+\hat{\ell}+\ell\cdot\tau}) \in R_{q_2}^{\ell\times(\ell\cdot\tau)}$, $\boldsymbol{u} = \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1} \in R_{q_2}^{\ell}$.
2. Sample $\mathbf{R}_0 \xleftarrow{\$} S_1^{(\ell+\hat{\ell})\times(\ell\cdot\tau)}$, and set $\mathbf{B} = \mathbf{D}\cdot\mathbf{R}_0 - m^*\cdot\mathbf{G} \in R_{q_2}^{\ell\times(\ell\cdot\tau)}$, where $\mathbf{G} = \mathbf{I}_n \otimes (1, \delta, \ldots, \delta^{\tau-1})$ and $\tau = \lfloor q_2^{1/\tau} \rceil$;
3. Send $\mathsf{pk} := (\mathbf{A}_0, \mathbf{B}, \boldsymbol{u})$ to $\mathcal{A}$.

According to the uniformity of $\boldsymbol{x}_{\ell+\hat{\ell}+1}, \ldots, \boldsymbol{x}_{\ell+\hat{\ell}+\ell\cdot\tau}$ and the distribution of $\mathbf{R}_0$, pk is a valid public key of our commit-transferrable signature, which follows from the $\mathsf{M\text{-}LWE}$ assumption.

Then, the $\mathcal{A}$ can conduct signature queries and get responses from $\mathcal{B}$. In particular, after receiving the signature query $(\mathsf{comm}, m, \mathbf{R})$ from $\mathcal{A}$, where $\mathsf{comm} := \mathbf{C} = \begin{bmatrix}\mathbf{C}_1\\ \mathbf{C}_2\end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$ and

$$\mathbf{C}_1 := \mathbf{A}\cdot\mathbf{R},$$

$$\mathbf{C}_2 := \mathbf{A}\cdot\mathbf{R} + m\cdot\mathbf{G}.$$

$\mathcal{B}$ can compute

$$\begin{aligned}\mathbf{F}_{\mathsf{comm}} &= \left[[\mathbf{D}|\mathbf{A}_0]\middle|[\mathbf{B}+\mathbf{C}_2]\middle|\mathbf{A}_2\right]\\ &= \left[[\mathbf{D}|\mathbf{A}_0]\middle|[\mathbf{D}\cdot\mathbf{R}_0 - m^*\cdot\mathbf{G} + \mathbf{C}_2]\middle|\mathbf{A}_2\right]\\ &= \left[[\mathbf{D}|\mathbf{A}_0]\middle|[\mathbf{D}\cdot\mathbf{R}_0 + \mathbf{A}_2\cdot\mathbf{R}_2 + (m-m^*)\cdot\mathbf{G}]\middle|\mathbf{A}_2\right],\end{aligned}$$

where we denote $\mathbf{R} = \begin{bmatrix}\mathbf{R}_1\\ \mathbf{R}_2\end{bmatrix} \in R^{k\times(\ell\cdot\tau)}$ with $\mathbf{R}_2 \in R^{(k-n)\times(\ell\cdot\tau)}$.

For any $m \neq m^*$, we know that $m - m^*$ is invertible in R_{q_2}, due to the fact that $\|m-m^*\|_\infty$ is small enough. According to the algorithm in Lemma A.7, the challenger can get a short vector $\boldsymbol{z} \in R^{\ell\cdot(2\tau+1)+\hat{\ell}+k-n}$ such that $\mathbf{F}_{\mathsf{comm}}\cdot\boldsymbol{z} = \boldsymbol{u}$. □

From above Claim B.5, we know that $\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully.

Next, for the challenge query, the adversary sends randomness $\mathbf{R}^*$ to the challenger, such that the final challenge query is of the form $(\mathsf{comm}^*, m^*, \mathbf{R}^*)$, where where $\mathsf{comm}^* := \mathbf{C}^* = \begin{bmatrix}\mathbf{C}_1^*\\ \mathbf{C}_2^*\end{bmatrix} \in R_{N,q_2}^{(n+\ell)\times(\ell\cdot\tau)}$ and

$$\mathbf{C}_1^* := \mathbf{A}\cdot\mathbf{R}^*,$$

$$\mathbf{C}_2^* := \mathbf{A}\cdot\mathbf{R}^* + m^*\cdot\mathbf{G}.$$

In this case, we have

$$\mathbf{F}_{\mathsf{comm}^*} = \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{D}\cdot\mathbf{R}_0+\mathbf{A}_2\cdot\mathbf{R}_2^*]\big|\mathbf{A}_2\big],$$

where we denote $\mathbf{R}^* = \begin{bmatrix}\mathbf{R}_1^*\\ \mathbf{R}_2^*\end{bmatrix} \in R^{k\times(\ell\cdot\tau)}$ with $\mathbf{R}_2^* \in R^{(k-n)\times(\ell\cdot\tau)}$.

Below, according to the fact that the adversary's forgery is for original signature or transferred one, we need to separate the following proof into two cases.

For the case of original one. If the adversary can forge a valid signature $\mathsf{Sig}_{\mathsf{comm}^*} := \begin{bmatrix}\boldsymbol{s}_{1,1}^*\\ \boldsymbol{s}_{1,2}^*\\ \boldsymbol{s}_2^*\\ \boldsymbol{s}_3^*\end{bmatrix}$, such that

$$\begin{aligned}\mathbf{F}_{\mathsf{comm}^*}\cdot\mathsf{Sig}_{\mathsf{comm}^*} &= \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{D}\cdot\mathbf{R}_0+\mathbf{A}_2\cdot\mathbf{R}_2^*]\big|\mathbf{A}_2\big]\cdot\begin{bmatrix}\boldsymbol{s}_{1,1}^*\\ \boldsymbol{s}_{1,2}^*\\ \boldsymbol{s}_2^*\\ \boldsymbol{s}_3^*\end{bmatrix}\\ &= \mathbf{D}\cdot\boldsymbol{s}_{1,1}^*+\mathbf{A}_0\cdot\boldsymbol{s}_{1,2}^*+(\mathbf{D}\cdot\mathbf{R}_0+\mathbf{A}_2\cdot\mathbf{R}_2^*)\cdot\boldsymbol{s}_2^*+\mathbf{A}_2\cdot\boldsymbol{s}_3^*\\ &= \boldsymbol{u},\end{aligned}$$

then $\mathcal{B}$ can compute $\boldsymbol{y} = \begin{bmatrix}\boldsymbol{s}_{1,1}^*+\mathbf{R}_0\cdot\boldsymbol{s}_2^*\\ \boldsymbol{s}_{1,2}^*\\ -1\\ \mathbf{R}_2^*\cdot\boldsymbol{s}_2^*+\boldsymbol{s}_3^*\end{bmatrix}$ as a solution to the $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ problem defined by

$$\mathbf{X} = [\boldsymbol{x}_1,\ldots,x_{\ell(\tau+1)+\hat{\ell}+1},\mathbf{I}_\ell,\boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+2},\ldots,\boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1+k-n-\ell}].$$

And the ℓ_2 norm of this solution is less than

$$\begin{aligned}\|\boldsymbol{y}\| \le \nu =& \alpha\sqrt{2\cdot\Big(\ell(\tau+1)+\hat{\ell}+k-n\Big)\cdot N}\\ &+\left(\sqrt{\ell+\hat{\ell}}+\sqrt{k-n}+2\sqrt{\ell\cdot\tau}\right)\cdot\alpha\cdot N\sqrt{2\cdot\ell\cdot\tau}+1.\end{aligned}$$

For the case of transferred one. If the adversary can forge a valid proof for the language $L_{\gamma',q_2,\bar{\mathcal{C}}}$, then the reduction algorithm $\mathcal{B}$ can run the extractor of the $\mathsf{NIZKPoK}$ system Π_2, and get a ℓ_2 norm short vector $\mathsf{Sig}'_{\mathsf{comm}^*} := \begin{bmatrix}\boldsymbol{s}_{1,1}^*\\ \boldsymbol{s}_{1,2}^*\\ \boldsymbol{s}_2^*\\ \boldsymbol{s}_3^*\end{bmatrix}$, such that

$$\begin{aligned}\mathbf{F}_{\mathsf{comm}^*}\cdot\mathsf{Sig}_{\mathsf{comm}^*} &= \big[[\mathbf{D}|\mathbf{A}_0]\big|[\mathbf{D}\cdot\mathbf{R}_0+\mathbf{A}_2\cdot\mathbf{R}_2^*]\big|\mathbf{A}_2\big]\cdot\begin{bmatrix}\boldsymbol{s}_{1,1}^*\\ \boldsymbol{s}_{1,2}^*\\ \boldsymbol{s}_2^*\\ \boldsymbol{s}_3^*\end{bmatrix}\\ &= \mathbf{D}\cdot\boldsymbol{s}_{1,1}^*+\mathbf{A}_0\cdot\boldsymbol{s}_{1,2}^*+(\mathbf{D}\cdot\mathbf{R}_0+\mathbf{A}_2\cdot\mathbf{R}_2^*)\cdot\boldsymbol{s}_2^*+\mathbf{A}_2\cdot\boldsymbol{s}_3^*\\ &= \bar{c}\cdot\boldsymbol{u},\end{aligned}$$

then $\mathcal{B}$ can compute $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{s}^*_{1,1} + \mathbf{R}_0 \cdot \boldsymbol{s}^*_2 \\ \boldsymbol{s}^*_{1,2} \\ -\bar{c} \\ \mathbf{R}^*_2 \cdot \boldsymbol{s}^*_2 + \boldsymbol{s}^*_3 \end{bmatrix}$ as a solution to the $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ problem defined by

$$\mathbf{X} = \left[\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1}, \mathbf{I}_\ell, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+2}, \ldots, \boldsymbol{x}_{\ell(\tau+1)+\hat{\ell}+1+k-n-\ell}\right]$$

And the ℓ_2 norm of this solution is less than

$$\begin{aligned} \|\boldsymbol{y}\| \leq \nu' =& \alpha' \sqrt{2 \cdot \left(\ell(\tau+1) + \hat{\ell} + k - n\right) \cdot N} \\ &+ \left(\sqrt{\ell + \hat{\ell}} + \sqrt{k-n} + 2\sqrt{\ell \cdot \tau}\right) \cdot \alpha' \cdot N\sqrt{2 \cdot \ell \cdot \tau} + 2\sqrt{\kappa}, \end{aligned}$$

with $\alpha' = \gamma' / \sqrt{2 \cdot \left(\ell \cdot (2\tau+1) + \hat{\ell} + k - n\right) \cdot N}$.

Furthermore, according to the forking lemma of [8, 50], $\mathcal{R}$ can complete the above reduction with probability at least $\varepsilon(\frac{\varepsilon}{h} - 2^{-\lambda})$.

Summing up all above arguments, we conclude that our commit transferrable signature satisfies unforgeability in the selective way. □

□

C Parameter Settings of Construction 4.1 and NIZKPoK system in Section 5

In this section, we set the concrete parameters for Construction 4.1 and the straight-line extractable NIZKPoK system, according to the related requirements in correctness and security. For clarity, we denote the straight-line extractable NIZKPoK system for $\hat{L}_{q_1,q_2}$ in Section 5 as Π_1, and denote the NIZKPoK system for $L_{\gamma',q_2,\bar{c}}$ in Section 4.2 as Π_2.

Requirements for Correctness. We require the following:

- The SamplePre in the Sign step needs to work properly. According to Lemma A.3, we need to set $\alpha \geq 2\sqrt{\delta^2+1} \cdot ((\sqrt{2\ell} + \sqrt{\ell \cdot \tau})\sqrt{N} + 1)$.
- The valid original signature $\mathsf{Sig}_{\mathsf{comm}}$ can be verified successfully. According to Lemma A.2, we need to set $\gamma = \alpha\sqrt{2 \cdot (\ell \cdot (2\tau+1) + \hat{\ell} + k - n) \cdot N}$.
- The valid transferred signature $\mathsf{Sig}'_{\mathsf{comm}'}$ can be verified successfully. According to Lemma 4.2 and the relaxed language in Theorem 4.3, we need to set

$$\begin{aligned} \gamma' = & 2\sqrt{2 \cdot \left(\ell \cdot (2\tau+1) + \hat{\ell} + k - n\right) \cdot N} \cdot \eta \cdot \kappa \\ & \cdot \left((\sqrt{k-n} + \sqrt{\ell \cdot \tau}) \cdot N \cdot \alpha\sqrt{2 \cdot \ell \cdot \tau} + \alpha\sqrt{2 \cdot (\ell \cdot (2\tau+1) + \hat{\ell} + k - n) \cdot N}\right). \end{aligned}$$

– The message space of Construction 5.1 is large enough to encrypt all randomness. And, the related Ciphertexts can be decrypted correctly. According to the corresponding analysis, we need set $k_{\mathsf{PKE}} = \ell \cdot \tau \cdot N/d \cdot k + 1$, $m_{\mathsf{PKE}} = 2 \cdot n_{\mathsf{PKE}} + k_{\mathsf{PKE}}$, $q_{\mathsf{PKE}}/2 > 12 \cdot m_{q_{\mathsf{PKE}}} \cdot d + 1$, with $\mathbf{A}_{\mathsf{PKE}} \in R_{d,q_{\mathsf{PKE}}}^{n_{\mathsf{PKE}} \times m_{\mathsf{PKE}}}$, $\mathbf{B}_{\mathsf{PKE}} \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}} \times m_{\mathsf{PKE}}}$.

Requirements for Security. We require the following:

– The ring R_N, R_d are cyclotomic rings, i.e., $R_N = \mathbb{Z}[X]/(X^N + 1)$, $R_d = \mathbb{Z}[X]/(X^d + 1)$, with $d|N$. In this case, according to the efficiently computable ring isomorphism between R_N and $R_d^{N/d}$, any relations we need to prove over R_N can be proven by showing the corresponding relations over R_d is set up.
– For the fixed security parameter λ, we require that the output distribution of the rejection sampling algorithm is within statistical distance of $\frac{2^{-\lambda}}{M}$ of the related product distribution, according to Lemma A.9. Thus, we need to set η satisfying $M = \exp\left(\sqrt{\frac{2(\lambda+1)}{\log e}} \cdot \frac{1}{\eta} + \frac{1}{2\eta^2}\right) = O(1)$.
– There exists a multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ system Π_1 for the commitment relation $\hat{L}_{q_1,q_2}$. Hence, in order to make Construction 5.1 to be $\mathsf{IND\text{-}CPA}$ security, $\mathsf{M\text{-}LWE}_{q_{\mathsf{PKE}},n_{\mathsf{PKE}},m_{\mathsf{PKE}},\hat{S}_2}$ need to be hard. Other hard problems for the concrete instantiation of LNP proof are implicitly considered in the parameter setting of Table 11.
– There exists a $\mathsf{NIZKPoK}$ system Π_2 for the language $L_{\gamma',q_2,\bar{\mathcal{C}}}$, according to Theorem 4.3. Thus, in order to make this language is hard, the problem $\mathsf{M\text{-}SIS}_{q_2,\ell,(\ell\cdot(2\tau+1)+\hat{\ell}+k-n),\gamma'}$ needs to be hard.
– The constructed CTS satisfies unforgeability in Definition 3.3. Hence, For Definition 3.3 with respect to the exact commitment relation $\hat{L}_{q_1,q_2}$, according to Lemma 4.6, and Claim B.5, we need to set $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu}$ problem and $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$ problem are hard with

$$\begin{aligned}\nu =& \alpha\sqrt{2 \cdot \Big(\ell(\tau+1) + \hat{\ell} + k - n\Big) \cdot N} \\ &+ \left(\sqrt{\ell + \hat{\ell}} + \sqrt{k-n} + 2\sqrt{\ell \cdot \tau}\right) \cdot \alpha \cdot N\sqrt{2 \cdot \ell \cdot \tau} + 1\end{aligned}$$

$$\begin{aligned}\nu' =& \alpha'\sqrt{2 \cdot \Big(\ell(\tau+1) + \hat{\ell} + k - n\Big) \cdot N} \\ &+ \left(\sqrt{\ell + \hat{\ell}} + \sqrt{k-n} + 2\sqrt{\ell \cdot \tau}\right) \cdot \alpha' \cdot N\sqrt{2 \cdot \ell \cdot \tau} + 2\sqrt{\kappa},\end{aligned}$$

where $\alpha' = \gamma'/\sqrt{2 \cdot \Big(\ell \cdot (2\tau+1) + \hat{\ell} + k - n\Big) \cdot N}$.
– The underlying BDLOP satisfies hiding and binding. Hence, according to Section 2.3, we need to set $\mathsf{M\text{-}LWE}_{q_2,k-n-\ell,n+\ell,S_1}$ and $\mathsf{M\text{-}SIS}_{q_1,n,k,8\sqrt{2}\cdot\eta\cdot\kappa\cdot\beta\cdot k\cdot N}$ being hard.

