Block Ciphers – Focus On The Linear Layer (feat. PRIDE)*

Full Version

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Abstract. The linear layer is a core component in any substitution-permutation network block cipher. Its design significantly influences both the security and the efficiency of the resulting block cipher. Surprisingly, not many general constructions are known that allow to choose trade-offs between security and efficiency. Especially, when compared to Sboxes, it seems that the linear layer is crucially understudied. In this paper, we propose a general methodology to construct good, sometimes optimal, linear layers allowing for a large variety of trade-offs. We give several instances of our construction and on top underline its value by presenting a new block cipher. PRIDE is optimized for 8-bit micro-controllers and significantly outperforms all academic solutions both in terms of code size and cycle count.
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1 Introduction

Block ciphers are one of the most prominently used cryptographic primitives and probably account for the largest portion of data encrypted today. This was facilitated by the introduction of Rijndael as the Advanced Encryption Standard (AES) [2], which was a major step forward in the field of block cipher design. Not only does AES offer strong security, but its structure also inspired many cipher designs ever since. One of the merits of AES (and its predecessor SQUARE [20]) was demonstrating that a well-chosen linear layer is not only crucial for the security (and efficiency) of a block cipher, but also allows to argue in a simple and thereby convincing way about its security.

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There are two main design strategies that can be identified for block ciphers: Sbox-based constructions and constructions without Sboxes, most prominently those using addition, rotation, and XORs (ARX designs). Furthermore, Sbox-based designs can be split into *Feistel-ciphers and substitution-permutation networks (SPN)*. Both concepts have been successfully used in practice, the most prominent example of an SPN cipher being AES and the most prominent Feistel-cipher being the former Data Encryption Standard (DES) [22].

It is also worth mentioning that the concept of SPN has not only been used in the design of block ciphers but also for designing cryptographic permutations, most prominently for the design of several sponge-based hash functions including SHA-3 [11]. In SP networks, the round function consists of a non-linear layer composed of small Sboxes working in parallel on small chunks of the state and a linear layer that mixes those chunks. Thus, designing an SPN block cipher essentially reduces to choosing one (or several) Sboxes and a linear layer.

A lot of research has been devoted to the study of Sboxes. All Sboxes of size up to 4 bits have been classified (indeed, more than once – cf. [14,38,48]). Moreover, Sboxes with optimal resistance against differential and linear attacks have been classified up to dimension 5 [17]. In general, several constructions are known for good and optimal Sboxes in arbitrary dimensions. Starting with the work of Nyberg [45], this has evolved into its own field of research in which those functions are studied in great detail. A nice survey of the main results of this line of study is provided by Carlet [18].

The situation for the other main design part, the linear layer, is less clear.

1.1 The Linear Layer

For the design of the linear layer, two general approaches can be identified. One still widely-used method is to design the linear layer in a rather ad-hoc fashion, without following general design guidelines. While this might lead to very secure and efficient algorithms (cf. Serpent [3] and SHA-3 as prominent examples), it is not very satisfactory from a scientific point-of-view. The second general design strategy is the wide-trail strategy introduced by Daemen in [19] (see also [21]). Especially for the security against linear [43] and differential [12] attacks, the wide-trail strategy usually results in simple and strong security arguments. It is therefore not surprising that this concept has found its way in many recent designs (e.g. Khazad [9], Anubis [8], Grøstl [27], PHOTON [31], LED [32], PRINCE [16], mCrypton [41] to name but a few). In a nutshell, the main idea of the wide-trail strategy is to link the number of active Sboxes for linear and differential cryptanalysis to the minimal distance of a certain linear code associated with the linear layer of the cipher. In turn, choosing a good code (with some additional constraints) results in a large number of active Sboxes.

While the wide-trail strategy does provide a powerful tool for arguing about the security of a cipher, it does not help in actually designing an efficient linear layer (or the corresponding linear code) with a suitable number of active Sboxes. Here, with the exception of early designs in [19] and later PRINCE and mCrypton, most ciphers following the wide-trail strategy simply choose an MDS matrix

as the core component. This might guarantee an (partially) optimal number of active Sboxes, but usually comes at the price of a less efficient implementation. The only exception here is that, in the case of MDS matrices, the authors of PHOTON and LED made the observation that implementing such matrices in a serialized fashion improves hardware-efficiency. This idea was further generalized in [49,56], and more recently in [5].

It is our belief that, in many cases, it is advantageous to use a near-MDS matrix (or in general a matrix with a sub-optimal branch number) for the overall design. Furthermore, it is, in our opinion, utmost surprising that there are virtually no general constructions or guidelines that would allow an SPN design to benefit from security vs. efficiency trade-offs. This is in particular important when it comes to ciphers where specific performance characteristics are crucial, e.g. in lightweight cryptography.

1.2 The Current State of Lightweight Cryptography

In recent years, the field of lightweight cryptography has attracted a lot of attention from the cryptographic community. In particular, designing lightweight block ciphers has been a very active field for several years now. The dominant metric according to which the vast majority of lightweight ciphers have been optimized was and still is the chip area. While this is certainly a valid optimization objective, its relevance to real-world applications is limited. Nowadays, there are several interesting and strong proposals available that feature a very small area but simultaneously neglect other, important real-world constraints. Moreover, recent proposals achieve the goal of a small chip area by sacrificing execution speed to such an extent that even in applications where speed is supposedly uncritical, the ciphers are getting too slow[1].

Note that software solutions, i.e. low-end embedded processors, actually dominate the world of embedded systems and dedicated hardware is a comparably small fraction. Considering this fact, it is quite puzzling that efficiency on low-cost processors was disregarded for so long. Certainly, there were a few exceptions: Several theoretical and practical studies have already been done in this field. Practical examples include several proposals for instruction set extensions [40,44,50,39]. Among these, the Intel AES instruction set [33] is the most well-known and practically relevant one. There have also been attempts to come up with ciphers that are (partially) tailored for low-cost processors [54,53,57,28,10,34]. Of these, execution times of both SEA and ITUbee are rather high, mostly due to the high number of rounds. Furthermore, ITUbee uses 8-bit Sboxes, which occupy a vast amount of program memory storage. SPECK, on the other hand, seems to be an excellent *lightweight software cipher* in terms of both speed and program memory.

It is obvious that there are quite some challenges to be overcome in this relatively untouched area of lightweight software cryptography. The software cipher for embedded devices of the future should not only be compact in terms

[1] See also [37] asking "Is lightweight = light + wait?"

of program memory, but also be relatively fast in execution time. It should clearly be secure and, preferably, its security should be easily analysed and verified. The latter can possibly be achieved by building on conservative structures, which are conventionally costly in software implementation, thereby posing even harder challenges.

One major component influencing all or at least most of those criteria outlined above is the linear layer. Thus, it is important to have general constructions for linear layers that allow to explore and make optimal use of the possible trade-offs.

1.3 Our Contribution

In this paper, we take steps towards a better understanding of possible trade-offs for linear layers. After introducing necessary concepts and notation in Section 2, we give a general construction that allows to combine several strong linear mappings on a few number of bits into a strong linear layer for a larger number of bits (cf. Section 3). From a coding theory perspective, this construction corresponds to a construction known as block-interleaving (see [42], pages 131-132). While this idea is rather simple, its applicability is powerful. Implicitly, a specific instance of our construction is already implemented in AES. Furthermore, special instances of this construction are recently used in [7] and [30].

We illustrate our approach by providing several linear layers with an optimal or almost optimal trade-off between hardware-efficiency and number of active Sboxes in Section 4. Along similar lines, we present a classification of all linear layers fulfilling the criteria of the block cipher PRINCE in Appendix C. Those examples show in particular that the construction given in Section 3 allows the construction of non-equivalent codes even when starting from equivalent ones. Secondly, we show that our construction also leads to very strong linear layers with respect to efficiency on embedded 8-bit micro-controllers. For this, we adopt a search strategy from [55] to find the *most efficient linear layer possible* within our constraints. We implemented this search on an FPGA platform to overcome the big computational effort involved and to have the advantage of reconfigurability. Details are described in Section 5.1.

With this, and as a second main contribution of our paper, we make use of our construction to design a new block cipher named PRIDE that significantly outperforms all existing block ciphers of similar key-sizes, with the exception of SIMON and SPECK [10]. One of the key-points here is that our construction of strong linear layers is nicely in line with a bit-sliced implementation of the Sbox layer. Our cipher is comparable, both in speed and memory size, to the new NSA block ciphers SIMON and SPECK, dedicated for the same platform. We conclude the paper in Section 6 with some open problems and pressing topics for further investigation. Finally, we note that while in this paper we focus on SPN ciphers, most of the results translate to the design of Feistel ciphers as well.

2 Notation and Preliminaries

In this section, we fix the basic notation and furthermore recall the ideas of the wide-trail strategy.

We deal with SPN block ciphers where the Sbox layer consist of n Sboxes of size b each. Thus the block size of the cipher is $n \times b$. The linear layer will be implemented by applying k binary matrices in parallel.

We denote by $\mathbb{F}_2$ the field with two elements and by $\mathbb{F}_2^n$ the n-dimensional vector space over $\mathbb{F}_2$. Note that any finite extension field $\mathbb{F}_{2^b}$ over $\mathbb{F}_2$ can be viewed as the vector space $\mathbb{F}_2^b$ of dimension b. Along these lines, the vector space $(\mathbb{F}_{2^b})^n$ can be viewed as the (nested) vector space $\left(\mathbb{F}_2^b\right)^n$.