– The successful simulation of the adversary in Claim B.5. Here, according to the Lemma A.7, we need to set $\alpha \geq 2\sqrt{\delta^2+1}\cdot((\sqrt{\ell+\hat{\ell}+k-n}+\sqrt{\ell\cdot\tau})\cdot\sqrt{N}+1)$.

Concrete Parameter instantiations. From the above analysis, according to unforgeability for exact commitment relation $\hat{L}_{q_1,q_2}$, we give the specific parameter setting as in Table 10. Moreover, we use LNP techniques, i.e., Figure 10 in [41], to instantiate the multi-theorem straight-line extractable NIZKPoK, as presented in Section 5. Thus, we set the concrete related parameters as in Table 11, and denote the related proof size as $\mathsf{size}_{\mathsf{LNP}}$ in the final computation on the pseudonym size of our Anonymous Credentials system.

	description	Params 1	Params 2
N	dimension of ring for CTS	2048	4096
d	dimension of ring for LNP	512	1024
t	$N = t \cdot d$	4	4
q_1	top modulus for BDLOP	$2^{24}-75$	$2^{26}-371$
q_2	bottom modulus for BDLOP	$2^{79}-67$	$2^{88}-299$
n	row number $\mathbf{A}_1$ for BDLOP	1	1
k	column number $\mathbf{A}_1$ for BDLOP	4	4
ℓ	row number of $\mathbf{A}_2$	1	1
$\hat{\ell}$	column number of $\mathbf{D}$	2	2
τ	dimension of $\boldsymbol{g} = (1, \delta, \ldots, \delta^{\tau-1})$ with $\delta = q_2^{1/\tau}$	7	5
ω	ℓ_1-norm of message $m \in \mathcal{M}$	22	18
ζ	2^{ζ} is the size of message space $\mathcal{M}$	≈ 128	≈ 128
λ	security target	128	256
$\hat{k}$	repetition times for attribute disclosure	2	3
M	abort time for rejection sampling	6	6
η	$\sigma = \eta \cdot T$ is the standard deviation for rejection sampling	7.6	10.6
κ	ℓ_1-norm of c in $\mathcal{C}$	14	27
α	standard deviation of original signature	$2^{20.14}$	$2^{26.83}$
γ	ℓ_2-norm of original signature	$2^{28.3}$	$2^{35.33}$
γ'	parameter for the relaxed language $L_{\gamma',q_2,\bar{\mathcal{C}}}$	$2^{51.07}$	$2^{60.14}$
ν'	bound for $\mathsf{M\text{-}SIS}_{q_2,\ell,\ell(\tau+1)+\hat{\ell}+k-n+1,\nu'}$	$2^{58.95}$	$2^{68.30}$
δ_0	Root-hermite factor	1.003729	1.002245
Bit-sec	classcial bit security	128.35	257.72

Table 10. Concrete Settings for the Parameters and the Related Security in the case of selective unforgeability with exact relation.

Below, we roughly explain about the calculations of these two tables.

– According to the used rejection sampling algorithms in Lemma A.9, we need to set the parameter η to satisfy the $M = \exp\left(\sqrt{\frac{2(\lambda+1)}{\log e}} \cdot \frac{1}{\eta} + \frac{1}{2\eta^2}\right)$, for any fixed M and λ.

– Given the concrete value of N, we need to fix κ such that the size of the challenge sets are larger than 2^λ, i.e., $\binom{N}{\kappa} \times 2^\kappa \geq 2^\lambda$.
– According to the used $\mathsf{NIZKPoK}$ system Π_2 in Theorem 4.3, we need to set

$$\gamma' = 2\sqrt{2 \cdot \Big(\ell \cdot (2\tau+1) + \hat{\ell} + k - n\Big) \cdot N} \cdot \eta \cdot \kappa$$
$$\cdot \left((\sqrt{k-n} + \sqrt{\ell \cdot \tau}) \cdot N \cdot \alpha\sqrt{2 \cdot \ell \cdot \tau} + \alpha\sqrt{2 \cdot (\ell \cdot (2\tau+1) + \hat{\ell} + k - n) \cdot N} \right).$$

, such that the assumption $\mathsf{M\text{-}SIS}_{q_2,\ell,(\ell(\tau+1)+\hat{\ell}+k-n),\gamma'}$ is hard, and a $\mathsf{NIZKPoK}$ system Π_2 exists for the relaxed language $L_{\gamma',q_2,\bar{c}}$.
– Given N, q_2, κ, we can calculate the values of α, γ, γ' (all these parameters need to be used in the description of our CTS in Construction 4.1), according to the above parameter analysis for correctness and security.
– We can further compute the values of ν, ν' (all these parameters need to be used to ensure the security proof of our CTS in Construction 4.1), as the requirement of security proof.

During the above calculation process, we use the Root-Hermite Factor δ_0 to estimate bit-hardness of the underlying assumptions, i.e., $\mathsf{M\text{-}SIS}$ and $\mathsf{M\text{-}LWE}$, according to the best known attacks, and δ_0 can be determined given N, q_1, q_2, α. Generally, we can use the work [3, 4, 30] to estimate δ_0 and its corresponding hardness of the assumptions.

Our reduction from each building block is essentially tight (by calling the adversary a constant number of times), so the attained security of our construction is essentially the same as that of the underlying $\mathsf{M\text{-}LWE}$ and $\mathsf{M\text{-}SIS}$ problems.

Size Computation. Based on the above parameters on CTS and multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ listed in Tables 10 and 11, the size of public parameter of CTS is about

$$|\mathsf{para}_{\mathsf{CTS}}| := n\cdot(k-n)\cdot N\lceil\log q_1\rceil + \ell\cdot(\hat{\ell}+k-n)\cdot N\lceil\log q_2\rceil + \log(\alpha\cdot N\cdot q_1\cdot q_2\cdot\kappa\cdot\gamma\cdot\gamma') \text{ bits.}$$

The additional size of public parameter of multiple-theorem straight-line extractable $\mathsf{NIZKPoK}$ consists of $|\mathsf{para}_{\mathsf{ABDLOP}}|$ and $|\mathsf{para}_{\mathsf{PKE}}|$, where

$$|\mathsf{para}_{\mathsf{ABDLOP}}| := (n^*\cdot(m_1+m_2+v_e)+(\ell^*+2)\cdot m_2)\cdot d\lceil\log(q_{\mathsf{LNP}})\rceil + 512\cdot m_2\cdot\lceil\log(q_{\mathsf{LNP}})\rceil,$$

$$|\mathsf{para}_{\mathsf{PKE}}| := (n_{\mathsf{PKE}} + k_{\mathsf{PKE}}) \cdot m_{\mathsf{PKE}} \cdot d \cdot \lceil\log(q_{\mathsf{PKE}})\rceil \text{ bits.}$$

Besides, the size of public parameter for the disclosure of attributes is about

$$|\mathsf{para}_{\mathsf{Disclosure}}| := k \cdot \hat{k} \cdot N \cdot \lceil\log q_2\rceil.$$

Thus, the total size of public parameter is about

$$|\mathsf{para}_{\mathsf{CTS}}| + |\mathsf{para}_{\mathsf{ABDLOP}}| + |\mathsf{para}_{\mathsf{PKE}}| + |\mathsf{para}_{\mathsf{Disclosure}}|.$$

variable	description	Para. 1	Para. 2
q_1	modulus for the BDLOP commitment	$2^{24}-75$	$2^{26}-371$
q_2		$2^{79}-67$	$2^{88}-299$
n	dimensions for the BDLOP commitment	1	1
ℓ		1	1
k		4	4
τ		7	5
d	dimension for the underlying ring of LNP	512	1024
t	$N = t \cdot d$ is the dimension for the ring of CTS	4	4
ω	# 1's in the identity $m \in \mathcal{M}$	22	18
$\lvert\mathcal{M}\rvert$	size of the user space	$\approx 2^{128}$	$\approx 2^{128}$
q_{PKE}	encryption modulus	1437757	2039837
n_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	2	1
m_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	117	83
k_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	113	81
δ_0^{PKE}	root-hermite factor of PKE	1.003433	1.00351
$\mathsf{Bit\text{-}sec}^{\mathsf{PKE}}$	classcial/quantum security of PKE	144.25/130.91	139.87/126.93
q_{LNP}	modulus for the proof system	$\approx 2^{103}$	$\approx 2^{114}$
l	# factors X^d+1 splits into mod q_{LNP}	2	2
γ_1	rejection sampling constant for $c\boldsymbol{s}_1$	17	17
γ_2	rejection sampling constant for $c\boldsymbol{s}_2$	1.2	1.2
$\gamma^{(e)}$	rejection sampling constant exact ARP	2.5	2.5
$\gamma^{(d)}$	rejection sampling constant non-exact ARP	12	12
η^*	upper bound of $\sqrt[2k]{\lVert c^{2k}\rVert}$ for $k=32$	23	19
κ^*	maximum coefficient of a challenge in $\mathcal{C}$	1	1
n^*	height of matrices $\mathbf{A}_1, \mathbf{A}_2$ in ABDLOP	2	1
m_1	length of the message $\boldsymbol{s}_1$ in the"Ajtai" part	581	413
ℓ^*	length of the message $\boldsymbol{m}$ in the"BDLOP " part	0	0
m_2	length of the message $\boldsymbol{s}_2$ in ABDLOP	18	13
ν^*	randomness $\boldsymbol{s}_2$ is sampled from $S_{\nu^*}^{m_2}$	1	1
γ^*	parameters to cut low-order bits from $\boldsymbol{w}$	$\approx 2^{35.69}$	$\approx 2^{37.81}$
D^*	number of low-order bits cut from $\boldsymbol{t}_A$	27	28
	repetition rate	7	7
$\lvert\mathsf{size}_{\mathsf{LNP}}\rvert$	proof size of the straight-line extractable NIZKPoK	1007.41 KB	1469.25 KB
$\lvert\mathsf{para}_{\mathsf{ABDLOP}}\rvert$	public parameter size of ABDLOP commitment	8162.75 KB	6594.19 KB
$\lvert\mathsf{para}_{\mathsf{PKE}}\rvert$	public parameter size of PKE encryption	17659.69 KB	17865.75 KB
$\lvert\mathsf{ct}_{\mathsf{PKE}}\rvert$	ciphertext size of PKE encryption	150.94 KB	215.25 KB

Table 11. Parameter selection and concrete sizes for for multi-theorem straight-line extractable NIZKPoK, where the setting root-hermite factor is 1.003735, with 128-bit classical security.

Besides, the sizes of public key and secret key of CTS or the final Anonymous Credentials are about

$$|\mathsf{pk}_{\mathsf{CTS}}| := (2\ell^2 \cdot \tau) \cdot N\lceil \log q_2 \rceil \text{ bits and } |\mathsf{sk}_{\mathsf{CTS}}| := ((\ell + \hat{\ell}) \cdot \ell \cdot \tau) \cdot N\lceil \log 3 \rceil \text{ bits},$$

respectively. Furthermore, the size of signature is about

$$|\mathsf{Sig}_{\mathsf{CTS}}| := (\ell \cdot (2\tau + 1) + \hat{\ell} + k - n) \cdot N \cdot \log(12\alpha) \text{ bits},$$

which can be further optimized by using the Huffman coding as in [41] to get the signature size as

$$|\mathsf{Sig}_{\mathsf{CTS}}| := (\ell \cdot (2\tau + 1) + \hat{\ell} + k - n) \cdot N \cdot (2.57 + \lceil \log(\alpha) \rceil) \text{ bits},$$

Moreover, the pseudonym consists of three parts: BDLOP commitments, the related multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ system and the disclosure of chosen attributes for attribute-based setting. And the size of commitment is about

$$|\mathsf{comm}_{\mathsf{CTS}}| := n \cdot \ell \cdot \tau \cdot N\lceil \log q_1 \rceil + \ell^2 \cdot \tau \cdot N\lceil \log q_2 \rceil \text{ bits}.$$

the size of proof is denoted by $\mathsf{size}_{\mathsf{LNP}}$, which is presented in Section 5. And according to Table 17 in Section 6.3, $\mathsf{size}_{\mathsf{Disclosure}}$ is about

$$\mathsf{size}_{\mathsf{Disclosure}} := 2 \cdot \hat{k} \cdot N \cdot \lceil \log q_2 \rceil + \hat{k} \cdot \log q_2 + k \cdot N(2.57 + \lceil \log(\eta \cdot \kappa\sqrt{k \cdot N}) \rceil) + \lambda.$$

Thus, the total size of pseudonym $|\mathsf{pseudonym}|$ is about

$$|\mathsf{pseudonym}| := |\mathsf{comm}_{\mathsf{CTS}}| + \mathsf{size}_{\mathsf{LNP}} + \mathsf{size}_{\mathsf{Disclosure}} \text{ bits}.$$

Finally, the credential size is about

$$|\mathsf{Cred}| := (\ell \cdot (2\tau + 1) + \hat{\ell} + k - n) \cdot N \cdot \log(12\eta \cdot \kappa \cdot \gamma) \text{ bits},$$

which can be optimized to get the credential size as

$$|\mathsf{Cred}| := (\ell \cdot (2\tau + 1) + \hat{\ell} + k - n) \cdot N \cdot (2.57 + \lceil \log(\eta \cdot \kappa \cdot \gamma) \rceil) \text{ bits},$$

D Adaptively Secure CTS

In this section, we present an adaptively secure CTS scheme. Our construction follows the partitioning approach as [1], and results in some additional reduction loss compared with the selective construction in section 4. We first introduce the additional preliminaries for this adaptive construction, and then present the construction, and show the correctness and security of the scheme. Finally, we provide some parameter settings for concrete instantiations.

D.1 Pairwise Independent Hash Function

We give a lemma which shows that pairwise independent hash function family which is denoted as $\mathcal{H}$ has the isolation property as long as a conditional probability defined as below approximates $1/|Q|$.

Lemma D.1 *Let $Q \subseteq \mathcal{M}$, A, B be integers such that $B \leq A$, $|Q| \leq \varsigma B$ for some $\varsigma \in (0,1)$, and let $\mathcal{H} : \mathcal{M} \to \mathcal{Y}$ be an pairwise independent hash function family which has the following properties:*

- $\forall \boldsymbol{a} \in \mathcal{M}$, $\mathsf{Pr}_{H \leftarrow \mathcal{H}}[H(\boldsymbol{a}) = 0] = 1/A$;
- $\forall \boldsymbol{a} \neq \boldsymbol{b} \in \mathcal{M}$, $\mathsf{Pr}_{H \leftarrow \mathcal{H}}[H(\boldsymbol{a}) = 0 | H(\boldsymbol{b}) = 0] \leq 1/B$.

Then for any element $\boldsymbol{a} \notin Q$, we have

$$\mathsf{Pr}_{H \in \mathcal{H}}[H(\boldsymbol{a}) = 0 \bigwedge H(\boldsymbol{a}') \neq 0, \forall \boldsymbol{a}' \in Q] \in \left[\frac{1-\varsigma}{A}, \frac{1}{A}\right].$$

An Explicit Almost Pairwise Independent Hash Construction. Let $q \in \mathbb{N}$ be a prime, $N, \theta \in \mathbb{N}$, $R_q = \mathbb{Z}_q[X]/\langle x^N + 1\rangle$, $S_q \subset R_q$ be a subfield of R_q with order $q^{\zeta'}$. We define the hash function family $\mathcal{H} : (S_q)^\theta \to S_q$ as follows: $\forall H \in \mathcal{H}$, H is indexed by $(\alpha, h_1, ...h_\theta) \in (S_q)^{\theta+1}$, $\forall \boldsymbol{x} = (x_1, ..., x_\theta) \in (S_q)^\theta$, $H(\boldsymbol{x}) = \alpha + \langle \boldsymbol{x}, \boldsymbol{h}\rangle \in S_q$. We have the following lemma.

Lemma D.2 ([1]) *The function family $\mathcal{H}$ defined above is an pairwise independent hash function. Moreover, we have*

- $\forall H \leftarrow \mathcal{H}$ *and* $\forall \boldsymbol{x} \in (S_q)^\theta$, $\mathsf{Pr}[H(\boldsymbol{x}) = 0] = 1/q^{\zeta'}$.
- $\forall H \leftarrow \mathcal{H}$ *and* $\forall \boldsymbol{x} \neq \boldsymbol{y} \in (S_q)^\theta$, $\mathsf{Pr}[H(\boldsymbol{y}) = 0 | H(\boldsymbol{x}) = 0] \leq 1/q^{\zeta'}$.

D.2 Adaptively Secure Construction

Our construction uses the following building blocks: (1) the BDLOP commitment scheme $\Gamma = \Gamma.\{\mathsf{CKeyGen}, \mathsf{Commit}, \mathsf{Open}, \mathsf{Combine}, \mathsf{Randomize}\}$, and (2) a $\mathsf{NIZKPoK}$ system $\Pi^{(3)} = \Pi^{(3)}.\{\mathsf{Setup}, \mathsf{Prove}, \mathsf{VerifyProve}, \mathsf{SimProve}\}$ for the following language (parameterized by $\gamma', q \in \mathbb{N}$)

$$L_{\gamma',q,\bar{\mathcal{C}}} = \Big\{(\mathbf{B}, u) \in R_q^{1\times(2\tau+6)} \times R_q : \exists\ \boldsymbol{x} \in R_q^{(2\tau+6)} \text{ and } f \in \bar{\mathcal{C}} \text{ such that } \|\boldsymbol{x}\|_2 \leq \gamma' \text{ and } \mathbf{B} \cdot \boldsymbol{x} = f \cdot \boldsymbol{u}\Big\},$$

Similar to the presentation of Construction 4.1 in Section 4.1, we first describe the required parameters in Table 12. Notice that for the adaptive security, we need to set the message space $\mathcal{M}$ as the concatenation of several independent and identical spaces $\bar{\mathcal{M}}$, i.e., $\mathcal{M} = \bar{\mathcal{M}}^\theta$. Moreover, $\bar{\mathcal{M}}$ should satisfy two requirements: (1) $\bar{\mathcal{M}}$ is a subfield of R_{q_2}; (2) the ℓ_2 norm of all elements in $\bar{\mathcal{M}}$ should be upper bounded by B. Here, for simplicity, we directly set the parameters of underlying BDLOP commitment Γ as $n = 1, \ell = 1, k = 4$.

Particularly, all parameters are in the following table.

Parameters	Description
N	Ring dimension
d	Ring dimension for the straight-line extractable NIZKPoK
R	Cyclotomic Ring used in this work
q_1 q_2	Moduli used for BDLOP commitment scheme
$\mathcal{M}$ $\bar{\mathcal{M}}$ θ, ζ'	Message space $\mathcal{M}$ of the commitment, which consists of θ subspace $\bar{\mathcal{M}}$. And $\bar{\mathcal{M}}$ consists of ζ' polynomials
δ τ	the basis and dimension of the gadget vector $\boldsymbol{g}$ i.e., $\boldsymbol{g}^\top = (1, \delta, \ldots, \delta^{\tau-1})$, with $\delta = q_2^{1/\tau}$
S_β	Set of all elements in R with ℓ_∞ norm at most β
α	Parameter used in SamplePre
γ	ℓ_2 norm parameter used in Verify algorithm for original signature
$\mathcal{C}$ κ	Challenge set of the NIZKPoK system Π $\mathcal{C} = \{c \in R : \|c\|_1 = \kappa, \|c\|_\infty = 1\}$
$\bar{\mathcal{C}}$	The set of differences $\mathcal{C} - \mathcal{C}$ except 0
γ'	ℓ_2 norm parameter for "short" vectors in the language of Π
δ_0	Root-Hermite Factor
Bit-sec	Bit-security in time

Table 12. Parameters of Adaptive Commit-Transferrable Signature Scheme

Construction D.3 (Commit-Transferrable Signature) *Our adaptive* CTS *is constructed as follow.*

- $\mathsf{Setup}(1^\lambda, \ell, B)$*: On input the security parameter* 1^λ*, the algorithm does the following.*
  1. *Run* $\Gamma.\mathsf{CKeyGen}$ *to get* $\mathbf{A} := \begin{bmatrix} 1, & \boldsymbol{a}_1'^\top \\ 0, 1, & a_2' \end{bmatrix} \leftarrow \Gamma.\mathsf{CKeyGen}(1^\lambda)$*, where* $[1, \boldsymbol{a}_1'^\top] \in R_{q_1}^{1\times 4}$ *and* $[0, 1, a_2'] \in R_{q_2}^{1\times 4}$*, with* $\boldsymbol{a}_1' \in R_{q_1}^3$*,* $\boldsymbol{a}_2^\top = (1, \boldsymbol{a}_2') \in R_{q_2}^3$*. Note that the commitment scheme sets message space* $\mathcal{M} \subseteq (R_{q_2})^\theta$ *with randomness space* $(\mathcal{R})^{\tau\theta} = (S_1^4)^{\tau\theta}$*, where* $\mathcal{M} = (\bar{\mathcal{M}})^\theta$*, and* $\bar{\mathcal{M}}$ *is a subfield of* R_{q_2} *with* $|\bar{\mathcal{M}}| = q_2^{\zeta'}$*. And the* ℓ_∞ *norm of all elements in* $\bar{\mathcal{M}}$ *is set be at most 1.*
  2. *Sample* $\boldsymbol{d} \xleftarrow{\$} R_{q_2}^3$*;*
  3. *Run* $\Pi.\mathsf{Setup}(1^\lambda)$ *to get common reference string* crs*;*
  4. *Output* $\mathsf{params} := (\mathbf{A}, \boldsymbol{d}, \mathcal{M}, \mathcal{R}, \mathsf{crs})$*.*
- $\mathsf{Commit}(\mathsf{params}, \boldsymbol{m}; \mathsf{Rand})$*: On input* params*, message* $\boldsymbol{m} \in \mathcal{M}$*, and randomness* $\mathsf{Rand} \in \mathcal{R}^{\tau\theta}$*, the algorithm does the following.*
  1. *Parse* Rand *as* θ *vectors* $\{(\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \cdots, \boldsymbol{r}_{i,\tau})\}_{i\in[\theta]}$*, where* $\boldsymbol{r}_{i,j} \in \mathcal{R} = S_1^4$ *for* $i \in [\theta], j \in [\tau]$*.*

2. *Parse* $\boldsymbol{m}$ *as* $(m_1, \cdots, m_\theta)$. *For* $i \in [\theta]$, *run* $\mathsf{comm}_{i,1} = \Gamma.\mathsf{Commit}(\mathbf{A}, m_i; \boldsymbol{r}_{i,1})$, $\mathsf{comm}_{i,2} = \Gamma.\mathsf{Commit}(\mathbf{A}, m_i\delta; \boldsymbol{r}_{i,2}), \cdots, \mathsf{comm}_{i,\tau} = \Gamma.\mathsf{Commit}(\mathbf{A}, m_i\delta^\tau; \boldsymbol{r}_{i,\tau})$.
3. *Output* $\mathsf{comm} = \{(\mathsf{comm}_{i,1}, \mathsf{comm}_{i,2}, \cdots, \mathsf{comm}_{i,\tau})\}_{i\in[\theta]}$ *as the commitment of* $\boldsymbol{m}$.

– $\mathsf{Randomize}(\mathsf{params}, \mathsf{comm}, \boldsymbol{m}, \mathsf{Rand}, \mathsf{Rand}')$*: On input* params, $\mathsf{Rand}, \mathsf{Rand}' \in \mathcal{R}^{\tau\theta}$, *and* $\mathsf{comm} = \{(\mathsf{comm}_{i,1}, \mathsf{comm}_{i,2}, \cdots, \mathsf{comm}_{i,\tau})\}_{i\in[\theta]}$, *the algorithm does the following.*
1. *Parse* Rand' *as* θ *vectors* $\{(\tilde{\boldsymbol{r}}_{i,1}, \tilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau})\}_{i\in[\theta]}$, *where* $\tilde{\boldsymbol{r}}_{i,j} \in \mathcal{R} = S_1^4$ *for* $i \in [\theta], j \in [\tau]$.
2. *For* $i \in [\theta]$, *run* $\mathsf{comm}'_{i,1} = \Gamma.\mathsf{Randomize}(\mathbf{A}, \mathsf{comm}_{i,1}, \tilde{\boldsymbol{r}}_{i,1})$, $\mathsf{comm}'_{i,2} = \Gamma.\mathsf{Randomize}(\mathbf{A}, \mathsf{comm}_{i,2}, \tilde{\boldsymbol{r}}_{i,2}), \cdots, \mathsf{comm}'_{i,\tau} = \Gamma.\mathsf{Randomize}(\mathbf{A}, \mathsf{comm}_{i,\tau}, \tilde{\boldsymbol{r}}_{i,\tau})$. *Set* $\mathsf{comm}' = \{(\mathsf{comm}'_{i,1}, \mathsf{comm}'_{i,2}, \cdots, \mathsf{comm}'_{i,\tau})\}_{i\in[\theta]}$.
3. *Output* comm' *as the rerandomized commitment of* $\boldsymbol{m}$.

– $\mathsf{Combine}(\mathsf{Rand}, \mathsf{Rand}')$*: Taking as input two randomness* $\mathsf{Rand} = \{(\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \cdots, \boldsymbol{r}_{i,\tau})\}_{i\in[\theta]} \in S_1^{4\times\tau\theta}$, *and* $\mathsf{Rand}' = \{(\tilde{\boldsymbol{r}}_{i,1}, \tilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau})\}_{i\in[\theta]} \in S_1^{4\times\tau\theta}$, *the algorithm computes and outputs* $\{(\hat{\boldsymbol{r}}_{i,1}, \hat{\boldsymbol{r}}_{i,2}, \cdots, \hat{\boldsymbol{r}}_{i,\tau})\}_{i\in[\theta]} \in S_2^{4\times\tau\theta}$, *where* $\hat{\boldsymbol{r}}_{i,j} = \boldsymbol{r}_{i,j} + \tilde{\boldsymbol{r}}_{i,j}$ *for* $i \in [\theta], j \in [\tau]$.

– $\mathsf{KeyGen}(\mathsf{params})$*: On input* params, *the algorithm does:*
1. *Sample* $\mathbf{T} \xleftarrow{\$} S_1^{3\times\tau}$, *and set* $\boldsymbol{a}^\top = \boldsymbol{d}^\top \cdot \mathbf{T} + \boldsymbol{g}_\delta^\top \in R_{q_2}^{1\times\tau}$, *where* $\boldsymbol{g}_\delta^\top = (1, \delta, \delta^2, \cdots, \delta^{\tau-1}) \in R_{q_2}^{1\times\tau}$.
2. *Sample* $(\boldsymbol{b}_0, \boldsymbol{b}_1, \cdots, \boldsymbol{b}_\theta) \xleftarrow{\$} R_{q_2}^{\tau(\theta+1)}$;
3. *Sample* $u \xleftarrow{\$} R_{q_2}$;
4. *Output* $\mathsf{pk} := (\boldsymbol{a}, \boldsymbol{b}_0, \{\boldsymbol{b}_i\}_{i\in[\theta]}, u)$, *and* $\mathsf{sk} := \mathbf{T}$.

– $\mathsf{Sign}(\mathsf{params}, \mathsf{pk}, \mathsf{sk}, \mathsf{comm})$*: On input* params, pk, sk *and* comm, *the algorithm does the following:*
1. *For* $i \in [\theta]$, *parse* $\mathsf{comm}_i = (\mathsf{comm}_{i,1}, \ldots, \mathsf{comm}_{i,\tau})$ *as* $\mathsf{comm}_{i,1} = \begin{bmatrix} t_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix}$, $\mathsf{comm}_{i,2} = \begin{bmatrix} t_{1,2}^{(i)} \\ t_{2,2}^{(i)} \end{bmatrix}, \cdots, \mathsf{comm}_{i,\tau} = \begin{bmatrix} t_{1,\tau}^{(i)} \\ t_{2,\tau}^{(i)} \end{bmatrix}$;
2. *Set* $\mathbf{F}_{\mathsf{comm}} = \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| \boldsymbol{b}_{\mathsf{comm}}^\top \middle| \boldsymbol{a}_2^\top \right] = \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| [\boldsymbol{b}_0 + \sum_{i\in[\ell]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \right)] \middle| \boldsymbol{a}_2^\top \right]$, *and sample* $\mathsf{Sig}_{\mathsf{comm}} := \begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{bmatrix} \leftarrow \mathsf{SamplePre}(\left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| \boldsymbol{b}_{\mathsf{comm}}^\top \middle| \boldsymbol{a}_2^\top \right], \mathbf{T}, u, \alpha)$, *and output* $\mathsf{Sig}_{\mathsf{comm}}$ *as the signature of* comm, *where* $\boldsymbol{s}_1 = \begin{bmatrix} \boldsymbol{s}_{1,1} \\ \boldsymbol{s}_{1,2} \end{bmatrix}$, *and* $\boldsymbol{s}_{1,1} \in R^3, \boldsymbol{s}_{1,2} \in R^\tau, \boldsymbol{s}_2 \in R^\tau$, $\boldsymbol{s}_3 \in R^3$.

– $\mathsf{Transfer}(\mathsf{params}, \mathsf{pk}, \mathsf{Sig}_{\mathsf{comm}}, \boldsymbol{m}, (\mathsf{Rand}, \mathsf{Rand}'))$*: On input* $\mathsf{params}, \mathsf{pk}$, *a signature* $\mathsf{Sig}_{\mathsf{comm}}$, *message* $\boldsymbol{m}$, *randomness* Rand *for generating the commitment* comm *for* $\boldsymbol{m}$, *the additional randomness* Rand' *for the rerandomization of* comm, *the algorithm does the followings:*
1. *Parse* $\mathsf{Sig}_{\mathsf{comm}}$ *as vector* $\begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{bmatrix}$, *where* $\boldsymbol{s}_1 \in R^{\tau+3}$, $\boldsymbol{s}_2 \in R^\tau$, $\boldsymbol{s}_3 \in R^3$.