Given a vector $x = (x_1, \ldots, x_n) \in \left(\mathbb{F}_2^b\right)^n$ where each $x_i \in \mathbb{F}_2^b$ we define its weight[2] as

$$\mathrm{wt}_b(x) = |\{1 \leq i \leq n \mid x_i \neq 0\}|.$$

Following [21], given a linear mapping $L : (\mathbb{F}_2^b)^n \to (\mathbb{F}_2^b)^n$ its differential branch number is defined as

$$\mathcal{B}_d(L) := \min\{\mathrm{wt}_b(x) + \mathrm{wt}_b(L(x)) \mid x \in \left(\mathbb{F}_2^b\right)^n \ , \ x \neq 0\}.$$

The cryptographic significance of the branch number is that the branch number corresponds to the minimal number of active Sboxes in any two consecutive rounds. Here an Sbox is called active if it gets a non-zero input difference in its input.

Given an upper bound p on the differential probability for a single Sbox along with a lower bound of active Sboxes immediately allows to deduce an upper bound for any differential characteristic[3] using

$$\text{average probability for any non-trivial characteristic } \leq p^{\#\text{active Sboxes}}.$$

For linear cryptanalysis, the linear branch number is defined as

$$\mathcal{B}_l(L) := \min\{\mathrm{wt}_b(x) + \mathrm{wt}_b(L^*(x)) \mid x \in \left(\mathbb{F}_2^b\right)^n, x \neq 0\}$$

where L^* is the adjoint linear mapping. That is, with respect to the standard inner product, L^* corresponds to the transposed matrix of L.

In terms of correlation (cf., for example, [19]), an upper bound c on the absolute value of the correlation for a single Sbox results in a bound for any linear trail (or linear characteristic, linear path) via

$$\text{absolute correlation for a trail } \leq c^{\#\text{active Sboxes}}.$$

The differential branch number corresponds to the minimal distance of the $\mathbb{F}_2$-linear code C over $\mathbb{F}_2^b$ with generator matrix

$$G = [I \mid L^T]$$

[2] Of course $\left(\mathbb{F}_2^b\right)^n$ is isomorphic to $\mathbb{F}_2^{nb}$, but the weight is defined differently on each.
[3] Averaging over all keys, assuming independent round keys.

where I is the $n \times n$ identity matrix. The length of the code is $2n$ and its dimension is n (here dimension corresponds to $\log_{2^b}(|C|)$ as it is not necessarily a linear code). Thus, C is a $(2n, 2^n)$ additive code over $\mathbb{F}_2^b$ with minimal distance $d = \mathcal{B}_d(L)$.

The linear branch number corresponds in the same way to the minimal distance of the $\mathbb{F}_2$-linear code $C^\perp$ with generator matrix

$$G^* = [L \mid I].$$

Note that $C^\perp$ is the dual code of C and in general the minimal distances of $C^\perp$ and C do not need to be identical.

Finally, given linear maps L_1 and L_2, we denote by $L_1 \times L_2$ the direct sum of the mappings, i.e.

$$(L_1 \times L_2)(x, y) := (L_1(x), L_2(y)).$$

3 The Interleaving Construction

Following the wide-trail strategy, we construct linear layers by constructing a $(2n, 2^n)$ additive codes with minimal distance d over $\mathbb{F}_2^b$. The code needs to have a generator matrix G in standard form, i.e.

$$G = [I \mid L^T]$$

where the submatrix L is invertible, and corresponds to the linear layer we are using.

Hence, the main question is how to construct "efficient" matrices L with a given branch number. Our construction allows to combine small matrices into bigger ones. We hereby drastically reduce the search-space of possible linear layers. This in turn makes it possible to construct efficient linear layers for various trade-offs, as demonstrated in the following sections.

As mentioned above, the construction described in [21] can be seen as a special case of our construction. The main difference (except the generalization) is that we shift the focus of the construction in [21] from the 4 round super-box view to a 2 round-view. While Daemen and Rijmen focused on the bounds for 4 rounds, we make use of their ideas to actually construct linear layers. Moreover, a particular instance of the general construction we elaborate on here, was already used in the linear layer of the hash function Whirlwind [7]. There, several small MDS matrices are used to construct a larger one.

We give a simple illustrative example of our approach in Appendix A.

3.1 The General Construction

We are now ready to give a formal description of our approach. First define the following isomorphism

$$\begin{aligned} P^n_{b_1,\dots b_k} : \left(\mathbb{F}_2^{b_1} \times \mathbb{F}_2^{b_2} \times \cdots \times \mathbb{F}_2^{b_k}\right)^n &\to \left(\mathbb{F}_2^{b_1}\right)^n \times \left(\mathbb{F}_2^{b_2}\right)^n \times \cdots \times \left(\mathbb{F}_2^{b_k}\right)^n \\ (x_1, \dots, x_n) &\mapsto \left(\left(x_1^{(1)}, \dots, x_n^{(1)}\right), \dots, \left(x_1^{(k)}, \dots, x_n^{(k)}\right)\right) \end{aligned}$$

$$\text{where } x_i = \left(x_i^{(1)}, \ldots, x_i^{(k)}\right) \text{ with } x_i^{(j)} \in \mathbb{F}_2^{b_j}.$$

This isomorphism performs the transformation of mapping Sbox outputs to our small linear layers L_i. For example, in Appendix A, we considered individual bits (i.e. $b_1, \ldots, b_k = 1$) from 4 (i.e., $k = 4$) 4-bit Sboxes (i.e $n = 4$).

Note that, for our purpose, there are in fact many possible choices for P. In particular, we may permute the entries within $(\mathbb{F}_2^{b_i})^n$. Given this isomorphism we can now state our main theorem. The construction of P follows the idea of a diffusion-optimal mapping as defined in [21, Definition 5].

Theorem 1. *Let $G_i = [I \mid L_i^T]$ be the generator matrix for an $\mathbb{F}_2$-linear $(2n, 2^n)$ code with minimal distance d_i over $\mathbb{F}_2^{b_i}$ for $0 \leq i < k$. Then the matrix $G = [I \mid L^T]$ with*

$$L = \left(P^n_{b_1,\ldots b_k}\right)^{-1} \circ (L_0 \times L_1 \times \cdots \times L_{k-1}) \circ P^n_{b_1,\ldots b_k}$$

is the generator matrix of an $\mathbb{F}_2$-linear $(2n, 2^n)$ code with minimal distance d over $\mathbb{F}_2^b$ where

$$d = \min_i d_i \quad \textit{and} \quad b = \sum_i b_i.$$

Proof. Since $P^n_{b_1,\ldots b_k}$ and $\left(P^n_{b_1,\ldots b_k}\right)^{-1}$ are permutation matrices, by construction L has full rank. To see that $\mathrm{wt}_b(w) + \mathrm{wt}_b(v) \geq \min_i d_i$ for any $v \in \mathbb{F}_2^b \setminus \{0\}$ and $w = L \cdot v$, observe that $\mathrm{wt}_b(w) + \mathrm{wt}_b(v)$ is minimal when all entries in v are zero except those mapped to the positions acted on by L_j where L_j is the matrix with the minimal branch number. □

Remark 1. The interleaving construction allows to construct non-equivalent codes even when starting with equivalent L_i's. This is shown in a particular case in Appendix C, where different choices of (equivalent) L_i's lead to different numbers of minimum-weight codewords.

A special case of the construction above is implicitly already used in AES. In the case of AES, it is used to construct a $[8, 4, 5]$ code over $\mathbb{F}_2^{32}$ from 4 copies of the $[8, 4, 5]$ code over $\mathbb{F}_2^8$ given by the *MixColumn* operation. In the Superbox view on AES, the *ShiftRows* operation plays the role of the mapping P (and its inverse) and *MixColumns* corresponds to the mappings L_i.[4]

In the following, we use this construction to design efficient linear layers. Besides the differential and linear branch number, we hereby focus mainly on three criteria:

- Maximize the diffusion (cf. Section 3.3)
- Minimize the density of the matrix (cf. Section 4)

[4] Note that the cipher PRINCE implicitly uses the construction twice. Once for generating the matrix M as in Appendix A and second for the improved bound on 4 rounds, just like in AES.

– Software-efficiency (cf. Section 5)

The strategy we employ is as follows. We first find candidates for L_0, i.e., $(2n, 2^n)$ additive codes with minimal distance d_0 over $\mathbb{F}_{2^{b_0}}$. In this stage, we ensure that the branch number is d_0 and our efficiency constraints are satisfied. We then apply permutations to L_0 to produce L_i for $i > 0$. This stage maximizes diffusion.

3.2 Searching for L_0

The following lemma (which is a rather straightforward generalization of Theorem 4 in [56]) gives a necessary and sufficient condition that a given matrix L has branch number d over $\mathbb{F}_2^b$.

Lemma 1. *Let L be a $bn \times bn$ binary matrix, decomposed into $b \times b$ submatrices $L_{i,j}$.*

$$L = \begin{pmatrix} L_{0,0} & L_{0,1} & \dots & L_{0,n-1} \\ L_{1,0} & L_{1,1} & \dots & L_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1,0} & L_{n-1,1} & \dots & L_{n-1,n-1} \end{pmatrix} \tag{1}$$

Then, L has differential branch number d over $\mathbb{F}_2^b$ if and only if all $i \times (n-d+i+1)$ block submatrices of L have full rank for $1 \leq i < d-1$. Moreover, L has linear branch number d if and only if all $(n-d+i+1) \times i$ block submatrices of L have full rank for $1 \leq i < d-1$.

Based on Lemma 1 we may instantiate various search algorithms which we will describe in Section 4 and Section 5. In our search we focus on cyclic matrices, i.e. matrices where row $i > 0$ is constructed by cyclic shifting row 0 by i indices. These matrices have the advantage of being efficient both in software and hardware. Furthermore, since these matrices are symmetric, considering the dual code $C^\perp$ to $C = [I \mid L^T]$ is straightforward.