2. *Parse* Rand *as* $\tau\theta$ *vectors* $\{(\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \cdots, \boldsymbol{r}_{i,\tau})\}_{i\in[\theta]}$, *where* $\boldsymbol{r}_{i,j} \in \mathcal{R} = S_1^4$.
3. *Parse* Rand' *as* $\tau\theta$ *vectors* $\{(\tilde{\boldsymbol{r}}_{i,1}, \tilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau})$, *where* $\tilde{\boldsymbol{r}}_{i,j} \in \mathcal{R} = S_1^4$.
4. *For* $i \in [\theta]$, *run* $\mathsf{Commit}(\mathsf{params}, m_i; (\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \cdots, \boldsymbol{r}_{i,\tau}))$ *and obtain:* $\mathsf{comm}_i = (\mathsf{comm}_{i,1}, \mathsf{comm}_{i,2}, \cdots, \mathsf{comm}_{i,\tau})$, *where* $\mathsf{comm}_{i,1} = \begin{bmatrix} t_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix}$, $\mathsf{comm}_{i,2} = \begin{bmatrix} t_{1,2}^{(i)} \\ t_{2,2}^{(i)} \end{bmatrix}, \cdots, \mathsf{comm}_{i,\tau} = \begin{bmatrix} t_{1,\tau}^{(i)} \\ t_{2,\tau}^{(i)} \end{bmatrix}$.
5. *For* $i \in [\theta]$, *run* $\mathsf{Randomize}\ (\mathsf{params}, \mathsf{comm}_i, (\tilde{\boldsymbol{r}}_{i,1}, \tilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau}))$ *and obtain* $\mathsf{comm}'_i = (\mathsf{comm}'_{i,1}, \mathsf{comm}'_{i,2}, \cdots, \mathsf{comm}'_{i,\tau})$, *where* $\mathsf{comm}'_{i,1} = \begin{bmatrix} \hat{t}_{1,1}^{(i)} \\ \hat{t}_{2,1}^{(i)} \end{bmatrix}$, $\mathsf{comm}'_{i,2} = \begin{bmatrix} \hat{t}_{1,2}^{(i)} \\ \hat{t}_{2,2}^{(i)} \end{bmatrix}, \cdots, \mathsf{comm}'_{i,k} = \begin{bmatrix} \hat{t}_{1,\tau}^{(i)} \\ \hat{t}_{2,\tau}^{(i)} \end{bmatrix}$.
6. *Compute a (temporary) signature* $\mathsf{Sig}_{\mathsf{comm}'}$ *as*

$$\begin{aligned} \mathsf{Sig}_{\mathsf{comm}'} :&= \begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 - \sum_{i\in[\theta]} \left(\tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\right) \cdot \boldsymbol{s}_2 \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{s}_{1,1} \\ \boldsymbol{s}_{1,2} \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 - \sum_{i\in[\theta]} \left(\tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\right) \cdot \boldsymbol{s}_2 \end{bmatrix} \in R^{2\tau+4}, \end{aligned}$$

where we denote $\tilde{\mathbf{R}}_i = \begin{bmatrix} \tilde{\mathbf{R}}_{i,1} \\ \tilde{\mathbf{R}}_{i,2} \end{bmatrix} = \left[\tilde{\boldsymbol{r}}_{i,1}, \tilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau}\right] \in R^{4\times\tau}$, *with* $\tilde{\mathbf{R}}_{i,1} \in R^{1\times\tau}$ *and* $\tilde{\mathbf{R}}_{i,2} \in R^{3\times\tau}$.
7. *Compute* $\mathbf{F}_{\mathsf{comm}'} := \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| \boldsymbol{b}_{\mathsf{comm}'}^\top \middle| \boldsymbol{a}_2^\top\right] = \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| [\boldsymbol{b}_0 + \sum_{i\in[\ell]} \left((\hat{t}_{2,1}^{(i)}, \hat{t}_{2,2}^{(i)}, \cdots, \hat{t}_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i)\right)] \middle| \boldsymbol{a}_2^\top\right]$.
8. *Run the prove algorithm, output* $\mathsf{Sig}'_{\mathsf{comm}'} := \pi \leftarrow \Pi^{(3)}.\mathsf{Prove}(\mathsf{crs}, (\mathbf{F}_{\mathsf{comm}'}, u), \mathsf{Sig}_{\mathsf{comm}'})$, *proving that* $\mathsf{Sig}_{\mathsf{comm}'}$ *is a short* ℓ_2 *norm vector and satisfies* $\mathbf{F}_{\mathsf{comm}'} \cdot \mathsf{Sig}_{\mathsf{comm}'} = u$, *through using the* NIZKPoK *system* Π *with the relaxed language* $L_{\gamma', q_2, \bar{\mathcal{C}}}$.

– $\mathsf{Verify}(\mathsf{params}, \mathsf{pk}, \mathsf{comm}, \mathsf{Sig})$*: On input* $\mathsf{params}, \mathsf{pk}, \mathsf{comm}, \mathsf{Sig}$, *the algorithm does the following.*

1. *Parse* $\mathsf{comm} = \{(\mathsf{comm}_{i,1}, \mathsf{comm}_{i,2}, \cdots, \mathsf{comm}_{i,\tau})\}_{i\in[\ell]}$ *as* $\mathsf{comm}_{i,1} = \begin{bmatrix} t_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix}$, $\mathsf{comm}_{i,2} = \begin{bmatrix} t_{1,2}^{(i)} \\ t_{2,2}^{(i)} \end{bmatrix}, \cdots, \mathsf{comm}_{i,\tau} = \begin{bmatrix} t_{1,\tau}^{(i)} \\ t_{2,\tau}^{(i)} \end{bmatrix}$;
2. *Based on the type of* Sig, *the verification works as follow.*
   - *If* Sig *is a short vector within* ℓ_2 *norm* γ, *then the algorithm does*

(a) *Set matrix*

$$\mathbf{F}_{\mathsf{comm}} := \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] | [\boldsymbol{b}_0 + \sum_{i \in [\theta]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \right)] | \boldsymbol{a}_2^\top \right].$$

(b) *Check whether* Sig *satisfies*

$$\mathbf{F}_{\mathsf{comm}} \cdot \begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{bmatrix} = u \in \mathcal{R}_{q_2}.$$

- *If* Sig *is a proof of the* NIZKPoK *system* $\Pi^{(3)}$,
  (a) *Set matrix*

$$\mathbf{F}_{\mathsf{comm}} := \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] | [\boldsymbol{b}_0 + \sum_{i \in [\theta]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \right)] | \boldsymbol{a}_2^\top \right].$$

  (b) *Run the verify algorithm (with respect to language* $L_{\gamma', q_2, \bar{\mathcal{C}}}$*)* $\Pi.\mathsf{VerifyProve}(\mathsf{crs}, (\mathbf{F}_{\mathsf{comm}}, u), \mathsf{Sig})$*, and output its result.*

Lemma D.4 (Correctness) *For parameters* $N, q_2, \alpha, \gamma = \alpha\sqrt{2 \cdot (2\tau + 6) \cdot N}$*, the* NIZKPoK *system* $\Pi^{(3)}$ *for the relaxed language* $L_{\gamma', q_2, \bar{\mathcal{C}}}$ *with* $\gamma' \geq (\sqrt{3\tau} + \tau)\sqrt{2}\tau\theta\alpha N^2\delta + \alpha\sqrt{2 \cdot (2\tau + 6)N}$*, Construction D.3 satisfies the correctness property as defined in Definition 3.1.*

Proof. The correctness according to Definition 3.1 requires to prove the following three statements: (1) four algorithms (Setup,Commit,Randomize, Combine) define a correct randomizable commitment scheme; (2) the signature by algorithm Sign passes the verification algorithm, i.e., Verify; and (3) the transferred signature (with respect to the randomized commitment) from Transfer also passes Verify.

The correctness of statement (1) and statement (2) are easy to verify. We just sketch the correctness of statement (3). Similar to the analysis of Lemma 4.2, it it suffices to show that $\mathbf{F}_{\mathsf{comm}'} \cdot \mathsf{Sig}_{\mathsf{comm}'} = u$ (as defined in the algorithm Transfer) $\mathsf{Sig}_{\mathsf{comm}'}$ is within ℓ_2 norm $(\sqrt{3\tau} + \tau)\sqrt{2}\tau\theta\alpha N^2\delta + \alpha\sqrt{2 \cdot (2\tau + 6)N}$.

Particularly, for all $\boldsymbol{m} \in \mathcal{M} = (\bar{\mathcal{M}})^\theta \subseteq (\mathcal{R}_{q_2})^\theta$, $\boldsymbol{r}_{i,j}, \tilde{\boldsymbol{r}}_{i,j} \in S_1^3, i \in [\theta], j \in [\tau]$, $(\mathsf{sk}, \mathsf{pk})$ output by KeyGen, and signature $\mathsf{Sig}_{\mathsf{comm}} = (\boldsymbol{s}_1^T, \boldsymbol{s}_2^T, \boldsymbol{s}_3^T) = ((\boldsymbol{s}_{1,1}^T, \boldsymbol{s}_{1,2}^T), \boldsymbol{s}_2^T, \boldsymbol{s}_3^T)$ output by Sign, it holds

$$\mathbf{F}_{\mathsf{comm}} \cdot \begin{bmatrix} \boldsymbol{s}_1 \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 \end{bmatrix} = u \in \mathcal{R}_{q_2},$$

where

$$\mathbf{F}_{\mathsf{comm}} = \left[[\boldsymbol{d}^T | \boldsymbol{a}^T] | [\boldsymbol{b}_0 + \sum_{i \in [\ell]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \right)] | \boldsymbol{a}_2^T \right].$$

And the ℓ_2 norm of the vector $(\boldsymbol{s}_{1,1}^T, \boldsymbol{s}_{1,2}^T, \boldsymbol{s}_2^T, \boldsymbol{s}_3^T)$ is less than $\alpha\sqrt{2\cdot(2\tau+4)N}$. This implies

$$\begin{aligned}\langle \boldsymbol{d}, \boldsymbol{s}_{1,1}\rangle + \langle \boldsymbol{a}, \boldsymbol{s}_{1,2}\rangle + \langle \boldsymbol{b}_0 + \sum_{i\in[\theta]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i)\right), \\ \boldsymbol{s}_2\rangle + \langle \boldsymbol{a}_2, \boldsymbol{s}_3\rangle = u \in \mathcal{R}_{q_2}.\end{aligned}$$

We notice that the above equation is equivalent to

$$\begin{aligned}u =& \langle \boldsymbol{d}, \boldsymbol{s}_{1,1}\rangle + \langle \boldsymbol{a}, \boldsymbol{s}_{1,2}\rangle + \langle \boldsymbol{b}_0 + \sum_{i\in[\theta]} (t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i), \\ & \boldsymbol{s}_2\rangle + \langle \boldsymbol{a}_2, \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\cdot \boldsymbol{s}_2\rangle - \langle \boldsymbol{a}_2, \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\cdot \boldsymbol{s}_2\rangle \\ & + \langle \boldsymbol{a}_2, \boldsymbol{s}_3\rangle \\ =& \langle \boldsymbol{d}, \boldsymbol{s}_{1,1}\rangle + \langle \boldsymbol{b}_0 + \sum_{i\in[\theta]} (t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) + \\ & \boldsymbol{a}_2^\top \cdot \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i), \boldsymbol{s}_2\rangle + \langle \boldsymbol{a}, \boldsymbol{s}_{1,2}\rangle + \langle \boldsymbol{a}_2, \\ & - \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\cdot \boldsymbol{s}_2\rangle + \langle \boldsymbol{a}_2, \boldsymbol{s}_3\rangle,\end{aligned}$$

which can be rewritten as

$$\begin{aligned}&[[\boldsymbol{d}^\top|\boldsymbol{a}^\top]|\boldsymbol{b}_0 + \sum_{i\in[\theta]} (t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) + \\ &\boldsymbol{a}_2^\top \cdot \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)|\boldsymbol{a}_2^\top] \cdot \begin{bmatrix} \boldsymbol{s}_{1,1} \\ \boldsymbol{s}_{1,2} \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 - \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\cdot \boldsymbol{s}_2 \end{bmatrix} = u.\end{aligned}$$

Here we denote $\tilde{\mathbf{R}}_i = \begin{bmatrix} \widetilde{\mathbf{R}}_{i,1} \\ \widetilde{\mathbf{R}}_{i,2} \end{bmatrix} = \left[\widetilde{\boldsymbol{r}}_{i,1}, \widetilde{\boldsymbol{r}}_{i,2}, \cdots, \tilde{\boldsymbol{r}}_{i,\tau}\right] \in R^{3\times\tau}$, with $\widetilde{\mathbf{R}}_{i,1} \in R^{1\times\tau}$ and $\widetilde{\mathbf{R}}_{i,2} \in R^{2\times\tau}$.

Then we observe that

$$\begin{aligned}\mathbf{F}_{\mathsf{comm}'} :=& \left[[\boldsymbol{d}^\top|\boldsymbol{a}^\top]|\boldsymbol{b}_0 + \sum_{i\in[\theta]} (\hat{t}_{2,1}^{(i)}, \hat{t}_{2,2}^{(i)}, \cdots, \hat{t}_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i)|\boldsymbol{a}_2^\top\right] \\ =& \Big[[\boldsymbol{d}^\top|\boldsymbol{a}^\top]|\boldsymbol{b}_0 + \sum_{i\in[\theta]} (t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \\ & + \boldsymbol{a}_2^\top \cdot \sum_{i\in[\theta]} \tilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)|\boldsymbol{a}_2^\top\Big],\end{aligned}$$

and $\mathsf{Sig}_{\mathsf{comm}'} := \begin{bmatrix} \boldsymbol{s}_{1,1} \\ \boldsymbol{s}_{1,2} \\ \boldsymbol{s}_2 \\ \boldsymbol{s}_3 - \sum_{i\in[\theta]} \widetilde{\mathbf{R}}_{i,2}\mathbf{G}^{-1}(\boldsymbol{b}_i)\cdot \boldsymbol{s}_2 \end{bmatrix}$. Now, it is easy to verify that the ℓ_2 norm of $\mathsf{Sig}_{\mathsf{comm}',1}$ and $\mathsf{Sig}_{\mathsf{comm}',2}$ are within $(\sqrt{3\tau}+\tau)\sqrt{2}\tau\theta\alpha N^2\delta + \alpha\sqrt{2\cdot(2\tau+6)N}$ and $\mathbf{F}_{\mathsf{comm}'}\cdot\mathsf{Sig}_{\mathsf{comm}'} = u$, since for such matrices $\widetilde{\mathbf{R}}_{i,2} \in S_1^{3\times\tau}$, its singular value $s_1(\widetilde{\mathbf{R}}_{i,2})$ is bounded by $(\sqrt{3}+\sqrt{\tau})\sqrt{N}$, and the singular value of $\mathbf{G}^{-1}(\boldsymbol{b}_i)$ is bounded by $\tau N\delta$ by Lemma A.4. This completes the proof. □

D.3 Instantiation of NIZKPoK system $\Pi^{(3)}$ in CTS

Before presenting the NIZKPoK system $\Pi^{(3)}$, we first specify the concrete language $L_{\gamma',q_2,\bar{\mathcal{C}}}$ in the algorithms Transfer and Verify,

$$L_{\gamma',q_2,\bar{\mathcal{C}}} = \Big\{(\mathbf{F}_{\mathsf{comm}'}, u) \in R_{N,q_2}^{1\times(2\tau+6)} \times R_{N,q_2} : \exists\ \boldsymbol{x} \in R^{2\tau+6} \text{ and } f \in \bar{\mathcal{C}} \\ \text{such that } 0 < \|\boldsymbol{x}\| \le \gamma' \text{ and } \mathbf{F}_{\mathsf{comm}'}\cdot\boldsymbol{x} = f\cdot u\Big\}.$$

Then, according to [6, 22], there exists such an efficient $\Pi^{(3)}$ for $L_{\gamma',q_2,\bar{\mathcal{C}}}$. The formal theorem is presented as follows.

Theorem D.5 ([6, 22]) *In the random oracle model, there exists a* NIZKPoK *system $\Pi^{(3)}$ for the relaxed language $L_{\gamma',q_2,\bar{\mathcal{C}}}$, with*

$$\gamma' = 2\sqrt{2(2\tau+6)N}\cdot\eta\kappa\cdot((\sqrt{3\tau}+\tau)\sqrt{2}\tau\theta\alpha N^2\delta + \alpha\sqrt{2\cdot(2\tau+6)N}).$$

Moreover, assuming a t-time adversary $\mathcal{A}$ forging a proof with probability ε, there exists a $O(t/\varepsilon)$-time extractor, who can successfully extract the witness $\boldsymbol{x}$ and $c \in \bar{\mathcal{C}}$ with probability $\frac{1}{2}$.

Remark D.6 *Notice that the concrete instantiation of* NIZKPoK *system $\Pi^{(3)}$ in Theorem D.5 is essentially a Fiat-Shamir signature, which is quite practical.*

D.4 Security Proof

In this section, we show the simulatability and unforgeability of the above Construction D.3.

Lemma D.7 (Simulatability) *The algorithm* Transfer *in Construction D.3 is simulatable.*

Proof. Similar to the proof of Lemma 4.5, we first construct a two-stage PPT simulator $\mathcal{S}$, and then prove that after running any polynomial $\varrho = \mathsf{poly}(\lambda)$ times, the distribution of $\{\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by $\mathcal{S}$ are statistically close to that of $\{\mathsf{Sig}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by Transfer.

The two-stage PPT simulator $\mathcal{S}$ can be constructed in the following way:

– First Stage: $\mathcal{S}$ conducts the following steps:
  1. Generate and output $\mathsf{params} := (\mathbf{A}, \boldsymbol{d}, \mathcal{M}, \mathcal{R}, \mathsf{crs})$.
– Second Stage: given params, and valid pk, comm', $\mathcal{S}$ conducts the following steps:
  1. Recognize pk as $(\boldsymbol{a}, \boldsymbol{b}_0, \{\boldsymbol{b}_i\}_{i\in[\theta]}, u)$.
  2. Parse $\mathsf{comm}' = (\{\mathsf{comm}'_{i,1}, \mathsf{comm}'_{i,2}, \cdots, \mathsf{comm}'_{i,\tau}\}_{i\in[\theta]})$ as $\mathsf{comm}_{i,1} = \begin{bmatrix} \hat{t}^{(i)}_{1,1} \\ \hat{t}^{(i)}_{2,1} \end{bmatrix}$, $\mathsf{comm}_{i,2} = \begin{bmatrix} \hat{t}^{(i)}_{1,2} \\ \hat{t}^{(i)}_{2,2} \end{bmatrix}, \cdots, \mathsf{comm}_{i,\tau} = \begin{bmatrix} \hat{t}^{(i)}_{1,\tau} \\ \hat{t}^{(i)}_{2,\tau} \end{bmatrix}$;
  3. Set matrix
$$\mathbf{F}'_{\mathsf{comm}'} := \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \Big| \boldsymbol{b}_0 + \sum_{i\in[\theta]} (\hat{t}^{(i)}_{2,1}, \hat{t}^{(i)}_{2,2}, \cdots, \hat{t}^{(i)}_{2,\tau}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \Big| \boldsymbol{a}_2^\top \right].$$
  4. With respect to the $\mathsf{NIZKPoK}$ system Π for the relaxed language $L_{\gamma',q_2,\bar{\mathcal{C}}}$,
$$L_{\gamma',q_2,\bar{\mathcal{C}}} = \Big\{ (\mathbf{F}'_{\mathsf{comm}'}, u) \in R_{q_2}^{1\times(2\tau+4)} \times R_{q_2} : \exists\, \boldsymbol{x} \in R_{q_2}^{2\tau+4} \text{ and } f \in \bar{\mathcal{C}} \text{ such that } \|\boldsymbol{x}\|_2 \le \gamma' \text{ and } \mathbf{F}'_{\mathsf{comm}'} \cdot \boldsymbol{x} = f \cdot u \Big\},$$
     we can run the corresponding simulation algorithm to generate a simulated proof π', whose distribution is statistically indistinguishable from that of the real proof π.
  5. Output $\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'} := \pi'$.

According to the zero knowledge property of the used $\mathsf{NIZKPoK}$ system Π, it is clear that after running any polynomial $\varrho = \mathsf{poly}(\lambda)$ times, the distribution of $\{\widetilde{\mathsf{Sig}}'_{\mathsf{comm}'_i}\}_{i\in[\varrho]}$ output by $\mathcal{S}$ are statistically close to that of $\{\mathsf{Sig}'_{\mathsf{comm}'_i}\}_{i\in[t]}$ output by $\mathsf{Transfer}$. □

Below, we analyse the unforgeability of Construction D.3. Before this, we first specify the exact commitment relation $\hat{L}_{q_1,q_2}$.

$$\hat{L}_{q_1,q_2} := \Big\{ \mathsf{comm} = \{\mathsf{comm}_{i,j}\}_{i\in[\theta],j\in[\tau]} : \exists((m_i)_{i\in[\theta]}, q_1, q_2, \{\boldsymbol{r}_{i,j}\}_{i\in[\theta],j\in[4]}) \text{such that } m_i \in \bar{\mathcal{M}}_i, \boldsymbol{r}_{i,j} \in S_1^3 \text{ and } \mathsf{comm}_{i,j} = \mathsf{Commit}(\mathsf{params}, m_i \cdot q_2^{\frac{j-1}{\tau}}, \boldsymbol{r}_{i,j}) \text{ for } i \in [\theta], j \in [\tau] \Big\}$$

Lemma D.8 (Unforgeability) *Assume that* $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu}$ *and* $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu'}$ *are hard with*

$$\nu = \alpha\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha\sqrt{(\ell B+1)^2 + \tau^2N^2\theta^2\delta^2}$$

and

$$\nu' = \alpha'\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha' \cdot \sqrt{(\theta\sqrt{N}+1)^2 + \tau^2N^2\delta^2},$$

with $\alpha' = \gamma'/\sqrt{(2\tau+6)N}$. *Then our above lattice-based commitment-transferrable signature scheme is adaptively unforgeable for the exact commitment relation* $\hat{L}_{q_1,q_2}$, *i.e., the advantage of any* PPT *adversary* $\mathcal{A}$ *against the selective unforgeability game of* CTS *is at most*

$$\mathsf{Adv}_{\mathcal{A}}^{unforge}(\lambda) \leq 2\mathsf{Adv}_{\mathcal{A}}^{\mathsf{RLWE}} + \mathsf{Adv}_{\mathcal{A}}^{unforge^*}(\lambda).$$

Proof. We argue the unforgeability using the series of hybrids.

$\mathsf{H_0}$: The challenger $\mathcal{B}$ runs the CTS honestly. He gives to the adversary $\mathcal{A}$ the public key pk and signatures with respect to the queried commitments comm_i. In this hybrid, we say $\mathcal{A}$ has advantage $\varepsilon = \mathsf{Adv}_{\mathcal{A}}^{\text{unforge}}(\lambda)$ in the unforgeability game. Then, it holds

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_0}(\lambda) = \mathsf{Adv}_{\mathcal{A}}^{\text{unforge}}(\lambda).$$

$\mathsf{H_1}$: The challenger $\mathcal{B}$ runs the identical procedures as H_0, except that he samples $\mathbf{R}_0 \xleftarrow{\$} S_1^{2\times k}$ and $\{\mathbf{R}_i\}_{i\in[\ell]} \xleftarrow{\$} S_1^{2\times k}$, and set $\boldsymbol{b}_i^\top = \boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \cdot \boldsymbol{g}_\delta^\top \in R_{q_2}^{1\times k}$ for $i \in \{0, 1, \ldots, \ell\}$, where h_i is included in a subfield $\mathcal{S}_{q_2}$ of R_{q_2} of order q_2. According to the Ring-LWE assumption, we know that H_0 and H_1 are computational indistinguishability. Then, it holds

$$|\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_0}(\lambda) - \mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_1}(\lambda)| \leq \ell \cdot \mathsf{Adv}_{\mathcal{A}}^{\text{Ring-}\mathsf{LWE}}(\lambda).$$

$\mathsf{H_2}$: The challenger $\mathcal{B}$ runs the identical procedures as H_1, except that except that we add an abort event that is independent of the adversary's view. Specifically, in the final challenge phase, the adversary outputs $(\boldsymbol{m}^*, \mathsf{Rand}^*, \sigma^*)$ as the forgery. $\mathcal{B}$ does the abort check: $h_0 + \langle \boldsymbol{m}_i, \boldsymbol{h} \rangle \neq 0 \bmod q_2 R$ and $h_0 + \langle \boldsymbol{m}_i, \boldsymbol{h} \rangle = 0 \bmod q_2 R$, where $\boldsymbol{h} = (h_1, \cdots, h_\ell) \in \mathcal{S}_{q_2}^\ell$. If the condition does not hold, $\mathcal{B}$ aborts the game.

The only difference between H_1 and H_2 is the abort event. We argue that the adversary still has non-negligible advantage in H_2 even though the abort event happens.

Lemma D.9 *Let* I *be a* $Q_1 + 1$ *tuple* $(\boldsymbol{m}^*, \boldsymbol{m}_1, ..., \boldsymbol{m}_{Q_1})$ *denoted the challenge message* $\boldsymbol{m}^*$ *along with the queried message's, and* $\varepsilon(I)$ *define the probability that an abort does not happen in hybrid* H_2. *Assuming* $\varepsilon(I) \in [\varepsilon_{min}, \varepsilon_{max}]$, *then we have*

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_2}(\lambda) \geq \varepsilon_{min} \cdot \mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_1}(\lambda) - \frac{1}{2}(\varepsilon_{max} - \varepsilon_{min}).$$

$\mathsf{H_3}$: The challenger $\mathcal{B}$ runs the identical procedures as H_2, except that he samples $\boldsymbol{a} \xleftarrow{\$} R^k$, and $\mathcal{B}$ answers the signature queries through using Lemma A.7, rather than Lemma A.8. According to the Ring-LWE assumption, we know that H_2 and H_3 are computational indistinguishability. Then, it holds

$$|\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_3}(\lambda) - \mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_2}(\lambda)| \leq \mathsf{Adv}_{\mathcal{A}}^{\text{Ring-}\mathsf{LWE}}(\lambda).$$

Besides, we denote the challenger in H_2 as $\mathcal{B}^*$. Thus, we have

$$\mathsf{Adv}_{\mathcal{A}}^{\mathsf{H}_3}(\lambda) = \mathsf{Adv}_{\mathcal{A}}^{\mathrm{unforge}^*}(\lambda).$$

Lemma D.10 *Let $\mathcal{A}$ be a* PPT *adversary with advantage ε in the adaptive unforgeability game with respect to $\mathcal{B}^*$ for the exact commitment relation $\hat{L}_{q_1,q_2}$, i.e., $\mathsf{Adv}_{\mathcal{A}}^{unforge^*}(\lambda) = \varepsilon$. Let Q_2 be a bound on the number of random oracle queries made by $\mathcal{A}$. Let*

$$\nu = \alpha\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha\sqrt{(\ell B+1)^2+\tau^2N^2\theta^2\delta^2}$$

and

$$\nu' = \alpha'\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha'\cdot\sqrt{(\theta\sqrt{N}+1)^2+\tau^2N^2\delta^2} + 2\sqrt{\kappa},$$

with $\alpha' = \gamma'/\sqrt{(2\tau+6)N}$. Then there exists a reduction algorithm $\mathcal{R}$ for $\mathsf{M\text{-}SIS}_{q_2,1,k+7,\nu}$ or $\mathsf{M\text{-}SIS}_{q_2,1,k+7,\nu'}$ such that

$$\mathsf{Adv}_{\mathcal{R}}^{\mathsf{M\text{-}SIS}}(\lambda) \geq \varepsilon(\frac{\varepsilon}{Q_2} - 2^{-\lambda}).$$

Proof. According to our construction, the verifier needs to consider two cases: original signature and transferred signature. Thus, we need to prove the unforgeability for both cases. Overall, both of them have the similar proof process, and are based on the hardness of $\mathsf{M\text{-}SIS}_{q_2,1,k+5,\nu}$ and $\mathsf{M\text{-}SIS}_{q_2,1,k+5,\nu'}$ problems, respectively. Below, we present the details for both cases in an unified form, and just separate in their different points.

Particularly, we prove that if the adversary $\mathcal{A}$ can forge a valid original/transferred signature in the selective way, then we can construct an efficient reduction algorithm $\mathcal{B}$ to solve the
$\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu}/\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu'}$ problem. In particular, $\mathcal{B}$ is given an uniformly random matrix $\boldsymbol{x}^\top = [x_1, x_2, \cdots, x_{\tau+7}] \in R_{q_2}^{\tau+7}$, and need to output a vector $\boldsymbol{y}$ such that $\langle \boldsymbol{x}, \boldsymbol{y}\rangle = 0 \bmod q_2$ and

$$\|\boldsymbol{y}\| \leq \alpha\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha\sqrt{(\ell B+1)^2+\tau^2N^2\theta^2\delta^2}$$

or

$$\|\boldsymbol{y}\| \leq \alpha'\sqrt{(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha'\cdot\sqrt{(\theta\sqrt{N}+1)^2+\tau^2N^2\delta^2} + 2\sqrt{\kappa},$$

with $\alpha' = \gamma'/\sqrt{(2\tau+6)N}$. Similar to the consideration in [22], we choose to use $\boldsymbol{x} = [x_1, x_2, x_3, \cdots, x_{\tau+3}, 1, x_{\tau+6}]$, since one of x_i will have an inverse with high probability.

In this case, $\mathcal{B}$ conducts the following steps:

1. Choose $\boldsymbol{x}_1' \xleftarrow{\$} R_{q_1}^3$ and set $\boldsymbol{a}_1^\top = (1, \boldsymbol{x}_1'^\top) \in R_{q_1}^4$.
2. Set $\boldsymbol{a}_2^\top = (1, x_{k+5}, x_{k+6}) \in R_{q_2}^3$.

3. Set $\mathbf{A} = \begin{bmatrix} \boldsymbol{a}_1^\top \\ 0, \boldsymbol{a}_2^\top \end{bmatrix}$ and send it to $\mathcal{A}$.

Clearly, $\mathbf{A}$ is a valid public parameter output by $\Gamma.\mathsf{CKeyGen}$.

Next, we need to argue that $\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully for the exact commitment relation $\hat{L}_{q_1,q_2}$. In particular, we use the following Claim B.5 to specify the case.

Claim D.11 *$\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully in the unforgeability game with respect to the exact commitment relation $\hat{L}_{q_1,q_2}$.*

Proof. $\mathcal{B}$ can set the public parameters in the following way:

1. Set $\boldsymbol{d}^\top = (x_1, x_2, x_3) \in R_{q_2}^3$, $\boldsymbol{a}^\top = (x_4, x_5, \cdots, x_{3+\tau}) \in R_{q_2}^\tau$, $u = x_{\tau+4}$.
2. For $i \in [\theta]$, sample $\mathbf{R}_i \xleftarrow{\$} S_1^{3\times\tau}$, and set $\boldsymbol{b}_i^\top = \boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \cdot (1, \delta, \cdots, \delta^{\tau-1}) \in R_{q_2}^{1\times\tau}$, where $h_i \in S_{q_2}$. Sample $\mathbf{R}_0 \xleftarrow{\$} S_1^{3\times\tau}$, and set $\boldsymbol{b}_0^\top = \boldsymbol{d}^\top \cdot \mathbf{R}_0 + h_0 \cdot (1, \delta, \cdots, \delta^{\tau-1}) \in R_{q_2}^{1\times\tau}$;
3. Send $\mathsf{pk} := (\boldsymbol{a}, \boldsymbol{b}_0, \{\boldsymbol{b}_i\}_{i\in[\theta]}, u)$ to $\mathcal{A}$.

According to the uniformity of $x_4, \cdots, x_{2+\tau}, x_{3+\tau}$ and the distribution of $\mathbf{R}_0$, pk is a valid public key of our commit-transferrable signature, which follows from the Ring-$\mathsf{LWE}_{q_2,2,1,S_1}$ assumption.

Then, the $\mathcal{A}$ can conduct signature queries and get responses from $\mathcal{B}$. In particular, after receiving the signature query $(\mathsf{comm}, \boldsymbol{m}, \{\boldsymbol{r}_{i,j}\}_{i\in[\theta],j\in[\tau]})$ from $\mathcal{A}$, where $\mathsf{comm} = \{(\mathsf{comm}_{i,1}, \mathsf{comm}_{i,2}, \cdots, \mathsf{comm}_{i,\tau})\}_{i\in[\theta]}$ and

$$\mathsf{comm}_{i,1} := \begin{bmatrix} t_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix} = \mathbf{A} \cdot \boldsymbol{r}_{i,1} + \begin{bmatrix} 0 \\ m_i \end{bmatrix},$$

$$\mathsf{comm}_{i,2} := \begin{bmatrix} t_{1,2}^{(i)} \\ t_{2,2}^{(i)} \end{bmatrix} = \mathbf{A} \cdot \boldsymbol{r}_{i,2} + \begin{bmatrix} 0 \\ m_i\delta \end{bmatrix},$$

$$\cdots$$

$$\mathsf{comm}_{i,\tau} := \begin{bmatrix} t_{1,\tau}^{(i)} \\ t_{2,\tau}^{(i)} \end{bmatrix} = \mathbf{A} \cdot \boldsymbol{r}_{i,\tau} + \begin{bmatrix} 0 \\ m_i\delta^{\tau-1} \end{bmatrix}.$$

$\mathcal{B}$ can compute

$$
\begin{aligned}
\mathbf{F}_{\mathsf{comm}} &= \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \big| \boldsymbol{b}_0 + \sum_{i\in[\theta]} \left((t_{2,1}^{(i)}, t_{2,2}^{(i)}, \cdots, t_{2,\tau}^{(i)}) \cdot \mathbf{G}^{-1}(\boldsymbol{b}_i) \right) \big| \boldsymbol{a}_2 \right] \\
&= \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \big| \boldsymbol{d}^\top \cdot \mathbf{R}_0 + h_0 \cdot \mathbf{G} + \sum_{i\in[\theta]} \left((\boldsymbol{a}_2^\top \cdot \mathbf{R}_{i,2} + m_i \mathbf{G}) \cdot \mathbf{G}^{-1}(\boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \mathbf{G}) \right) \big| \boldsymbol{a}_2 \right] \\
&= \Bigg[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \big| \boldsymbol{d}^\top \cdot \mathbf{R}_0 + h_0 \cdot \mathbf{G} + \sum_{i\in[\theta]} \Big(\boldsymbol{a}_2^\top \cdot \mathbf{R}_{i,2} \cdot \mathbf{G}^{-1}(\boldsymbol{d}^\top \cdot \mathbf{R}_i \\
&\quad + h_i \mathbf{G}) + \boldsymbol{d}^\top \cdot m_i \mathbf{R}_{i,2} + m_i h_i \mathbf{G} \Big) \big| \boldsymbol{a}_2 \Bigg] \\
&= \Bigg[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \big| \boldsymbol{a}_2^\top \cdot \sum_{i\in[\theta]} \left(\mathbf{R}_{i,2} \cdot \mathbf{G}^{-1}(\boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \mathbf{G}) \right) + \\
&\quad + \boldsymbol{d}^\top \cdot \left(\mathbf{R}_0 + \sum_{i\in[\theta]} m_i \mathbf{R}_{i,2} \right) + (h_0 + \langle \boldsymbol{m}, \boldsymbol{h} \rangle) \cdot \mathbf{G} \big| \boldsymbol{a}_2 \Bigg],
\end{aligned}
$$

where we denote $\mathbf{R}_i = \begin{bmatrix} \mathbf{R}_{i,1} \\ \mathbf{R}_{i,2} \end{bmatrix} = \left[\boldsymbol{r}_{i,1}, \boldsymbol{r}_{i,2}, \cdots, \boldsymbol{r}_{i,\tau} \right] \in R^{4\times\tau}$ with $\mathbf{R}_{i,2} \in R^{3\times\tau}$.