3.3 Ensuring High Dependency

In this section, we assume we are given a matrix L_0 and wish to construct $L_1, \dots, L_{k-1}$ that maximize the diffusion of the map $L = \left(P^n_{b_1,\dots b_k}\right)^{-1} \circ (L_0 \times L_1 \times \dots \times L_{k-1}) \circ P^n_{b_1,\dots b_k}$.

Given an $bn \times bn$ binary matrix L decomposed as in Eq. (1), we define its support as the $n \times n$ binary matrix $\mathrm{Supp}(L)$ where

$$\mathrm{Supp}(L)_{i,j} = \begin{cases} 1 & \text{if } L_{i,j} \neq 0 \\ 0 & \text{else} \end{cases}$$

Now assume that $\mathrm{Supp}(L_0)$ has a zero entry at index i', j'. If we apply the same L_i in all k positions this means that the outputs from the i'th Sbox have no

impact on the inputs of the j'th Sbox after the linear layer. In other words, a linear-layer following the construction of Theorem 1 ensure full dependency if and only if

$$\left(\bigvee_{0 \leq i < k} \mathrm{Supp}(L_i) \right)_{i',j'} = 1 \quad \forall\ 0 \leq i', j' < n.$$

Hence, we want to apply different matrices L_i in each of the k positions, such that in at least one $\mathrm{Supp}(L_i)$ has a non-zero entry at index i', j' for all $0 \leq i', j' < n$. In order to construct matrices L_i for $i > 0$ from a matrix L_0 we may apply block-permutation matrices from the left and right to L_0 as these clearly neither impact the density nor the branch number. Hence, we focus on finding permutation matrices P_i, Q_i such that the density of $\bigvee_{0 \leq i < b} \mathrm{Supp}(P_i \cdot L_0 \cdot Q_i)$ is maximized. In Appendix F, we give two strategies for finding such P_i,Q_i, one is heuristic but computationally cheap, the other is guaranteed to return an optimal solution – based on Constraint Integer Programming – but can be computationally intensive.

We note that the difficulty of the problem depends on the size of the Sbox and the density of L_i. As MDS matrices always have density 1, the problem of full dependency does not occur when combining such matrices. Finally, if the construction ensures full dependency for a given k, it is always possible to achieve full dependency for any $k' \geq k$.

In contrast with the branch number, if a linear layer ensures high dependency, its inverse does not necessarily achieve the same dependency. Thus, it is in general necessary to check the dependency of the inverse separately.

4 Optimizing for Hardware

In this section, we give examples of $[2n, n, d]$ codes over $\mathbb{F}_2^b$ and give algorithms for finding such instances. First, the following lemma gives a lower bound on the density of a matrix with branch number d. Our aim here is to find linear layers that are efficiently implementable in hardware. More precisely, we aim for an implementation in one cycle. PHOTON and LED demonstrated that there is a trade-off between clock cycles and number of gate equivalence for the linear layer. The trade-off we consider here is, complementary to PHOTON and LED, between efficient implementation in one clock cycle and the (linear and differential) branch number. Note that in our setting, the cost of implementation is directly connected to the number of ones in the matrix representation of the linear layer.

Lemma 2. *Let matrix $G = [I \mid L^T]$ be the generator matrix for an $\mathbb{F}_2$-linear $(2n, 2^n)$ code with minimal distance d such that the dual code has minimum distance d as well. Then L has at least $d-1$ ones per row and per column.*

Proof. Computing $w = L \cdot v$ where v is a vector with one non-zero entry 1, we have that w must be a vector with $d-1$ non-zero entries if the minimum distance

of $[I \mid L^T]$ is d. Hence, there must be at least $d-1$ ones per row. Applying the same argument to $w = L^T \cdot v = v \cdot L$ shows that at least $d-1$ entries per column must be non-zero. □

The main merit of the above lemma is that it allows to determine the optimal solutions in terms of efficiency. This is in contrast to the case for software implementation, where the optimal solution is unknown.

Lemmas 1 and 2 give rise to various search strategies for finding $(2n, 2^n)$ additive codes with minimal distance d over $\mathbb{F}_2^b$. We discuss those strategies in Appendix B and present results of those strategies next.

4.1 Hardware-Optimal Examples

Below we give some examples for our construction. We hereby focus on $[2n, n, d]$ codes over $\mathbb{F}_2$, i.e. we use $b_i = 1$.[5] Note that this naturally limits the achievable branch number. For binary linear codes the optimal minimal distance is known for small length (cf. [29] for more information). We give a small abridgement of the known bounds on the minimal distance for linear $[2n, n]$ codes over $\mathbb{F}_2, \mathbb{F}_4$, and $\mathbb{F}_8$ in Appendix E. As can be seen in this table, in order to achieve a high branch number, it might be necessary to consider linear codes over $\mathbb{F}_{2^m}$, or (more general) additive codes over $\mathbb{F}_2^m$ for some small $m > 1$.

The examples in Figure 1 are optimal in the sense that they achieve the best possible branch number (both linear and differential) for the given length (with the exception of $n = 11, 13$, and 14) with the least possible number of ones in the matrix (cf. Lemma 2). The number D corresponds to the average number of ones per row/column and D_{inv} to the average number of ones per row/column of the inverse matrix. The only candidate which does not satisfy $D = d - 1$ is $n = 8$. This candidate was found using the approach from Appendix B.3, which guarantees to return the optimal solution. Hence, we conclude that $4\frac{1}{8}$ is indeed the lowest density possible. That is, there is no 8×8 binary matrix with branch number 5 with only 32 ones, but the best we can do is 33 ones.

For each example we list the dimension (i.e the number of Sboxes), the achieved branch number and the minimal k such that it is possible to achieve full dependency with two Sbox layers interleaved with one linear layer. These values were found using the CIP approach in Section 3.3. Note that in this case (i.e. $b_i = 1$) the value k actually corresponds to the minimal Sbox size that allows full dependency. Finally, k_{inv} is the minimum Sbox size to achieve full diffusion for the inverse matrix. Note that for all these examples, the corresponding code is actually equivalent to its dual. In particular this implies that the linear and differential branch number are equal.

[5] We refer to Appendix C for an exemplary comparison of the set of linear layers constructed by Theorem 1 and the entire space with the same criteria for $[8, 4, 4]$ codes over $\mathbb{F}_2^4$.

n	max(d)	d	D	D_{inv}	k	k_{inv}	Technique	Matrix
2	2	2	1	1	2	2	App. B.1	cyclic shift (10) to the left.
3	2	2	1	1	3	3	App. B.1	cyclic shift (100) to the left.
4	4	4	3	3	2	2	App. B.1	cyclic shift (1110) to the left.
5	4	4	3	3	2	2	App. B.1	cyclic shift (11100) to the left.
6	4	4	3	3	2	2	App. B.1	cyclic shift (110100) to the left.
7	4	4	3	$4\frac{3}{7}$	3	2	App. B.3	in Figure 2
8	5	5	$4\frac{1}{8}$	$4\frac{7}{8}$	3	2	App. B.3	in Appendix F.3
9	6	6	5	$5\frac{6}{9}$	2	2	App. B.3	in Figure 2
10	6	6	5	5	2	2	App. B.1	cyclic shift (1111010000) to the left.
11	7	6	5	5	3	3	App. B.1	cyclic shift (11110100000) to the left.
12	8	8	7	7	2	2	App. B.1	cyclic shift (110111101000) to the left.
13	7	6	5	5	≤ 4	≤ 4	App. B.1	cyclic shift (1110110000000) to the left.
14	8	6	5	5	≤ 4	≤ 4	App. B.1	cyclic shift (11101010000000) to the left.
15	8	8	7	7	3	3	App. B.1	cyclic shift (101101110010000) to the left.
16	8	8	7	7	3	3	App. B.1	cyclic shift (1111011010000000) to the left.

Fig. 1. Examples of hardware efficient linear layers over $\mathbb{F}_2$

$$\begin{pmatrix} 0110001 \\ 1000011 \\ 0100011 \\ 0001110 \\ 1001100 \\ 0110100 \\ 1011000 \end{pmatrix} \quad \begin{pmatrix} 001110110 \\ 100101110 \\ 010010111 \\ 111101000 \\ 100110101 \\ 001111001 \\ 111010010 \\ 011001011 \\ 110001101 \end{pmatrix}$$

Fig. 2. Examples of $[14, 7, 4]$ and $[18, 9, 6]$ codes over $\mathbb{F}_2$.

5 Software-Friendly Examples and the Cipher PRIDE

In this section, we describe our new lightweight software-cipher PRIDE, a 64-bit block cipher that uses a 128-bit key. We refer to Appendix D for a sketch of the security analysis and to the full version for more details.

We chose to design an SPN block cipher because it seems that this structure is better understood than e.g. ARX designs. We are, unsurprisingly, making use of the construction given in Theorem 1. We here decided on a linear layer with high dependency and a linear & differential branch number of 4. One key-observation is that the construction of Theorem 1 fits naturally with a bit-sliced implementation of the cipher, in particular with the Sbox layer. As a bit-sliced implementation of the Sbox layer is advantageous on 8-bit micro-controllers, in any case this is a nice match.

The target platform of PRIDE is Atmel's AVR micro-controller [4], as it is dominating the market along with PIC [46] (see [47]). Furthermore, many implementations in literature are also implemented in AVR, we therefore opt for this platform to provide a better comparison to other ciphers (including SIMON and SPECK [10]). However, the reconfigurable nature of our search architecture (cf. Section 5.1) to find the basic layers of the cipher allows us to extend the search to various platforms in the future.