For any $h_0 + \langle \boldsymbol{m}, \boldsymbol{h} \rangle \neq 0 \bmod q_2 R$, we know that $h_0 + \langle \boldsymbol{m}, \boldsymbol{h} \rangle$ is invertible over the subfield $\mathcal{S}_{q_2}$ of ring R_{q_2}. According to the algorithm in Lemma A.7, the challenger can get a short vector $\boldsymbol{z} \in R^{2\tau+6}$ such that $\mathbf{F}_{\mathsf{comm}} \cdot \boldsymbol{z} = u$. □

From above Claim B.5, we know that $\mathcal{B}$ can simulate the environment of $\mathcal{A}$ successfully.

Next, for the challenge query of the form $(\mathsf{comm}^*, \boldsymbol{m}^*, \{\boldsymbol{r}_{i,j}^*\}_{i\in[\theta], j\in[\tau]})$, we have

$$
\mathbf{F}_{\mathsf{comm}^*} = \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \big| \boldsymbol{a}_2^\top \cdot \sum_{i\in[\theta]} \left(\mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \mathbf{G}) \right) + \boldsymbol{d}^\top \cdot (\mathbf{R}_0 + \sum_{i\in[\theta]} m_i^* \mathbf{R}_{i,2}^*) \big| \boldsymbol{a}_2^\top \right].
$$

Below, according to the fact that the adversary's forgery is for original signature or transferred one, we need to separate the following proof into two cases.

For the case of original one. If the adversary can forge a valid signature $\mathsf{Sig}_{\mathsf{comm}^*} := \begin{bmatrix} \boldsymbol{s}_{1,1}^* \\ \boldsymbol{s}_{1,2}^* \\ \boldsymbol{s}_2^* \\ \boldsymbol{s}_3^* \end{bmatrix}$ with $\boldsymbol{s}_3^* = (s_{3,1}^*, s_{3,2}^*, s_{3,3}^*)^T \in R^3$, such that

$$\begin{aligned}
&\mathbf{F}_{\mathsf{comm}^*} \cdot \mathsf{Sig}_{\mathsf{comm}^*} = \\
&\left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| \boldsymbol{a}_2^\top \cdot \sum_{i \in [\ell]} \mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{p}) + \boldsymbol{d}^\top \cdot \mathbf{R}^* \middle| \boldsymbol{a}_2 \right] \cdot \begin{bmatrix} \boldsymbol{s}_{1,1}^* \\ \boldsymbol{s}_{1,2}^* \\ \boldsymbol{s}_2^* \\ \boldsymbol{s}_3^* \end{bmatrix} \\
=&\langle \boldsymbol{d}, \boldsymbol{s}_{1,1}^* \rangle + \langle \boldsymbol{a}, \boldsymbol{s}_{1,2}^* \rangle + \langle \boldsymbol{a}_2^\top \cdot \sum_{i \in [\theta]} \mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{p}) + \boldsymbol{d}^\top \cdot \mathbf{R}^*, \boldsymbol{s}_2^* \rangle \\
&+ \langle \boldsymbol{a}_2, \boldsymbol{s}_3^* \rangle \\
=& u,
\end{aligned}$$

where $\boldsymbol{p} = \boldsymbol{d}^\top \cdot \mathbf{R}_i + h_i \mathbf{G}, \mathbf{R}^* = \mathbf{R}_0 + \sum_{i \in [\ell]} m_i^* \mathbf{R}_{i,2}^*$, then $\mathcal{B}$ can compute $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{s}_{1,1}^* + \mathbf{R}^* \cdot \boldsymbol{s}_2^* \\ \boldsymbol{s}_{1,2}^* \\ -1 \\ \boldsymbol{s}_3^* + \sum_{i \in [\theta]} \mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{p}) \cdot \boldsymbol{s}_2^* \end{bmatrix}$ as a solution to the $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu}$ problem defined by $[x_1, x_2, x_3, \cdots, x_{\tau+3}, x_{\tau+4}, 1, x_{\tau+5}, x_{\tau+6}]$. And the ℓ_2 norm of this solution is less than $\|\boldsymbol{y}\| \leq \alpha\sqrt{2(\tau+6)N} + (\tau + \sqrt{3\tau})\sqrt{2}N\alpha\sqrt{(\ell B+1)^2 + \tau^2 N^2 \theta^2 \delta^2}$.

For the case of transferred one. If the adversary can forge a valid proof for the language $L_{\gamma', q_2, \bar{\mathcal{C}}}$, then the reduction algorithm $\mathcal{B}$ can run the extractor of the $\mathsf{NIZKPoK}$ system Π_2, and get a ℓ_2 norm short vector $\mathsf{Sig}'_{\mathsf{comm}^*} := \begin{bmatrix} \boldsymbol{s}_{1,1}^* \\ \boldsymbol{s}_{1,2}^* \\ \boldsymbol{s}_2^* \\ \boldsymbol{s}_3^* \end{bmatrix}$ with $\boldsymbol{s}_3^* = (s_{3,1}^*, s_{3,2}^*, s_{3,3}^*)^\top \in R^3$, such that

$$\begin{aligned}
&\mathbf{F}_{\mathsf{comm}^*} \cdot \mathsf{Sig}_{\mathsf{comm}^*} \\
&= \left[[\boldsymbol{d}^\top | \boldsymbol{a}^\top] \middle| \boldsymbol{a}_2^\top \cdot \sum_{i \in [\theta]} \mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{p}) + \boldsymbol{d}^\top \cdot \mathbf{R}^* \middle| \boldsymbol{a}_2 \right] \cdot \begin{bmatrix} \boldsymbol{s}_{1,1}^* \\ \boldsymbol{s}_{1,2}^* \\ \boldsymbol{s}_2^* \\ \boldsymbol{s}_3^* \end{bmatrix} \\
&= \langle \boldsymbol{d}, \boldsymbol{s}_{1,1}^* \rangle + \langle \boldsymbol{a}, \boldsymbol{s}_{1,2}^* \rangle + \langle \boldsymbol{a}_2^\top \cdot \sum_{i \in [\theta]} \mathbf{R}_{i,2}^* \mathbf{G}^{-1}(\boldsymbol{p}) + \boldsymbol{d}^\top \cdot \mathbf{R}^*, \boldsymbol{s}_2^* \rangle \\
&\quad + \langle \boldsymbol{a}_2, \boldsymbol{s}_3^* \rangle \\
&= \bar{c} u,
\end{aligned}$$

then $\mathcal{B}$ can compute $\boldsymbol{y} = \begin{bmatrix} \boldsymbol{s}^*_{1,1} + \mathbf{R}^* \cdot \boldsymbol{s}^*_2 \\ \boldsymbol{s}^*_{1,2} \\ -\bar{c} \\ \sum_{i\in[\theta]} \mathbf{R}^*_{i,2}\mathbf{G}^{-1}(\boldsymbol{p}) \cdot \boldsymbol{s}^*_2 + \boldsymbol{s}^*_3 \end{bmatrix}$ as a solution to the $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu'}$ problem defined by $[x_1, x_2, x_3, \cdots, x_{\tau+3}, x_{\tau+4}, 1, x_{\tau+5}, x_{\tau+6}]$. And the ℓ_2 norm of this solution is less than $\|\boldsymbol{y}\| \leq \alpha'\sqrt{2(\tau+6)N}+(\tau+\sqrt{3\tau})\sqrt{2}N\alpha' \cdot \sqrt{(\theta\sqrt{N}+1)^2+\tau^2N^2\delta^2}+2\sqrt{\kappa}$, with $\alpha' = \gamma'/\sqrt{(2\tau+6)N}$.

Furthermore, according to the forking lemma of [8, 50], $\mathcal{R}$ can complete the above reduction with probability at least $\varepsilon(\frac{\varepsilon}{Q_2} - 2^{-\lambda})$.

Summing up all above arguments, we conclude that our commit transferrable signature satisfies unforgeability in the adaptive way. □

Completing the Proof. Recall that Q_1 is the upper bound of the number of the adversary's signing queries, and ε_1 is the advantage of the adversary in H_1. By Lemma D.1 and D.2, we can know that

$$\Pr_H\left[H(\boldsymbol{m}^*) = 0 \bigwedge H(\boldsymbol{m}_1) \neq 0 \bigwedge \ldots \bigwedge H(\boldsymbol{m}_{Q_1}) \neq 0\right] \in \left[\frac{1}{|\bar{\mathcal{M}}|}(1-\frac{Q_1}{|\bar{\mathcal{M}}|}), \frac{1}{|\bar{\mathcal{M}}|}\right].$$

Thus, we know that for any (Q_1+1)-tuple I denoting a challenge $\boldsymbol{m}^*$ along with signing queries, we have $\varepsilon(I) \in \left[\frac{1}{|\bar{\mathcal{M}}|}(1-\frac{Q_1}{|\bar{\mathcal{M}}|}), \frac{1}{|\bar{\mathcal{M}}|}\right]$. Then by setting $[\varepsilon_{min}, \varepsilon_{max}] = \left[\frac{1}{|\bar{\mathcal{M}}|}(1-\frac{Q_1}{|\bar{\mathcal{M}}|}), \frac{1}{|\bar{\mathcal{M}}|}\right]$ in Lemma D.9, we have

$$\mathsf{Adv}^{\mathsf{H}_2}_{\mathcal{A}}(\lambda) \geq \frac{1}{|\bar{\mathcal{M}}|}(1-\frac{Q_1}{|\bar{\mathcal{M}}|})\varepsilon_1 - \frac{Q_1}{2|\bar{\mathcal{M}}|^2}.$$

By our parameter setting, $|Q| \leq \frac{1}{2}\varepsilon_1|\bar{\mathcal{M}}|$, we have that

$$\mathsf{Adv}^{\mathsf{H}_2}_{\mathcal{A}}(\lambda) \geq \frac{1}{|\bar{\mathcal{M}}|}(1-\frac{Q_1}{|\bar{\mathcal{M}}|})\varepsilon_1 - \frac{Q_1}{2|\bar{\mathcal{M}}|^2} \geq \frac{1}{4|\bar{\mathcal{M}}|} \cdot \mathsf{Adv}^{\mathsf{H}_1}_{\mathcal{A}}(\lambda).$$

In summary, we have that

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{H}_0}_{\mathcal{A}}(\lambda) &\leq \mathsf{Adv}^{\mathsf{H}_1}_{\mathcal{A}}(\lambda) + \ell \cdot \mathsf{Adv}^{\text{Ring-LWE}}_{\mathcal{A}}(\lambda) \\
&\leq \theta \cdot \mathsf{Adv}^{\text{Ring-LWE}}_{\mathcal{A}}(\lambda) + 4|\bar{\mathcal{M}}|\mathsf{Adv}^{\mathsf{H}_2}_{\mathcal{A}}(\lambda) \\
&\leq (\theta + 4|\bar{\mathcal{M}}|) \cdot \mathsf{Adv}^{\text{Ring-LWE}}_{\mathcal{A}}(\lambda) + 4|\bar{\mathcal{M}}|\mathsf{Adv}^{\mathsf{H}_3}_{\mathcal{A}}(\lambda) \\
&\leq (\theta + 4|\bar{\mathcal{M}}|) \cdot \mathsf{Adv}^{\text{Ring-LWE}}_{\mathcal{A}}(\lambda) + 4|\bar{\mathcal{M}}|\sqrt{(\mathsf{Adv}^{\mathsf{M\text{-}SIS}}_{\mathcal{R}}(\lambda) + \frac{1}{2^\lambda})Q_1},
\end{aligned}$$

which completes the proof. □

D.5 Instantiation of NIZKPoK for Construction D.3

In this part, we instantiate the NIZKPoK involved in Construction D.3. We need to prove the following equations over R_{N,q_1} and R_{N,q_2}:

$$\begin{cases} \mathsf{comm}_{i,1} := \begin{bmatrix} \boldsymbol{t}_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix} = \begin{bmatrix} 1, \boldsymbol{a}_1^\top \\ 0, \boldsymbol{a}_2^\top \end{bmatrix} \tilde{\boldsymbol{r}}_{i,1} + \begin{bmatrix} \boldsymbol{0} \\ m_i \end{bmatrix} & \begin{matrix} \bmod q_1 \\ \bmod q_2 \end{matrix}, \\ \mathsf{comm}_{i,2} := \begin{bmatrix} \boldsymbol{t}_{1,2}^{(i)} \\ t_{2,2}^{(i)} \end{bmatrix} = \begin{bmatrix} 1, \boldsymbol{a}_1^\top \\ 0, \boldsymbol{a}_2^\top \end{bmatrix} \tilde{\boldsymbol{r}}_{i,2} + \begin{bmatrix} \boldsymbol{0} \\ m \cdot \delta \end{bmatrix} & \begin{matrix} \bmod q_1 \\ \bmod q_2 \end{matrix}, \\ \quad \vdots \quad \vdots \quad \vdots \\ \mathsf{comm}_{i,\tau} := \begin{bmatrix} \boldsymbol{t}_{1,\tau}^{(i)} \\ t_{2,\tau}^{(i)} \end{bmatrix} = \begin{bmatrix} 1, \boldsymbol{a}_1^\top \\ 0, \boldsymbol{a}_2^\top \end{bmatrix} \tilde{\boldsymbol{r}}_{i,\tau} + \begin{bmatrix} \boldsymbol{0} \\ m \cdot \delta^{\tau-1} \end{bmatrix} & \begin{matrix} \bmod q_1 \\ \bmod q_2 \end{matrix}. \end{cases} \tag{8}$$

Furthermore, we can easily transfer the above Equation (8) into the following equations.

$$\begin{cases} \boldsymbol{t}_{1,1}^{(i)} = \left[1, \boldsymbol{a}_1^\top\right] \cdot \tilde{\boldsymbol{r}}_{i,1} \mod q_1 \\ \boldsymbol{t}_{1,2}^{(i)} = \left[1, \boldsymbol{a}_1^\top\right] \cdot \tilde{\boldsymbol{r}}_{i,2} \mod q_1 \\ \quad \vdots \quad \vdots \quad \vdots \\ \boldsymbol{t}_{1,\tau}^{(i)} = \left[1, \boldsymbol{a}_1^\top\right] \cdot \tilde{\boldsymbol{r}}_{i,\tau} \mod q_1 \end{cases}, \quad \begin{cases} t_{2,1} = \langle 0 \| \boldsymbol{a}_2, \tilde{\boldsymbol{r}}_{i,1} \rangle + m_i \mod q_2 \\ t_{2,2} = \langle 0 \| \boldsymbol{a}_2, \tilde{\boldsymbol{r}}_{i,2} \rangle + m_i \cdot \delta \mod q_2 \\ \quad \vdots \quad \vdots \quad \vdots \\ t_{2,\tau} = \langle 0 \| \boldsymbol{a}_2, \tilde{\boldsymbol{r}}_{i,\tau} \rangle + m_i \cdot \delta^{k-1} \mod q_2 \end{cases}, \tag{9}$$

where $\tilde{\boldsymbol{r}}_{i,j} \in S_1^3$ for $i \in [\theta], j \in [\tau]$, and $m \in \bar{\mathcal{M}}$, as defined in the setup algorithm of Construction 4.1. Moreover, $(\mathrm{mod} q_1)$ and $(\mathrm{mod} q_2)$ means the computations are conducted over R_{N,q_1} and R_{N,q_2}, respectively.