5.1 The Search for The Linear Layer

A natural choice in terms of Theorem 1 is to choose $k = 4$ and $b_1 = b_2 = b_3 = b_4 = 1$. Thus, the task reduces to find four 16×16 matrices forming one 64×64 matrix (to permute the whole state) of the following form:

$$\begin{pmatrix} L_0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 \\ 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & L_3 \end{pmatrix}$$

Each of these four 16×16 matrices should provide branch number 4 and together achieve high dependency with the least possible number of instructions. Instead of searching for an efficient implementation for a given matrix, we decided to search for the most efficient solution fulfilling our criteria.

To find such matrices (L_i) that could be implemented very efficiently given the AVR instruction set, we performed an extensive and hardware-aided tree search. Our search engine was optimized to look for AVR assembly code segments utilizing a limited set of instructions that would result in linear behaviour at matrix level. These are namely CLC, EOR, MOV, MOVW, CLR, SWAP, ASR, ROR, ROL, LSR, and LSL instructions. As we are looking for 16×16 matrices, the state to be multiplied with each L_i is stored in two 8-bit registers, which we call X and Y. We also allowed utilization of four temporary registers, namely $T0$, $T1$, $T2$, and $T3$. We designed and optimized our search engine according to these registers. Our search engine checks the resulting matrix L_i after N instructions to see if it provides the desired characteristics. While trying to reach instruction N, we try all possible instruction-register combinations in each step. This of course comes with an impractical time complexity, especially when N is increased further. To deal with this time complexity, we came up with several optimizations. As a first step, we limited the utilization of certain instruction-register combinations. For example, we excluded CLC and CLR instructions from the combinations for the first and last instructions. Also, EOR is not considered in the first instruction. Again, for the first and last instructions, SWAP, ASR, ROR, ROL, LSR, and LSL instructions are only used with X and Y. Furthermore, we did not allow temporary registers as the destination while trying MOV and MOVW instructions in the last instruction and $X - Y$ registers as the destination while trying MOV and MOVW instructions in the first instruction.

However, such optimizations were not enough to reduce the time complexity. We therefore applied further optimizations, i.e., when the matrices of all registers do not give full rank, we stop the search as we know that we cannot find an invertible linear layer any more.

In the end, we found matrices that fulfil all of our criteria starting from 7 instructions.

We implemented our search architecture on a Xilinx ML605 (Virtex-6 FPGA) evaluation board. The reconfigurable nature of the FPGA allowed us to change easily between different parameters, i.e. the number of instructions. The details of this search engine can be found in [35].

5.2 An Extremely Efficient Linear Layer

As a result of the search explained in Section 5.1, we achieved an extremely efficient linear layer. The cheapest solution provided by our search needed 36 cycles for the complete linear layer, which is what we opted for. The optimal matrices forming the linear layer are given in the Appendix G. Of these four matrices, L_0 and L_3 are involutions with the cost of 7 instructions (in turn, clock cycles), while L_1 and L_2 require 11 and 13 instructions for true and inverse matrices, respectively. The assembly codes are given in Appendix H to show the claimed number of instructions.

Comparing to linear layers of other SPN-based ciphers clearly demonstrated the benefit of our approach. Note however, that these comparisons have to be taken with care as not all linear layers operate on the same state size and do not offer the same security level. The linear layer of the ISO-standard lightweight cipher PRESENT [15] costs 144 cycles (derived from the total cycle count given in [25]). MixColumns operation of NIST-standard AES[6] costs 117 instructions (but 149 cycles because of 3-cycle data load instruction utilizations, as MixColumns constants are implemented as look-up table – which means additional 256 bytes of memory, too) [6]. Note that ShiftRows operation was merged with the look-up table of Sbox in this implementation, so we take only MixColumns cost as the linear layer cost. The linear layer of another ISO-standard lightweight cipher CLEFIA [51] (again 128-bit cipher) costs 146 instructions and 668 cycles. Bit-sliced oriented design Serpent (AES finalist, 128-bit cipher) linear layer costs 155 instructions and 158 cycles. Other lightweight proposals, KLEIN [28] and mCrypton linear layers cost 104 instructions (100 cycles) and 116 instructions (342 cycles), respectively [24]. Finally, the linear layer cost of PRINCE is 357 instructions and 524 cycles[7], which is even worse than AES. One of the reasons for this high cost is the non-cyclic 4×4 matrices forming the linear layer. The other reason is the ShiftRows operation applied on 4-bit state words, which makes coding much more complex than that of AES on an 8-bit micro-controller.

[6] It is of course not fair to compare a 128-bit cipher with a 64-bit cipher. However, we provide AES numbers as a reference due to the fact that it is a widely-used standard cipher and its cost is much better compared to many lightweight ciphers.

[7] We implemented this cipher on AVR, as we could not find any AVR implementations in the literature.

5.3 Sbox Selection

For our bit-sliced design, we decided to use a very simple (in terms of software-efficiency – the formulation is given in Appendix I) 10-instruction Sbox (which makes $10 \times 2 = 20$ clock cycles in total for the whole state). It is at the same time an involution Sbox, which prevents the encryption/decryption overhead. Besides being very efficient in terms of cycle count, this Sbox is also optimal with respect to linear and differential attacks. The maximal probability of a differential is 1/4 and the best correlation of any linear approximation is 1/2. The PRIDE Sbox is given below.

x	0x0	0x1	0x2	0x3	0x4	0x5	0x6	0x7	0x8	0x9	0xa	0xb	0xc	0xd	0xe	0xf
$\mathcal{S}(x)$	0x0	0x4	0x8	0xf	0x1	0x5	0xe	0x9	0x2	0x7	0xa	0xc	0xb	0xd	0x6	0x3

The assembly codes are given in Appendix H to show the claimed number of instructions.

5.4 Description of PRIDE

Similar to PRINCE, the cipher makes use of the FX construction [36,13]. A pre-whitening key k_0 and post-whitening key k_2 are derived from one half of k, while the second half serves as basis k_1 for the round keys, i.e.,

$$k = k_0||k_1 \quad \text{with} \quad k_2 = k_0.$$

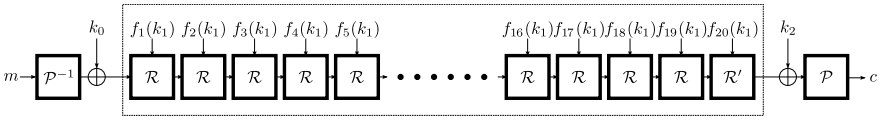
Moreover, in order to allow an efficient bit-sliced implementation, the cipher starts and ends with a bit-permutation. This clearly does not influence the security of PRIDE in any way. Note that in a bit-sliced implementation, none of the permutations P nor P^{-1} used in PRIDE has to be actually implemented explicitly. The cipher has 20 rounds, of which the first 19 are identical. Subkeys are different for each round, i.e., the subkey for round i is given by $f_i(k_1)$. We define

$$f_i(k_1) = k_{1_0}||g_i^{(0)}(k_{1_1})||k_{1_2}||g_i^{(1)}(k_{1_3})||k_{1_4}||g_i^{(2)}(k_{1_5})||k_{1_6}||g_i^{(3)}(k_{1_7})$$

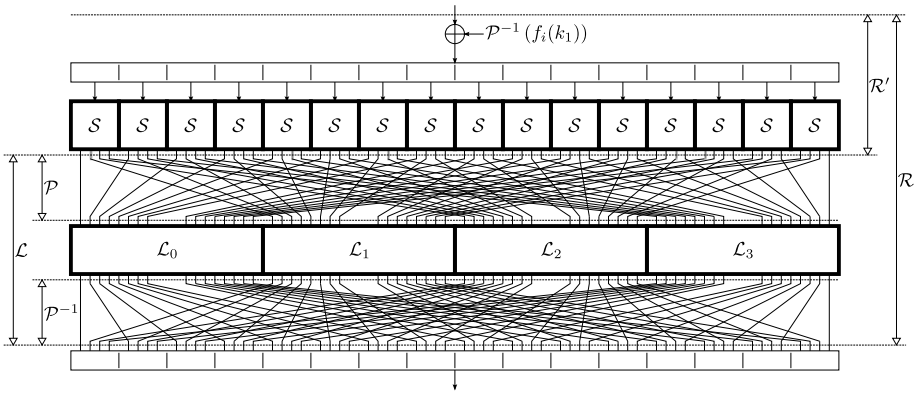
as the subkey derivation function with four byte-local modifiers of the key as

$$g_i^{(0)}(x) = (x + 193i) \bmod 256, \quad g_i^{(1)}(x) = (x + 165i) \bmod 256,$$
$$g_i^{(2)}(x) = (x + 81i) \bmod 256, \quad g_i^{(3)}(x) = (x + 197i) \bmod 256,$$

which simply add one of four constants to every other byte of k_1. The overall structure of the cipher is depicted here:



The round function $\mathcal{R}$ of the cipher shows a classical substitution-permutation network: The state is XORed with the round key, fed into 16 parallel 4-bit Sboxes and then permuted and processed by the linear layer.



The difference between $\mathcal{R}$ and $\mathcal{R}'$ is that in the latter no more diffusion is necessary, therefore the last round ends after the substitution layer. With the software-friendly matrices we have found as described above, the linear layer is defined as follows (cf. Theorem 1 and Appendix G):

$$L := P^{-1} \circ (L_0 \times L_1 \times L_2 \times L_3) \circ P \quad \text{where} \quad P := P^{16}_{1,1,1,1}.$$

The test vectors for the cipher are provided in the Appendix J.

5.5 Performance Analysis

As depicted above, one round of our proposed cipher PRIDE consists of a linear layer, a substitution layer, a key addition, and a round constant addition (key update). In a software implementation of PRIDE on a micro-controller, we also perform branching in each round of the cipher in addition to the previously listed layers. Adding up all these costs gives us the total implementation cost for one round of the cipher. The total cost can roughly be calculated by multiplying the number of rounds with the cost of each round. Note that we should subtract the cost of one linear layer from the overall cost, as PRIDE has no linear layer in the last round. The software implementation cost of the round function of PRIDE on Atmel AVR ATmega8 8-bit micro-controller [4] is presented in the following:

	Key update	Key addition	Sbox Layer	Linear Layer	Total
Time (cycles)	4	8	20	36	**68**
Size (bytes)	8	16	40	72	**136**

Comparing PRIDE to existing ciphers in literature, we can see that it outperforms many of them significantly both in terms of cycle count and code size. Note that we are not using any look-up tables in our implementation, in turn no RAMs[8]. The comparison with existing implementations is given below:

	AES-128 [25]	SERPENT-128 [25]	PRESENT-128 [25]	CLEFIA-128 [25]	SEA-96 [53]	NOEKEON-128 [24]	PRINCE-128	ITUbee-80 [34]	SIMON-64/128 [10]	SPECK-64/96 [10]	SPECK-64/128 [10]	PRIDE
t(cyc)	3159	49314	10792	28648	17745	23517	3614	2607	2000	1152	1200	**1514**
bytes	1570	7220	660	3046	386	364	1108	716	282	182	186	**266**
eq.r.	5/10	1/32	4/31	1/18	8/92	1/16	5/12	12/20	33/44	34/26	34/27	

In the table, the first row is the time (performance) in clock cycles, the second row is the code size in bytes, and the third row is the *equivalent rounds*. The third row expresses the number of rounds for the given ciphers that would result in a total running time similar to PRIDE.

Note that, as we did not come across to any reference implementations in the literature, we implemented PRINCE in AVR for comparison. We also do not list the RAM utilization for the ciphers under comparison in the table.

In the implementation of PRIDE, our target was to be fast and at the same time compact. Note that we do not exclude data & key read and data write back as well as the whitening steps in our results (these are omitted in SIMON and SPECK numbers). Although the given numbers are just for encryption, decryption overhead is also acceptable: It costs 1570 clock cycles and 282 bytes.

A cautionary note is indicated for the above comparison for several reasons. AES, SERPENT, CLEFIA, and NOEKOEN are working on 128-bit blocks; so, for a cycle per byte comparison, their cycle count has to be divided by a factor of two. Moreover, the ciphers differ in the claimed security level and key-size. PRIDE does not claim any resistance against related-key attacks (and actually can be distinguished trivially in this setting) and also generic time-memory trade-offs are possible against PRIDE in contrast to most other ciphers. Besides those restrictions, the security margin in PRIDE in terms of the number of rounds is (in our belief) sufficient.

One can see that PRIDE is comparable to SPECK-64/96 and SPECK-64/128 (members of NSA's *software*-cipher family), which are based on a Feistel structure and use modular additions as the main source of non-linearity.

In addition to the above table, the recent work of Grosso et al. [30] presents LS-Designs. This is a family of block ciphers that can systematically take ad-

[8] Which has the additional advantage of increased resistance against cache-timing attacks.

vantage of bit-slicing in a principled manner. In this paper, the authors make use of look-up tables. Therefore, a direct comparison with PRIDE is not fair as the use of look-up tables does not minimize the linear layer cost. However, to have an idea, we can try to estimate the cost of the 64-bit case of this family. They suggest two options: The first uses 4-bit Sbox with 16-bit Lbox, and the second uses 8-bit Sbox with 8-bit Lbox. The first option has 8 rounds, which results in 64 non-linear operations, 128 XORs, and 128 table look-ups in total. The second one has 6 rounds, which takes 72 non-linear operations, 144 XORs, and 48 table look-ups. For linear layer cost, we consider the XOR cost together with table look-ups. Unfortunately, it is not easy to estimate the overall cost of the given two options on AVR platform as the table look-ups take more than one cycle compared to the non-linear and linear operations. Another important point here to mention is that the use of look-up tables result in a huge memory utilization.

Finally, we note that, despite its target being software implementations, PRIDE is also efficient in hardware. It can be considered a hardware-friendly design, due to its cheap linear and Sbox layers.

6 Conclusion

In this work, we have presented a framework for constructing linear layers for block ciphers which allows to trade security against efficiency. For a given security level, in our case we focused on the branch number, we demonstrated techniques to find very efficient linear layers satisfying this security level. Using this framework, we presented a family of linear layers that are efficient in hardware. Furthermore, we presented a new cipher PRIDE dedicated for 8-bit micro-controllers that offers competitive performance due to our new techniques for finding linear layers.

One important question is on the optimality of a given construction for a linear layer. In particular, in the case of our construction, the natural question is if the reduction of the search space excludes optimal solutions and only sub-optimal solutions remain. For the hardware-friendly examples presented in Section 4 and Appendix C, it is easy to argue that those constructions are optimal. Thus, in this case the reduction of the search space clearly did not have a negative influence on the results. In general, and for the linear layer constructed in Section 5 in particular, the situation is less clear. The main reason is that, again, the construction of linear layers is understudied and hence we do not have enough prior work to answer this question satisfactorily at the moment. Instead we view the PRIDE linear layer as a strong benchmark for efficient linear layers with the given parameters and encourage researchers to try to beat its performance.

Along these lines, we see this work as a step towards a more rigorous design process for linear layers. Our hope is that this framework will be extended in future. In particular, we would like to mention the following topic for further investigations. It seems that using an Sbox with a non-trivial branch number

has the potential to significantly increase the number of active Sboxes when combined with a linear layer based on Theorem 1. Finding ways to easily prove such a result is worth investigating.

Finally, regarding PRIDE, we obviously encourage further cryptanalysis.

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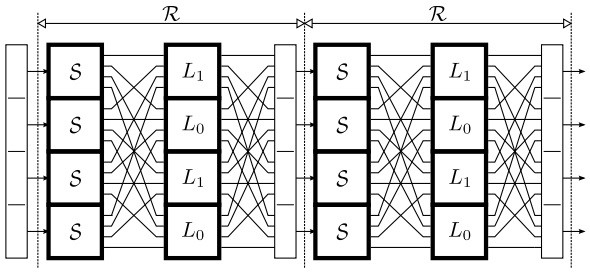
Appendices

A An Example for the Interleaving Construction

Our example takes its cue from the cipher PRINCE. Assume we want to construct a linear layer L working on 4 chunks of 4 bits with linear- and differential branch number 4. That is, we want to construct an $(8, 2^4)$ additive code with minimal distance 4 over $\mathbb{F}_2^4$ such that the dual code has minimum distance 4 as well. As a further requirement in this example, we want to focus on the hardware-efficiency, i.e. we would like to reduce the number of ones in the corresponding matrix to a minimum. Lastly, we also would like to ensure good diffusion. More precisely, after two Sbox layers interleaved with one linear layer we require that each bit of the output depends on each bit of the input.

It is not hard to see that (as a matrix) L needs to have at least 3 ones in each row and column (cf. Lemma 2 in Section 4). We thus face the problem of finding an invertible 16×16 binary matrix with branch number 4 and exactly 3 ones in each row and column[9]. As there are 2^{256} 16×16 matrices, the search space is a priori huge.

The basic idea of our construction (depicted below) is simply to first re-group the output bits of the Sbox layer.



We collect all first output bits of each Sbox, all second bits, all third bits, and all fourth bits. Next, we apply independently 4 linear mappings on 4 bits, i.e. we multiply each 4-bit chunk with a 4×4 binary matrix. Afterwards the bits are again re-grouped, and the process is repeated.

The key point (cf. Theorem 1 for the general statement) is that the linear (resp. differential) branch number of the entire linear layer (using wt_4) equals the minimal linear (resp. differential) branch number of the 4 small binary matrices (using wt_1). Moreover, the number of ones in each row and column in the entire linear layer is the same as in the small matrices. Thus an optimal solution for the small binary matrices extends to an optimal solution for the entire linear layer.

This simple observation allows us to focus on 4×4 binary matrices instead of 16×16 matrices. As there are only 2^{16} such matrices (and clearly only 4! of them

[9] As a side-note, it is easy to see that no 4×4 matrix over $\mathbb{F}_{16}$ fulfills our requirements one the number of ones per row and column.

have branch number 4), investigating all of them is easily possible. Two examples of such binary matrices fulfilling both the branch number and the requirement on the number of ones are

$$L_0 = \begin{pmatrix} 1\,1\,1\,0 \\ 0\,1\,1\,1 \\ 1\,0\,1\,1 \\ 1\,1\,0\,1 \end{pmatrix} \text{ and } L_1 = \begin{pmatrix} 0\,1\,1\,1 \\ 1\,0\,1\,1 \\ 1\,1\,0\,1 \\ 1\,1\,1\,0 \end{pmatrix}.$$

Using not only one but different matrices (L_0 and L_1 twice each in this example) furthermore allows us to achieve our second requirement, namely maximal diffusion (cf. Section 3.3 for the general setup).

Finally, it can be seen already in this small example that the described regrouping of bits goes naturally nice together with a bit-sliced implementation of the Sbox layer. This is an observation we heavily make use of in Section 5.

B Optimizing for Hardware

As mentioned above, Lemmas 1 and 2 give rise to various search strategies for finding $(2n, 2^n)$ additive codes with minimal distance d over $\mathbb{F}_{2^b}^n$ that we describe in the following.

B.1 Exhaustive Search on a Subspace

A first approach is to, again, consider circulant matrices, i.e., matrices where row $i > 0$ is constructed by cyclic shifting row 0. If z is the number of ones per row/column, we consider all possible $\binom{n}{z}^b$ choices of $b \times b$ matrices $L_{0,0}, \ldots, L_{0,n-1}$ over $\mathbb{F}_2$ and consider

$$L = \begin{pmatrix} L_{0,0} & L_{0,1} & \ldots & L_{0,n-1} \\ L_{0,1} & L_{0,2} & \ldots & L_{0,0} \\ \vdots & \vdots & \ddots & \vdots \\ L_{0,n-1} & L_{0,0} & \ldots & L_{0,n-2} \end{pmatrix},$$

and test whether it satisfies the conditions of Lemma 1.