Notice that due to the usage of pair-wise independent hash function for adaptive security, we can not directly apply the LNP approach to prove the validness of the committed message. This is because with the LNP approach, we direct prove the committed message is a binary polynomial with certain constant ℓ_1-norm. But such type of message space can not be easily proven to be a field. Notice also that field is a necessary condition for the usage of pair-wise independent hash function in Lemma D.2.

In order to conquer this dilemma, we use the property of the algebraic structure of m-th cyclotomic ring $R = \mathbb{Z}_q[X]/\langle \Phi_m(X) \rangle$ in Lemmas A.13 and A.14. Particularly, this property means that a polynomial $m \in R$ is contained in a subfield, if and only if m is fixed under automorphisms σ_{-1} and $\sigma_5^{\zeta'}$, i.e., $m = \sigma_{-1}(m) = \sigma_5^{\zeta'}(m)$. Here the size of this subfield is $q^{\zeta'}$, and q is prime such that $q = 3$ or $5 \mod 8$. The concrete protocol $\Pi_{\mathsf{well\text{-}formedness}}$ is presented in the following Table 13. So compared with the multi-theorem straight-line extractable NIZKPoK in Section 5, we need to consider the additional overhead due to Table 13. And this overhead will be mild when ℓ is a small constant.

Interactive proof system $\Pi_{\mathsf{well\text{-}formedness}}$

Public Parameter for Commitment Scheme:

$\mathbf{A} = \begin{bmatrix} \boldsymbol{a}_1^\top \\ \boldsymbol{a}_2^\top \end{bmatrix} = \begin{bmatrix} 1, & \boldsymbol{a}_1'^\top \\ 0, 1, & a_2' \end{bmatrix}$ as in Construction D.3, $\zeta' | N$, $B_1 = \xi \cdot \sqrt{6N}$,

Prover's Witness: for $i \in [\theta]$, $\boldsymbol{r}_{i,1} \in S_1^3$, $m_i = \sum_{\hat{i} \in [N]} m_{\hat{i}-1} X^{\hat{i}-1}$ is in a subfield of R_{q_2},

Commitment: $(\mathsf{comm}_{i,1})_{i \in [\theta]}$, $\mathsf{comm}_{i,1} := \begin{bmatrix} t_{1,1}^{(i)} \\ t_{2,1}^{(i)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_1^\top \\ \boldsymbol{a}_2^\top \end{bmatrix} \cdot \boldsymbol{r}_{i,1} + \begin{bmatrix} 0 \\ m_i \end{bmatrix}$.

Prover		Verifier
$\forall i \in [\theta], \boldsymbol{y}_i \leftarrow \mathcal{D}_\xi^3, \boldsymbol{y}_{-1,i}, \boldsymbol{y}_{5,i} \leftarrow \mathcal{D}_\xi^3$		
$\forall i \in [\theta], w_{i,1} = \boldsymbol{a}_1^\top \cdot \boldsymbol{y}_i,$		
$w_{1,-1}^{(i)} = \sigma_{-1}(\boldsymbol{a}_1^\top) \cdot \boldsymbol{y}_{-1,i}$, $w_{1,5}^{(i)} = \sigma_5^{\zeta'}(\boldsymbol{a}_1^\top) \cdot \boldsymbol{y}_{5,i}$		
$w_{2,-1}^{(i)} = \boldsymbol{a}_2^\top \cdot \boldsymbol{y}_i - \sigma_{-1}(\boldsymbol{a}_2^\top) \cdot \boldsymbol{y}_{-1,i}$		
$w_{2,5}^{(i)} = \boldsymbol{a}_2^\top \cdot \boldsymbol{y}_i - \sigma_5^{\zeta'}(\boldsymbol{a}_2^\top) \cdot \boldsymbol{y}_{5,i}$		
	$\xrightarrow{w_{i,1}, w_{1,-1}^{(i)}, w_{1,5}^{(i)}, w_{2,-1}^{(i)}, w_{2,5}^{(i)}}$	
		$d \xleftarrow{\$} \mathcal{C}$
	$\xleftarrow{d}$	
$\forall i \in [\theta], \boldsymbol{z}_{i,1} = \boldsymbol{y}_i + d \cdot \boldsymbol{r}_{i,1}$		
$\boldsymbol{z}_{-1,i} = \boldsymbol{y}_{-1,i} + d \cdot \sigma_{-1}(\boldsymbol{r}_{i,1})$		
$\boldsymbol{z}_{5,i} = \boldsymbol{y}_{5,i} + d \cdot \sigma_5^{\zeta'}(\boldsymbol{r}_{i,1})$		
$\mathsf{Rej}(\boldsymbol{z}_{i,1} \| \boldsymbol{z}_{-1,i} \| \boldsymbol{z}_{5,i}, d \cdot (\boldsymbol{r}_i \| \sigma_{-1}(\boldsymbol{r}_{i,1}) \| \sigma_5^{\zeta'}(\boldsymbol{r}_{i,1})), \xi)$		
	$\xrightarrow{\boldsymbol{z}_{i,1}, \boldsymbol{z}_{-1,i}, \boldsymbol{z}_{5,i}}$	
		Check:
		1. for $i \in [\theta], \Vert\boldsymbol{z}_{i,1}\Vert \overset{?}{\leq} B_1,$
		$\Vert z_{-1}\Vert \overset{?}{\leq} B_1, \Vert z_5\Vert \overset{?}{\leq} B_1$
		2. for $i \in [\theta], \boldsymbol{a}_1^\top \cdot \boldsymbol{z}_{i,1} \overset{?}{=} w_{i,1} + d \cdot t_{1,i}$
		$\sigma_{-1}(\boldsymbol{a}_1^\top) \cdot \boldsymbol{z}_{-1,i} \overset{?}{=} w_{1,-1}^{(i)} + d \cdot \sigma_{-1}(t_{1,1}^{(i)})$
		$\sigma_5^{\zeta'}(\boldsymbol{a}_1^\top) \cdot \boldsymbol{z}_{5,i} \overset{?}{=} w_{1,5}^{(i)} + d \cdot \sigma_5^{\zeta'}(t_{1,1}^{(i)})$
		$\boldsymbol{a}_2^\top \cdot \boldsymbol{z}_{i,1} - \sigma_{-1}(\boldsymbol{a}_2^\top) \cdot \boldsymbol{z}_{-1,i}$
		$\overset{?}{=} w_{2,-1}^{(i)} + d \cdot (t_{2,1}^{(i)} - \sigma_{-1}(t_{2,1}^{(i)}))$
		$\boldsymbol{a}_2^\top \cdot \boldsymbol{z}_{i,1} - \sigma_5^{\zeta'}(\boldsymbol{a}_2^\top) \cdot \boldsymbol{z}_{5,i}$
		$\overset{?}{=} w_{2,5}^{(i)} + d \cdot (t_{2,1}^{(i)} - \sigma_5^{\zeta'}(t_{2,1}^{(i)}))$
		Accept if all the above conditions hold.

Table 13. Proof of $m = (m_i)_{i \in [\theta]}$ is included into the message space.

Overall, the Equations (9) can be proven through instantiating LNP proof in the following Table 14, which is similar to Section 5. Additionally, the concrete parameters for LNP are presented in Table 15.

D.6 Parameter Settings of Construction D.3

In this part, we set the concrete parameters for Construction D.3 and the straight-line extractable $\mathsf{NIZKPoK}$ system, according to the related requirements

variable	description	instantiation
ρ	# of equations to prove	$t_0 = 2 \cdot \tau \cdot \theta \cdot N/d$
ρ_{eval}	# of evaluations with const. coeff. zero	0
v_e	# of exact norm proofs	$\tau \cdot \theta + 2$
v_d	# non-exact norm proofs	1
k_{bin}	length of the binary vector to prove	0
$\boldsymbol{s}_1$	committed message in the Ajtai part	$(\boldsymbol{r}_{\mathsf{PKE}}, \{\tilde{\boldsymbol{r}}_{i,j}\}_{i\in[\theta],j\in[\tau]}, m)$
$\boldsymbol{m}$	committed message in the BDLOP part	$\emptyset$ (no message)
$f_1, \ldots, f_{t_0}$	equations to prove	Equations (9)
F_1	evaluation to prove const coeff. zero	$\emptyset$
$\mathbf{E}_1$	public matrix for proving $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$[\mathbf{I}_{m_{\mathsf{PKE}}}\ \mathbf{0}\ \ldots\ \mathbf{0}\ 0]$
$\boldsymbol{v}_1$	public vector for proving $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$\mathbf{0}$
$\beta_1^{(e)}$	upper-bound on $\|\mathbf{E}_1\boldsymbol{s} - \boldsymbol{v}_1\| \leq \beta_1^{(e)}$	$2\sqrt{d \cdot m_{\mathsf{PKE}}}$
$\mathbf{E}_2$	public matrix for proving $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$\left[\mathbf{0}\ \mathbf{I}_{3\cdot N/d}\ \ldots \mathbf{0}\ 0\right]$
$\boldsymbol{v}_2$	public vector for proving $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$\mathbf{0}$
$\beta_2^{(e)}$	upper-bound on $\|\mathbf{E}_2\boldsymbol{s} - \boldsymbol{v}_2\| \leq \beta_2^{(e)}$	$\sqrt{3 \cdot N}$
$\vdots$	$\vdots$	$\vdots$
$\mathbf{E}_{\tau\cdot\theta+1}$	public matrix for proving $\|\mathbf{E}_{\tau\cdot\theta+1}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+1}\| \leq \beta_{\tau\cdot\theta+1}^{(e)}$	$\left[\mathbf{0}\ \ldots\ \mathbf{I}_{3\cdot N/d}\ 0\right]$
$\boldsymbol{v}_{\tau\cdot\theta+1}$	public vector for proving $\|\mathbf{E}_{\tau\cdot\theta+1}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+1}\| \leq \beta_{\tau\cdot\theta+1}^{(e)}$	$\mathbf{0}$
$\beta_{\tau\cdot\theta+1}^{(e)}$	upper-bound on $\|\mathbf{E}_{\tau\cdot\theta+1}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+1}\| \leq \beta_{\tau\cdot\theta+1}^{(e)}$	$\sqrt{3 \cdot N}$
$\mathbf{E}_{\tau\cdot\theta+2}$	public matrix for proving $\|\mathbf{E}_{\tau\cdot\theta+2}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+2}\| \leq \beta_{\tau\cdot\theta+2}^{(e)}$	$[\mathbf{0}\ \ldots\ 1]$
$\boldsymbol{v}_{\tau\cdot\theta+2}$	public vector for proving $\|\mathbf{E}_{\tau\cdot\theta+2}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+2}\| \leq \beta_{\tau\cdot\theta+2}^{(e)}$	$\mathbf{0}$
$\beta_{\tau\cdot\theta+2}^{(e)}$	upper-bound on $\|\mathbf{E}_{\tau\cdot\theta+2}\boldsymbol{s} - \boldsymbol{v}_{\tau\cdot\theta+2}\| \leq \beta_{\tau\cdot\theta+2}^{(e)}$	$\sqrt{N}$
$\mathbf{D}_1$	public matrix for proving $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$q_{\mathsf{PKE}}^{-1} \cdot \begin{bmatrix} \mathbf{A}_{\mathsf{PKE}}, \mathbf{0} \\ \mathbf{B}_{\mathsf{PKE}}, \mathbf{I}_{k_{\mathsf{PKE}}} \end{bmatrix}$
$\boldsymbol{u}_1$	public vector for proving $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$q_{\mathsf{PKE}}^{-1} \cdot \begin{bmatrix} \boldsymbol{t}_0 \\ \boldsymbol{t}_1 \end{bmatrix}$
$\beta_1^{(d)}$	upper-bound on $\|\mathbf{D}_1\boldsymbol{s} - \boldsymbol{u}_1\| \leq \beta_1^{(d)}$	$(d \cdot m_{\mathsf{PKE}} + 1)\sqrt{(n_{\mathsf{PKE}} + 1) \cdot d}$

Table 14. Instantiation of LNP proof for multi-theorem straight-line extractable NIZKPoK.

variable	description	Para. 1
q_1	modulus for the BDLOP commitment	$2^{35}-451$
q_2		$2^{175}-267$
d	dimension for the underlying ring of LNP	4096
t	$N = t \cdot d$ is the dimension for the ring of CTS	4
ζ'	$m \in \mathcal{M}$ are fixed under $\langle \sigma_{-1}, \sigma_5^{\zeta'} \rangle$	1
$\lvert\mathcal{M}\rvert$	size of the user space	$\approx 2^{175}$
q_{PKE}	encryption modulus	472154173
n_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	1
m_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	4803
k_{PKE}	height of $\mathbf{A}_{\mathsf{PKE}}$	4801
δ_0^{PKE}	root-hermite factor of PKE	1.00129048
$\mathsf{Bit\text{-}sec}^{\mathsf{PKE}}$	classcial/quantum security of PKE	528.81/479.91
q_{LNP}	modulus for the proof system	$\approx 2^{210}$
l	# factors X^d+1 splits into mod q_{LNP}	2
γ_1	rejection sampling constant for $c\boldsymbol{s}_1$	17
γ_2	rejection sampling constant for $c\boldsymbol{s}_2$	1.2
$\gamma^{(e)}$	rejection sampling constant exact ARP	2.5
$\gamma^{(d)}$	rejection sampling constant non-exact ARP	12
η^*	ℓ_∞-norm upper bound of challenge	14
κ^*	maximum coefficient of a challenge in $\mathcal{C}$	1
n^*	height of matrices $\mathbf{A}_1, \mathbf{A}_2$ in ABDLOP	1
m_1	length of the message $\boldsymbol{s}_1$ in the"Ajtai" part	24013
ℓ^*	length of the message $\boldsymbol{m}$ in the"BDLOP " part	0
m_2	length of the message $\boldsymbol{s}_2$ in ABDLOP	9
ν^*	randomness $\boldsymbol{s}_2$ is sampled from $S_{\nu^*}^{m_2}$	1
γ^*	parameters to cut low-order bits from $\boldsymbol{w}$	$\approx 2^{123.74}$
D^*	number of low-order bits cut from $\boldsymbol{t}_A$	112
	repetition rate	7
$\lvert\mathsf{size}_{\mathsf{LNP}}\rvert$	proof size of the straight-line extractable NIZKPoK	354.84 MB
$\lvert\mathsf{para}_{\mathsf{ABDLOP}}\rvert$	public parameter size of ABDLOP commitment	2.447 GB
$\lvert\mathsf{para}_{\mathsf{PKE}}\rvert$	public parameter size of PKE encryption	318.935 GB
$\lvert\mathsf{ct}_{\mathsf{PKE}}\rvert$	ciphertext size of PKE encryption	67.997 MB

Table 15. Parameter selection and concrete sizes for for multi-theorem straight-line extractable NIZKPoK, where the setting root-hermite factor is 1.003735, with 128-bit classical security.

in correctness and security. For clarity, we denote the straight-line extractable NIZKPoK system for $\hat{L}_{q_1,q_2}$ in Section D.4 as Π_1, and denote the NIZKPoK system for $L_{\gamma',q_2,\bar{\mathcal{C}}}$ in Section D.3 as Π_2.