B.2 Complete Exhaustive Search

We may expand the search space by considering not only circulant matrices but arbitrary matrices, i.e., we consider

$$L = \begin{pmatrix} L_{0,0} & L_{0,1} & \ldots & L_{0,n-1} \\ L_{1,0} & L_{1,1} & \ldots & L_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1,0} & L_{n-1,1} & \ldots & L_{n-1,n-1} \end{pmatrix}.$$

and check whether the conditions of Lemma 1 are satisfied. Indeed, the search space can be reduced by prunning search trees. This is because of the requirement that many small submatrices (such as $1 \times (n-d)$) must have full rank, ruling out many candidates and allowing search trees based on them to be cut.

B.3 System Solving Approaches

Instead of exhaustively searching over (all) possible matrices M to construct $(2n, 2^n)$ additive codes with minimal distance d over $\mathbb{F}_2^b$ given by the generator matrix $G = [I \mid L^T]$ where L has at most z ones per column and row, we may express the constraints on L as multivariate polynomials or integer constraints and use off-the-shelf solvers to find matrices satisfying them.

Polynomial System Solving We consider a matrix

$$L = \begin{pmatrix} L_{0,0} & L_{0,1} & \dots & L_{0,n-1} \\ L_{1,0} & L_{1,1} & \dots & L_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1,0} & L_{n-1,1} & \dots & L_{n-1,n-1} \end{pmatrix} \text{ with } L_{i,j} = \begin{pmatrix} \ell_{i,j,0,0} & \dots & \ell_{i,j,0,b-1} \\ \ell_{i,j,1,0} & \dots & \ell_{i,j,1,b-1} \\ \vdots & \ddots & \vdots \\ \ell_{i,j,b-1,0} & \dots & \ell_{i,j,b-1,b-1} \end{pmatrix}$$

where $\ell_{i,j,i',j'}$ for $0 \leq i, j < n$ and $0 \leq i', j' < b$ are variables over $\mathbb{F}_2$. We construct an equation system in the variables $\ell_{i,j,i',j'}$ with equations to enforce the following three conditions:

1. By Lemma 1 we require that all $i \times (n-d+i+1)$ block submatrices of L have full rank for $1 \leq i < d-1$. This is equivalent to requiring that at least one of the $\min(i, n-d+i+1) \times \min(i, n-d+i+1)$ minors must have determinant 1. Hence, if $t_0, \dots, t_s$ represent the determinants of all $\min(i, n-d+i+1) \times \min(i, n-d+i+1)$ minors, we require that $0 = \prod_{i=0}^{s-1}(t_i + 1)$.
2. We require that L has full rank by requiring that $\det(L) = 1$ is one.
3. Given a target number of ones per row/column z, we require that any product of $z+1$ variables in one row/column is zero.

We may then use any polynomial system solver to recover a solution if it exists or to recover a proof that no such matrix exists. Since we expect that many solutions exist SAT solvers, such as CryptoMiniSat [52], appear to be more appropriate solvers when compared with Gröbner basis algorithms such as F4 [26] that recover an algebraic description of all solutions.

Optimization – Constraint Integer Programming We may also express the problem as a Constraint Integer Program. A first approach is simply using a MIP solver to solve systems arising as in Section B.3. However, here we describe

a different approach. We again consider the matrix

$$L = \begin{pmatrix} L_{0,0} & L_{0,1} & \dots & L_{0,n-1} \\ L_{1,0} & L_{1,1} & \dots & L_{1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1,0} & L_{n-1,1} & \dots & L_{n-1,n-1} \end{pmatrix} \text{ with } L_{i,j} = \begin{pmatrix} \ell_{i,j,0,0} & \dots & \ell_{i,j,0,b-1} \\ \ell_{i,j,1,0} & \dots & \ell_{i,j,1,b-1} \\ \vdots & \ddots & \vdots \\ \ell_{i,j,b-1,0} & \dots & \ell_{i,j,b-1,b-1} \end{pmatrix}$$

where $0 \leq \ell_{i,j,i',j'} \leq 1$ for $0 \leq i, j < n$ and $0 \leq i', j' < b$ are boolean variables. We construct a Constraint Integer Program minimizing $\sum_{0 \leq i',j' < b, 0 \leq i,j < n} \ell_{i,j,i',j'}$, i.e., we are minimising the density subject to the following constraints

1. $d - 1 \leq \sum_{\ell_j \in L_{(i)}} \ell_j \leq z$, i.e., that at most z and at least $d-1$ entries per row are one.
2. $d - 1 \leq \sum_{\ell_j \in L^T_{(i)}} \ell_j \leq z$, i.e., that at most z and at least $d-1$ entries per column are one.
3. For all $w = L \cdot v$ for all $v \in \mathbb{F}_2^{nb}$, we model the relation between active input and output blocks. First, denote by $0 < ia \leq n$ the number of active input blocks (of size b) and let

$$z_{i,v} = \bigvee_{0 \leq i' < b} w_{(i \cdot b + i')} = \bigvee_{0 \leq i' < b} (L \cdot v)_{(i \cdot b + i')}$$

   be a binary variable indicating that a given output block is active under $L \cdot v$. We then require that $n - d + ia - 1 \leq \sum_{0 \leq i < n} z_{i,v}$ if $ia < d - 1$.
4. For all $w = L \cdot v$ for all $v \in \mathbb{F}_2^{nb} \backslash \{0\}$, we require that $\sum_{0 \leq i < n, 0 \leq i' < b} w_{(ib+i')} \geq 1$ to enforce that L has full rank.

C Classification of PRINCE-like linear layers

As mentioned above, the block cipher PRINCE uses a special instance of Theorem 1. More precisely, it uses a 16×16 binary matrix with (linear and differential) branch number 4 over $\mathbb{F}_2^4$ and exactly 3 ones per row and column. On top, this matrix is an involution, i.e. it is its own inverse.

In this section, we give a classification of all PRINCE-like linear layers, i.e. of all 16×16 binary matrices fulfilling all criteria mentioned above. Note that this implies in particular that the corresponding codes are near-MDS codes. There are quite some results known about near-MDS codes, see in particular [23].

Clearly, the construction from Theorem 1 covers only a subspace of all possible linear layers. This section can therefore also be seen as an exemplary study on how large this subspace is.

Definition 1. *We call $\mathcal{IV}_{n,b,d}$ any $n \times n$ block matrix over $b \times b$ matrices over $\mathbb{F}_2$ with linear and differential branch number d over $\mathbb{F}_2^b$ which is also an involution.*

For an efficient classification it is crucial to only consider *essentially different* matrices. For this we establish the following equivalence relation.

Lemma 3. *Let A be $\mathcal{IV}_{n,b,d}$. Then,*

$$B = P \cdot Q \cdot A \cdot Q^T \cdot P^T$$

is also $\mathcal{IV}_{n,b,d}$ for P being a block permutation matrix – permuting $b \times b$ blocks as units – and Q being a block diagonal matrix of $n \times n$ permutation matrices.

Definition 2. *We call two $\mathcal{IV}_{n,m,d}$ matrices A and B equivalent if $B = P \cdot Q \cdot A \cdot Q^T \cdot P^T$ for P, Q as in Lemma 3.*

Finally, we need to pick a representative for each family. For this the definition of a normal forms is helpful.

Definition 3. *We call a matrix in $\mathcal{IV}_{n,b,k}$* normal form$_<$ *under $<$ iff A*

1. *is $\mathcal{IV}_{n,b,k}$, and*
2. *is the smallest matrix under $<$ satisfying these conditions.*

There are many choices for the ordering $<$ used in this definition. For implementation reasons we picked an ordering which essentially is the natural ordering on an 256-bit integer representation of the binary matrices.

The Case $\mathcal{IV}_{4,1,4}$

Note that all those codes correspond to $[8, 4, 4]$ binary linear codes. It is known (cf. [23]) that this code is, up to code-equivalence, the extended Hamming code. However, the notion of equivalence we define above is different to code equivalence. In particular there is more than one solution up to equivalence. Moreover, different combinations of those matrices to $[8, 4, 4]$ codes over $\mathbb{F}_2^4$ using Th. 1 may lead to non-equivalent codes as is demonstrated in Table 3.

We ran exhaustive search over all $\mathcal{IV}_{4,1,4}$ matrices to find all optimal matrices (3 ones per row/column). There are 10 different matrices. These are:

$$\left(\begin{array}{c|c|c|c} & 1 & 1 & 1 \\ \hline 1 & & 1 & 1 \\ \hline 1 & 1 & & 1 \\ \hline 1 & 1 & 1 & \end{array}\right) \left(\begin{array}{c|c|c|c} 1 & 1 & 1 & \\ \hline 1 & & 1 & 1 \\ \hline 1 & 1 & & 1 \\ \hline & 1 & 1 & 1 \end{array}\right) \left(\begin{array}{c|c|c|c} & 1 & 1 & 1 \\ \hline 1 & 1 & & 1 \\ \hline 1 & & 1 & 1 \\ \hline 1 & 1 & 1 & \end{array}\right) \left(\begin{array}{c|c|c|c} & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \\ \hline 1 & 1 & & 1 \\ \hline 1 & & 1 & 1 \end{array}\right) \left(\begin{array}{c|c|c|c} 1 & 1 & 1 & \\ \hline 1 & 1 & & 1 \\ \hline 1 & & 1 & 1 \\ \hline & 1 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{c|c|c|c} 1 & 1 & & 1 \\ \hline 1 & & 1 & 1 \\ \hline & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \end{array}\right) \left(\begin{array}{c|c|c|c} 1 & & 1 & 1 \\ \hline & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \\ \hline 1 & 1 & & 1 \end{array}\right) \left(\begin{array}{c|c|c|c} 1 & & 1 & 1 \\ \hline & 1 & 1 & 1 \\ \hline 1 & & 1 & 1 \\ \hline 1 & 1 & & 1 \end{array}\right) \left(\begin{array}{c|c|c|c} & 1 & 1 & 1 \\ \hline 1 & & 1 & 1 \\ \hline 1 & 1 & & 1 \\ \hline 1 & 1 & 1 & \end{array}\right) \left(\begin{array}{c|c|c|c} 1 & 1 & & 1 \\ \hline 1 & 1 & 1 & \\ \hline & 1 & 1 & 1 \\ \hline 1 & & 1 & 1 \end{array}\right)$$

Those allow the construction of 10^4 different matrices in $\mathcal{IV}_{4,4,4}$. Those 10^4 matrices contain a set of 74 pairwise non-equivalent representatives.

The Case $\mathcal{IV}_{4,2,4}$

We then ran exhaustive search over all $\mathcal{IV}_{4,2,4}$ to find all optimal matrices (3 ones per row/column). There are 220 different matrices satisfying the conditions.

Again, those matrices allow the construction of 220^2 different matrices in $\mathcal{IV}_{4,4,4}$. The above mentioned equivalence relation reduced those to 470 essentially different matrices in $\mathcal{IV}_{4,4,4}$.

The Case $\mathcal{IV}_{4,4,4}$

Finally, we then ran exhaustive search on 16×16 matrices pruning any search tree not satisfying the conditions of Lemma 1. Moreover, taking the equivalence relation into account allows further pruning of search trees, resulting in a significant speed up. In total, the whole search took less than one day on a standard PC.

There are 739 $\mathcal{IV}_{4,4,4}$ normal forms. Of those, 74 are composed of $\mathcal{IV}_{4,1,4}$ matrices and 470 are composed of $\mathcal{IV}_{4,2,4}$ matrices. The remaining 196 matrices are either not composed of $\mathcal{IV}_{4,i,4}$ matrices for $i < 4$ or are composed of $\mathcal{IV}_{4,3,4}$ and $\mathcal{IV}_{4,1,4}$ matrices.

The Weight Distribution of $\mathcal{IV}_{4,4,4}$

One of the benefits of our classification is that the a-priori huge number of choices for a PRINCE-like linear layer is reduced to a much more manageable amount. This in particular allows to not only focus on the linear and differential branch number, but more detailed one the exact weight distribution of the corresponding additive codes. While this does not improve the (directly) provable bounds on the probabilities of linear and differential trails it has an impact on the number of trails with optimal bias resp. probability. Note that, as proven in [23], the weight distribution of the code and its dual are identical. Moreover, given the number of code-words with minimal weight, the whole weight distribution is fixed (cf. [23, Theorem 4.1]).

Below we give an exhaustive list of the possible weight distributions for all 739 PRINCE-like linear layers. A_i denotes the number of words with weight i. We also give the multiplicity, i.e. how many of the 739 cases lead to the given weight distribution. We furthermore note which of the possible weight distributions occur for linear layers constructed via Theorem 1 either by combining $\mathcal{IV}_{4,1,4}$ or $\mathcal{IV}_{4,2,4}$ matrices. As can be seen, there is a rather big variance in the number of code words with minimal weight, ranging from 52 up to 210. While intuitively it seems beneficial to select a code with a minimal number of code words of small weight, the exact benefits of a specific choice depend on cipher details outside the scope of this paper.

D Security Analysis of PRIDE

Due to following the interleaving construction it is straightforward to bound the probability of differential characteristics and the absolute bias of linear trails.

A_4	A_5	A_6	A_7	A_8	Multiplicity	$\mathcal{IV}_{4,1,4}$	$\mathcal{IV}_{4,2,4}$
52	632	4932	20792	39127	5	false	true
54	624	4944	20784	39129	3	false	true
56	616	4956	20776	39131	7	false	true
60	600	4980	20760	39135	25	true	true
62	592	4992	20752	39137	11	true	true
64	584	5004	20744	39139	26	false	true
66	576	5016	20736	39141	7	true	true
68	568	5028	20728	39143	43	true	true
70	560	5040	20720	39145	10	true	true
72	552	5052	20712	39147	30	true	true
74	544	5064	20704	39149	21	true	true
76	536	5076	20696	39151	34	true	true
78	528	5088	20688	39153	22	false	true
82	512	5112	20672	39157	40	true	true
84	504	5124	20664	39159	26	true	true
86	496	5136	20656	39161	42	true	true
88	488	5148	20648	39163	5	false	false
90	480	5160	20640	39165	40	true	true
92	472	5172	20632	39167	4	false	true
94	464	5184	20624	39169	49	true	true
96	456	5196	20616	39171	4	false	true
98	448	5208	20608	39173	43	false	true
102	432	5232	20592	39177	30	true	true
106	416	5256	20576	39181	24	true	true
110	400	5280	20560	39185	12	false	true
112	392	5292	20552	39187	4	true	true
114	384	5304	20544	39189	24	true	true
118	368	5328	20528	39193	10	false	true
122	352	5352	20512	39197	14	false	true
126	336	5376	20496	39201	22	true	true
130	320	5400	20480	39205	22	false	true
134	304	5424	20464	39209	8	false	true
138	288	5448	20448	39213	16	true	true
154	224	5544	20384	39229	18	true	true
162	192	5592	20352	39237	17	false	true
178	128	5688	20288	39253	12	false	true
210	0	5880	20160	39285	9	true	true

Fig. 3. The Weight Distribution of All PRINCE-like Codes

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We also investigated in detail if there is a significant linear-hull or differential effect.

Here, we justify our assumption of the resistance of PRIDE against classical analysis techniques, such as linear and differential cryptanalysis.

Classical Cryptanalysis. As mentioned above the differential and linear branch numbers of L_0 to L_3 are all 4. Thus, it follows by Theorem 1 that the same holds for the entire linear layer L. This means we have –at least– 4 active Sboxes per two rounds and thus 32 in 16 rounds. For the Sboxes we selected, the best non-zero differential has probability 1/4. Thus, assuming independent round keys, there is no single differential trail for 16 rounds of PRIDE with average probability better than $(1/4)^{32} = 2^{-64}$, which is too small to be usable for an attack even when using the full code-book. Similarly, for linear cryptanalysis, we can upper-bound the absolute correlation for any single trail by $(1/2)^{32} = 2^{-32}$. Again, this is too small to be of use in an attack.

We have computationally generated all optimal characteristics and trails for six rounds of PRIDE. We found 15 871 differentials with probability 2^{-24} and 5 632 trails with the expected absolute correlation of 2^{-12}. Each optimal characteristic (resp. trail) has a unique pair of input and output difference (resp. mask). In other words, there is no clustering of neither optimal characteristics nor optimal trails.

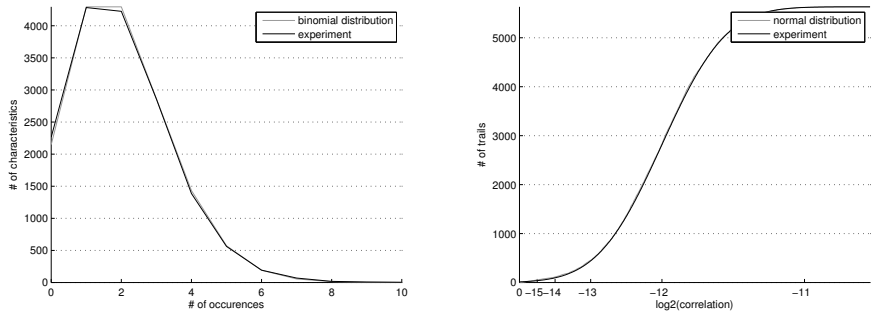
We experimentally verify these results with a round-reduced implementation of the cipher in order to check for linear-hull and differential effects. In the case of differential cryptanalysis, for each input difference we picked a random key and encrypted 2^{25} random plaintexts pairs while counting how often the predicted output differences were observed. While for some input/output differences the resulting counter was significantly higher than 2, this is not surprising but fits the theory. To illustrate this, Figure 4(a) shows how closely these results match a binomial distribution with parameters

$$p = 2^{-24} \quad \text{and} \quad N = 2^{25}.$$

In addition, for any characteristic that resulted in a counter above 6 in the previous experiment, we re-run the program with an increased amount of plaintext/ciphertext pairs. Using 2^{30} pairs, no significant differential effect was detected.

For the linear part, we performed a similar experiment using 2^{27} plaintexts. Figure 4(b) shows how many (out of the 5 632 trails) resulted in a correlation smaller than a given value. Again, even though some of the observed correlations are higher than 2^{-13}, the distribution fits nicely with the theoretically expected normal distribution. Again, we re-ran the experiments for the trails with experimentally high correlation with increased data complexity. No significant linear-hull effect was observed.

Finally, we have generated the best possible characteristics and linear trails for sixteen rounds of PRIDE (with max. 4 active Sboxes per two rounds) and



(a) Occurences of differential characteristics

(b) Correlation of linear trails

Fig. 4. Experimental results for differential and linear cryptanalysis over six rounds of PRIDE

found that there is no clustering of those optimal trails/characteristics. Additionally, our program shows that the bounds we have established are tight. However, PRIDE has 20 rounds; and in our belief, it should be sufficient.

Other Attacks. We also considered advanced variants of linear and differential attacks. Higher-order differentials, truncated differentials, impossible differentials, and zero-correlation attacks do not seem to pose a threat on PRIDE. It seems that the bit-wise structure of the linear-layer limit the applicability of these attacks.