Requirements for Correctness. We require the following:

- The SamplePre in the Sign step needs to work properly. according to Lemma A.3, we need to set

$$\alpha \geq 2\sqrt{\delta^2+1}\cdot((\sqrt{\tau}+\sqrt{3})\sqrt{N}+1),$$

  where $\delta = q_2^{1/\tau}$ and τ is an integer in $[2, \lfloor \log q_2 \rfloor]$.
- The valid original signature $\mathsf{Sig}_{\mathsf{comm}}$ can be verified successfully. According to Lemma A.2, we need to set

$$\gamma = \alpha\sqrt{2\cdot(2\tau+6)\cdot N}.$$

- The valid transferred signature $\mathsf{Sig}'_{\mathsf{comm}'}$ can be verified successfully. According to Lemma D.4 and the relaxed language in Theorem 4.3, we need to set

$$\gamma' = 2\sqrt{2(2\tau+6)N}\cdot\eta\kappa\cdot((\sqrt{3\tau}+\tau)\sqrt{2\tau}\theta\alpha N^2\delta + \alpha\sqrt{2\cdot(2\tau+6)N}).$$

- The message space of Construction 5.1 is large enough to encrypt all randomness. And, the related Ciphertexts can be decrypted correctly. According to the corresponding analysis, we need set $k_{\mathsf{PKE}} = 3\cdot N/d\cdot\tau\cdot\theta+1$, $q_{\mathsf{PKE}}/2 > 12\cdot m_{q_{\mathsf{PKE}}}\cdot d+1$, with $\mathbf{A}_{\mathsf{PKE}} \in R_{d,q_{\mathsf{PKE}}}^{n_{\mathsf{PKE}}\times m_{\mathsf{PKE}}}$, $\mathbf{B}_{\mathsf{PKE}} \in R_{d,q_{\mathsf{PKE}}}^{k_{\mathsf{PKE}}\times m_{\mathsf{PKE}}}$.

Requirements for Security. We require the following:

- The ring R_N, R_d are cyclotomic rings, i.e., $R_N = \mathbb{Z}[X]/(X^N+1)$, $R_d = \mathbb{Z}[X]/(X^d+1)$, with $d|N$. In this case, according to the efficiently computable ring isomorphism between R_N and $R_d^{N/d}$, any relations we need to prove over R_N can be proven by showing the corresponding relations over R_d is set up.
- For the fixed security parameter λ, we require that the output distribution of the rejection sampling algorithm is within statistical distance of $\frac{2^{-\lambda}}{M}$ of the related product distribution, according to Lemma A.9. Thus, we need to set η satisfying $M = \exp\left(\sqrt{\frac{2(\lambda+1)}{\log e}}\cdot\frac{1}{\eta}+\frac{1}{2\eta^2}\right) = O(1)$.
- There exists a multi-theorem straight-line extractable NIZKPoK system Π_1 for the commitment relation $\hat{L}_{q_1,q_2}$. Hence, in order to make Construction 5.1 to be IND-CPA security, $\mathsf{M\text{-}LWE}_{q_{\mathsf{PKE}},n_{\mathsf{PKE}},m_{\mathsf{PKE}},\hat{S}_2}$ need to be hard. Other hard problems for the concrete instantiation of LNP proof are implicitly considered in the parameter setting of Table 11.
- There exists an NIZKPoK system Π_2 for the language $L_{\gamma',q_2,\bar{c}}$. According to Theorem 4.3, the problem $\mathsf{M\text{-}SIS}_{q_2,1,2\tau+6,\gamma'}$ needs to be hard.
- The constructed CTS satisfies unforgeability in Definition 3.3. Particularly,
  - For Definition 3.3 with respect to the exact commitment relation $\hat{L}_{q_1,q_2}$, according to Lemma D.10, and Claim D.11, we need to set $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu'}$ being hard with

$$\nu' = \alpha'\sqrt{2(\tau+6)N} + (\tau+\sqrt{3\tau})\sqrt{2}N\alpha'\cdot\sqrt{(\theta\sqrt{N}+1)^2+\tau^2N^2\delta^2}+2\sqrt{\kappa},$$

with $\alpha' = \gamma'/\sqrt{(2\tau+6)N}$.

- The underlying BDLOP satisfies hiding and binding. Hence, according to Section 2.3, we need to set M-LWE$_{q_2,2,1}$ and M-SIS$_{q_1,1,4,8\sqrt{2}\cdot\eta\cdot\kappa\cdot\beta\cdot 4\cdot N}$ being hard.
- The successful simulation of the adversary in Claim D.11. Here, according to the Lemma A.7, we need to set

$$\alpha \geq 2\sqrt{\delta^2+1}\cdot((\sqrt{\tau}+\sqrt{6})\cdot\sqrt{N}(\theta\sqrt{N}+\tau\theta N\delta+1)+1).$$

More specifically, we have the concreted parameter setting in the following Table 16.

	Params Example
N	16384
d	4096
q_1	$2^{35}-451$
q_2	$2^{175}-267$
λ	580
κ	55
η	15.9
M	6
θ	4
τ	100
ζ'	1
δ	3.363
Q_1	2^{64}
α	$2^{37.8430}$
γ	$2^{49.186}$
γ'	$2^{105.727}$
ν'	$2^{139.87}$
δ_0	1.901183
Bit-sec of underlying assumptions	590.27
Bit-sec of concrete construction	128

Table 16. Concrete Settings for the Parameters and the Related Security in the case of unforgeability with exact relation.

Below, we roughly explain about the calculations of the above table.

- According to the used NIZKPoK system Π_2 in Theorem D.5, we need to set

$$\gamma' = 2\sqrt{2(2\tau+6)N}\cdot\eta\kappa\cdot((\sqrt{3\tau}+\tau)\sqrt{2}\tau\theta\alpha N^2\delta+\alpha\sqrt{2\cdot(2\tau+6)N}),$$

such that the assumption M-SIS$_{q_2,1,2\tau+6,\gamma'}$ is hard, and a NIZKPoK system $\Pi^{(3)}$ exists for the relaxed language $L_{\gamma',q_2,\bar{\mathcal{C}}}$.

– Given the concrete value of N, we need to fix κ such that the size of the challenge set is larger than 2^λ, i.e., $\binom{N}{\kappa} \times 2^\kappa \geq 2^\lambda$.
– Given N, q_2, κ, we can calculate the values of α, γ, γ' (all these parameters need to be used in the description of our CTS in Construction D.3), according to the above parameter analysis for correctness and security.
– As a reasonable setting, we assume the upper bound of the number of queries that the adversary can make to be $Q_1 = 2^{64}$.
– We can further compute the values of ν' (all these parameters need to be used to ensure the security proof of our CTS in Construction D.3), as the requirement of security proof.
– In order to obtain much better tradeoff between efficiency and security, we first choose modulus q_2 such that both the hiding (based on $\mathsf{M\text{-}LWE}_{q_2,2,1}$) and the unforgeability (based on $\mathsf{M\text{-}SIS}_{q_2,1,\tau+7,\nu'}$) properties have the sufficient security level.

During the above calculation process, we use the Root-Hermite Factor δ_0 to estimate bit-hardness of the underlying assumptions, i.e., $\mathsf{M\text{-}SIS}$ and $\mathsf{M\text{-}LWE}$, according to the best known attacks, and δ_0 can be determined given N, q_1, q_2, α. Generally, we can use the work [3, 4, 30] to estimate δ_0 and its corresponding hardness of the assumptions.

Size Computation. Based on the above parameters on the adaptive CTS in Construction D.3 and multi-theorem straight-line extractable $\mathsf{NIZKPoK}$ listed in Table 15, the size of public parameter of CTS is about

$$|\mathsf{para}_{\mathsf{CTS}}| := 3 \cdot N\lceil \log q_1 \rceil + 5 \cdot N\lceil \log q_2 \rceil + \log(\alpha \cdot N \cdot q_1 \cdot q_2 \cdot \kappa \cdot \gamma \cdot \gamma') \text{ bits.}$$

The additional size of public parameter of multiple-theorem straight-line extractable $\mathsf{NIZKPoK}$ consists of $|\mathsf{para}_{\mathsf{ABDLOP}}|$ and $|\mathsf{para}_{\mathsf{PKE}}|$, where

$$|\mathsf{para}_{\mathsf{ABDLOP}}| := (n^* \cdot (m_1 + m_2 + v_e) + (\ell^* + 2) \cdot m_2) \cdot d\lceil \log(q) \rceil + 512 \cdot m_2 \cdot \lceil \log(q) \rceil,$$

$$|\mathsf{para}_{\mathsf{PKE}}| := (n_{\mathsf{PKE}} + k_{\mathsf{PKE}}) \cdot m_{\mathsf{PKE}} \cdot d \cdot \lceil \log(q_{\mathsf{PKE}}) \rceil \text{ bits.}$$

Besides, the size of public parameter for the disclosure of attributes is about

$$|\mathsf{para}_{\mathsf{Disclosure}}| := 4 \cdot \hat{k} \cdot N \cdot \lceil \log q_2 \rceil.$$

Thus, the total size of public parameter is about

$$|\mathsf{para}_{\mathsf{CTS}}| + |\mathsf{para}_{\mathsf{ABDLOP}}| + |\mathsf{para}_{\mathsf{PKE}}| + |\mathsf{para}_{\mathsf{Disclosure}}|.$$

Besides, the sizes of public key and secret key of CTS or the final Anonymous Credentials are about

$$|\mathsf{pk}_{\mathsf{CTS}}| := (\tau \cdot (\theta + 2) + 1) \cdot N\lceil \log q_2 \rceil \text{ bits and } |\mathsf{sk}_{\mathsf{CTS}}| := 3 \cdot \tau \cdot N\lceil \log 3 \rceil \text{ bits,}$$

respectively. Furthermore, the size of signature is about

$$|\mathsf{Sig}_{\mathsf{CTS}}| := (2\tau + 6) \cdot N \cdot \log(12\alpha) \text{ bits,}$$

which can be further optimized by using the Huffman coding as in [41] to get the signature size as

$$|\mathsf{Sig}_{\mathsf{CTS}}| := (2\tau + 6) \cdot N \cdot (2.57 + \lceil \log(\alpha) \rceil) \text{ bits},$$

Moreover, the pseudonym consists of four parts: BDLOP commitments, the related multi-theorem straight-line extractable NIZKPoK system, the disclosure of chosen attributes for attribute-based setting, and the additional overhead for the validness proof of message. Concretely, the size of commitment is about

$$|\mathsf{comm}_{\mathsf{CTS}}| := \tau \cdot \theta \cdot (N\lceil \log q_1 \rceil + N\lceil \log q_2 \rceil) \text{ bits}.$$

the size of proof is denoted by $\mathsf{size}_{\mathsf{LNP}}$, which is presented in Section 5. As θ committed polynomials might need to be disclosed coefficients with respect to different positions, according to Table 17 in Section 6.3, $\mathsf{size}_{\mathsf{Disclosure}}$ is about

$$\theta \cdot (2 \cdot \hat{k} \cdot N \cdot \lceil \log q_2 \rceil + \hat{k} \cdot \log q_2 + 4 \cdot N(2.57 + \lceil \log(\eta \cdot \kappa \sqrt{3 \cdot N}) \rceil) + \lambda).$$

According to Table 13, $\mathsf{size}_{\mathsf{validness}}$ is about

$$\theta \cdot 4N(2.57 + \lceil \log(\eta \cdot \kappa \cdot \sqrt{3N}) \rceil).$$

Thus, the total size of pseudonym $|\mathsf{pseudonym}|$ is about

$$|\mathsf{pseudonym}| := |\mathsf{comm}_{\mathsf{CTS}}| + \mathsf{size}_{\mathsf{LNP}} + \mathsf{size}_{\mathsf{Disclosure}} + \mathsf{size}_{\mathsf{validness}} \text{ bits}.$$

Finally, the credential size is about

$$|\mathsf{Cred}| := (2\tau + 6) \cdot N \cdot \log(12\eta \cdot \kappa \cdot \gamma) \text{ bits},$$

which can be optimized to get the credential size as

$$|\mathsf{Cred}| := (2\tau + 6) \cdot N \cdot (2.57 + \lceil \log(\eta \cdot \kappa \cdot \gamma) \rceil) \text{ bits},$$

E Supplementary Materials for Section 6

In this section, we present the interactive protocol to disclose certain coefficients of a committed polynomial, which can extend the basic anonymous credentials system to attribute-based on supporting *chosen disclosure of attributes*.

Interactive proof system $\Pi_{\mathsf{Disclosure}}$

Public Parameter for Commitment Scheme:

$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \boldsymbol{a}_2^\top \end{bmatrix}$ as in Construction 4.1, $B = \xi \cdot \sqrt{2k \cdot N}$, $\mathbf{B}^\top = (\boldsymbol{b}_i) \in R_{q_2}^{k \times \hat{k}}$

Commitment: $\mathsf{comm}_1 := \begin{bmatrix} \boldsymbol{t}_{1,1} \\ t_{2,1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \boldsymbol{a}_2^\top \end{bmatrix} \cdot \boldsymbol{r}_1 + \begin{bmatrix} 0 \\ m \end{bmatrix}$, with $m = \sum_{i \in [N]} m_i X^{i-1}$.

Let $\mathcal{I} \subseteq [N]$ denote the subset of indices where the prover wants to disclose $m_{\mathcal{I}} = \{m_i\}_{i \in \mathcal{I}}$. Set $m_{\mathsf{att}} = \sum_{i \in \mathcal{I}} m_i X^{i-1}$, $m' = \sum_{i \in [N] \setminus \mathcal{I}} m_i X^{i-1}$, where any coefficient of m' with respect to X^i is 0 for $i \in \mathcal{I}$. The prover discloses m_{att} publicly.

Prover		Verifier
$\boldsymbol{g} := (g_1, \ldots, g_{\hat{k}})^\top \xleftarrow{\$} \{f \in R_{q_2} : f_i = 0 \text{ for } i \in \mathcal{I}\}^{\hat{k}}$, $\boldsymbol{t}_g = (t_{g,i}) = \mathbf{B} \cdot \boldsymbol{r}_1 + \boldsymbol{g}$		
	$\xrightarrow{\boldsymbol{t}_g}$	
	$\xleftarrow{(\gamma_i)_{i \in [\hat{k}]}}$	$(\gamma_i) \xleftarrow{\$} \mathbb{Z}_{q_2}^{\hat{k}}$
$\forall i \in [\hat{k}], h_i = g_i + \gamma_i \cdot m'$ $\boldsymbol{y} \leftarrow \mathcal{D}_\xi^k$, $\boldsymbol{w} = \mathbf{A}_1 \cdot \boldsymbol{y}$ $\forall i \in [\hat{k}]$, $w_i = (\gamma_i \cdot \boldsymbol{a}_2^\top + \boldsymbol{b}_i^\top) \cdot \boldsymbol{y}$		
	$\xrightarrow{\boldsymbol{w}, h_i, w_i}$	
		$d \xleftarrow{\$} \mathcal{C}$
	$\xleftarrow{d}$	
$\boldsymbol{z} = \boldsymbol{y} + d \cdot \boldsymbol{r}_1$ $\mathsf{Rej}(\boldsymbol{z}, d \cdot \boldsymbol{r}_1, \xi)$		
	$\xrightarrow{\boldsymbol{z}}$	
	Check:	
	1. $\|\boldsymbol{z}\| \overset{?}{\leq} B$, $\mathbf{A}_1 \cdot \boldsymbol{z} \overset{?}{=} \boldsymbol{w} + d \cdot t_{1,1}$	
	2. $\forall i \in [\hat{k}]$, whether the coefficients with respect to $\mathcal{I}$ in h_i are zero, and $(\gamma_i \cdot \boldsymbol{a}_2^\top + \boldsymbol{b}_i^\top) \cdot \boldsymbol{z} \overset{?}{=} w_i + d \cdot (\gamma_i \cdot (t_{2,1} - m_{\mathsf{att}}) + t_{g,i} - h_i)$	
	Accept if all the above conditions are set up.	

Table 17. The interactive version of $\Pi_{\mathsf{Disclosure}}$: Disclosure of certain coefficients in the committed message m. Even the whole pseudonym is the commitment to a matrix $m \cdot \mathbf{G}$, for disclosure purpose, we just need to focus on certain vectors $\boldsymbol{a}_2, \boldsymbol{r}_1$, such that $t_{2,1} = \langle \boldsymbol{a}_2, \boldsymbol{r}_1 \rangle + m$.