Finally, we have considered algebraic attacks, but did not find any serious issues.

E Code Table

The following table contains the best-known (bounds on the) minimal distance for a $[2n, n]$ code over $\mathbb{F}_{2^b}$ for $b = 1, 2, 3$. An entry with a single number means that a code with a minimal distance matching the upper bound exists.

n	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\mathbb{F}_2$	2	3	4	4	4	4	5	6	6	7	8	7	8	8	8
$\mathbb{F}_{2^2}$	3	4	4	5	6	6	7	8	8	8-9	9	10	11	12	11-12
$\mathbb{F}_{2^3}$	3	4	5	5	6	7	8	8	9	9-10	10-11	10-12	11-13	11-14	12-15

F Algorithms for Finding Permutations Maximizing Dependencies

F.1 Heuristic

We represent the permutations P, Q as integer vectors $(p_0, \ldots, p_{n-1})$ with $i' \leq p_{i'} < n$. This format is also known as LAPACK-style permutations and allows to apply permutations to matrices in place. This format is interpreted by looping over $0 \leq i' < n$ in increasing order and swapping row/column at index i' with row/column at index $p_{i'}$. Our first algorithm simply finds local optima and combines them. It fixes P_0, Q_0 to the identity (without loss of generality) and iterates over $1 \leq i < b$. For each level it proceeds index-wise. That is, for each index i', it tries all possible values for p'_i and q'_i in P_i and Q_i and keeps the value which maximizes fill-in locally. It then proceeds to the next index.

F.2 Complete – Constraint Integer Program

We consider matrices

$$P_i = \begin{pmatrix} p_{i,0,0} & p_{i,0,1} & \cdots & p_{i,0,n-1} \\ p_{i,1,0} & p_{i,1,1} & \cdots & p_{i,1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ p_{i,n-1,0} & p_{i,n-1,1} & \cdots & p_{i,n-1,n-1} \end{pmatrix}, Q_i = \begin{pmatrix} q_{i,0,0} & q_{i,0,1} & \cdots & q_{i,0,n-1} \\ q_{i,1,0} & q_{i,1,1} & \cdots & q_{i,1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ q_{i,n-1,0} & q_{i,n-1,1} & \cdots & q_{i,n-1,n-1} \end{pmatrix}.$$

and denote

$$\begin{pmatrix} p\ell q_{i,0,0} & p\ell q_{i,0,1} & \cdots & p\ell q_{i,0,n-1} \\ p\ell q_{i,1,0} & p\ell q_{i,1,1} & \cdots & p\ell q_{i,1,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ p\ell q_{i,n-1,0} & p\ell q_{i,n-1,1} & \cdots & p\ell q_{i,n-1,n-1} \end{pmatrix} = P_i \cdot L_0 \cdot Q_i.$$

1. We add quadratic constraints expressing $p\ell q_{i',i,j}$ in terms of $P_{i'} \cdot L_0 \cdot Q_{i'}$.
2. We add linear constraints expressing that all rows and columns of $P_{i'}$ and $Q_{i'}$ have exactly one 1 per row/column.
3. We add OR constraints $v_{i,j} = \left(\bigvee_{0 \leq i' < b} pa q_{i',i,j}\right)$.
4. We add a linear constraint $1 \leq \sum v_{i,j} \leq b \cdot (\sum_{0 \leq i,j < n} c_i)$ to encode that we only have a limited number of ones to use for fill-in.

We then maximize $\sum v_{i,j}$ using an off-the-shelf CIP solver such as SCIP [1].

F.3 Example

We finish by giving an example comparing the two approaches. We start with the 8×8 matrix over $\mathbb{F}_2$:

$$L_0 = \begin{pmatrix} 1&0&1&0&1&0&1&0 \\ 1&0&0&1&1&0&0&1 \\ 1&1&0&0&0&0&1&1 \\ 0&0&1&1&0&1&1&0 \\ 0&1&0&1&0&1&1&1 \\ 0&0&1&0&1&1&0&1 \\ 0&1&0&1&1&1&0&0 \\ 1&1&1&0&0&1&0&0 \end{pmatrix}$$

and search for one pair of permutation matrices P, Q such that the density of $L_0 \vee (P \cdot L_0 \cdot Q)$ is maximized. Our heuristic search returned

$$P = \begin{bmatrix} 2&5&6&3&4&5&6&7 \end{bmatrix}, \; Q = \begin{bmatrix} 4&7&6&5&4&5&6&7 \end{bmatrix}$$

in LAPACK-style notation. Under this choice for P and Q, we have that $L_0 \vee (P \cdot L_0 \cdot Q)$ has a density of $60/64$. On the other hand, applying Constraint Integer Programming, we recovered

$$P = \begin{bmatrix} 5&7&6&3&4&5&6&7 \end{bmatrix}, \; Q = \begin{bmatrix} 6&2&2&4&4&5&6&7 \end{bmatrix}$$

in LAPACK-style notation. Under this choice for P and Q we have that $L_0 \vee (P \cdot L_0 \cdot Q)$ has a density of $63/64$, which is the density achievable.

G Software-friendly Matrices

$$L_0 \;\&\; L_0^{-1} = \begin{pmatrix} 0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0 \\ 0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1 \\ 1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0 \\ 0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1 \\ 1&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&1&0&0&0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&1&0&0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&1 \\ 1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0 \end{pmatrix} \quad L_1 = \begin{pmatrix} 1&1&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&1&1&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&1&1&0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&1&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&1&0&1&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&1&1&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&1 \\ 0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0&1&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&1&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&1&1&0&0&0&0 \end{pmatrix}$$

$$L_2 = \begin{pmatrix}
0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&1&1&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&1&0&0&0&0&0&0\\
1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0\\
1&1&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&1&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&1&1&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&1&1&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1\\
0&0&0&0&0&1&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&1&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&1&1&0&0&0&0\\
1&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&1
\end{pmatrix}$$

L_3 & $L_3^{-1} =$

$$\begin{pmatrix}
1&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0\\
0&1&0&0&0&1&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&1\\
1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1\\
1&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0\\
0&1&0&0&0&0&0&0&0&1&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&1
\end{pmatrix}$$

$L_1^{-1} =$

$$\begin{pmatrix}
0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&0\\
1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1\\
1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&1&0&0&0&0&0&0&1&0&0\\
0&0&0&1&0&0&0&0&0&0&0&1&1&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&1\\
0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&1&1&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&1&0&0&0&0
\end{pmatrix}$$

$L_2^{-1} =$

$$\begin{pmatrix}
0&0&1&1&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&1&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&1&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&1&0&0&0&0&0&0&1&0\\
1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1\\
1&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&1&1&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&1&1&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&1&1&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&1&1&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&1
\end{pmatrix}$$

H AVR Assembly Codes for Linear and Substitution Layer

```
; State s0, s1, s2, s3, s4, s5, s6, s7
; Temporary registers t0, t1, t2, t3


; Linear Layer and Inverse Linear Layer: L0
movw t0, s0   ; t1:t0 = s1:s0
swap s0
swap s1
eor s0, s1
```

```
eor t0, s0
mov s1, t0
eor s0, t1

; Linear Layer: L1
swap s3
movw t0, s2   ; t1:t0 = s3:s2
movw t2, s2   ; t3:t2 = s3:s2
lsl t0
rol t2
lsr t1
ror t3
eor s2, t3
mov t0, s2
eor s2, t2
eor s3, t0

; Inverse Linear Layer: L1
movw t0, s2   ; t1:t0 = s3:s2
movw t2, s2   ; t3:t2 = s3:s2
lsr t0
ror t2
lsr t1
ror t3
eor t3, t2
eor s3, t3
swap s3
mov s2, t3
lsr t3
ror s2
eor s2, t2

; Linear Layer: L2
swap s4
movw t0, s4   ; t1:t0 = s5:s4
movw t2, s4   ; t3:t2 = s5:s4
lsl t0
rol t2
lsr t1
ror t3
eor s4, t3
mov  t0, s4
eor s4, t2
eor s5, t0

; Inverse Linear Layer: L2
movw t0, s4   ; t1:t0 = s5:s4
movw t2, s4   ; t3:t2 = s5:s4
lsr t0
ror t2
lsr t1
ror t3
eor t3, t2
eor s5, t3
mov s4, t3
lsr t3
ror s4
eor s4, t2
swap s4

; Linear Layer and Inverse Linear Layer: L3
movw t0, s6   ; t1:t0 = s7:s6
swap s6
swap s7
eor s6, s7
eor t1, s6
mov s7, t1
eor s6, t0
```

```
; Substitution Layer
movw t0, s0
movw t2, s2
and s0, s2
eor s0, s4
and s2, s4
eor s2, s6
and s1, s3
eor s1, s5
and s3, s5
eor s3, s7
movw s4, s0
movw s6, s2
and s4, s6
eor s4, t0
and s6, s4
eor s6, t2
and s5, s7
eor s5, t1
and s7, s5
eor s7, t3
```

I Sbox Formulation

$$A = c \oplus (a \& b)$$
$$B = d \oplus (b \& c)$$
$$C = a \oplus (A \& B)$$
$$D = b \oplus (B \& C)$$

J Testvectors for PRIDE

Plaintext	k_0	k_1	Ciphertext
0000000000000000	0000000000000000	0000000000000000	82b4109fcc70bd1f
ffffffffffffffff	0000000000000000	0000000000000000	d70e60680a17b956
0000000000000000	ffffffffffffffff	0000000000000000	28f19f97f5e846a9
0000000000000000	0000000000000000	ffffffffffffffff	d123ebaf368fce62
0123456789abcdef	0000000000000000	fedcba9876543210	d1372929712d336e