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3. *A mountain-chain farther to the north*, which can be continued from N. W. New-Guinea over Waigeoe and Salawati in the eastern Moluccas and in which the late-tertiary sediments are sometimes folded intensively.

Besides folding, very numerous fractures form the principal characteristic of the tectonics of the Eastern Moluccas. Along with the many that are known, we may e.g. mention a great number on the Soela-islands, which by the occurrence of hot springs and by topographical features are often easy to trace. Also along the Sibella-mountain on Batjan numerous hot springs occur. Some fractures are volcanic fissures, however, as has been observed above, the fissures must often be later than the older volcanic rocks, so that it is not allowed to connect the places, where these rocks are found in a region that for the greater part is covered by the sea, by volcanic fissures.

Physics. — *"An experiment of* MAXWELL *and* AMPÈRE's *molecular currents."* By Dr. W. J. DE HAAS and Dr. G. L. DE HAAS—LORENTZ. (Communicated by Prof. H. A. LORENTZ.)

(Communicated in the meeting of June 26, 1915).

EINSTEIN and DE HAAS, who proved experimentally the existence of AMPÈRE's molecular currents, mentioned in their paper [1]) that RICHARDSON has already tried, though unsuccessfully, to give a similar proof.

In connection with this it is interesting, that so early as 1861 MAXWELL [2]) made an experiment for the purpose of deciding whether a magnet contains any rotatory motion. This experiment was arranged as follows:

A coil can turn about a horizontal diameter BB' of a ring, which again can rotate about its vertical diameter. Let, in case the coil does not rotate, the axis CC' fixed in it coincide with the vertical one. If in the coil there are rotatory motions about an axis perpendicular to BB' and CC' and if the ring turns about its vertical diameter, the axis CC' must deviate from the vertical. Further particulars on the experiments are not known. MAXWELL mentions only that he has not been able to detect the deviation in question, even when the coil had an iron core.

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1) Proc. Acad. Amsterdam. **18**, p. 696.

2) MAXWELL, Electricity and Magnetism, Vol. II, p. 203.

We shall now treat this problem somewhat more in detail, specializing it more than MAXWELL did. In fact we do not seek for the effect of rotatory motions of some wholly unknown kind, but for that of the circular currents supposed by AMPÈRE, or, according to the conceptions of the theory of electrons, of (negative) elecrons circulating round the molecules. For the sake of definiteness we shall even imagine all electrons to move with the same velocity in fixed circular paths, all of the same radius.

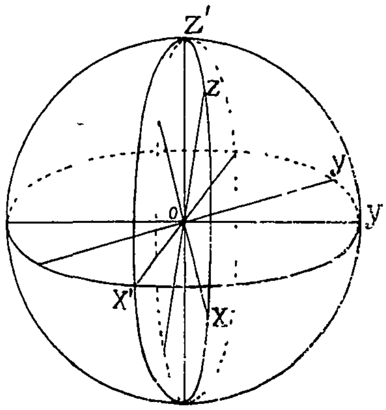

Fig. 1.

Fig. 1 represents this schematically.

Here OX', OY', OZ' are fixed axes, OZ' being vertical.

OX, OY, OZ are axes fixed in the body. We choose for these the principal axes of inertia and denote by A, B, C the moments of inertia with respect to them. OX is the axis of the coil.

The center of gravity of the body lies in O, so that gravity does not produce a rotating couple.

Further we shall suppose the terrestrial magnetic field to be compensated.

Both systems of coordinates are of the same kind, so that they may be made to coincide by a rotation.

In the experiment the axis OY was forced to move in a horizontal plane, OX, OY, OZ can therefore be brought from the positions OX' OY' OZ' into the actual positions by means of two rotations, one about OZ' through the angle $Y'OY = \varphi$ and one about OY through the angle $Z'OZ = \theta$.

If there are no molecular currents the kinetic energy T of the body is a function of $\dot\varphi$ and $\dot\theta$, viz.

$$2\,T = A\dot\varphi^2 \sin^2\theta + B\dot\theta^2 + C\dot\varphi^2\cos^2\theta \quad . \quad . \quad . \quad . \quad (1)$$

According to LAGRANGE's equation we get for the couple tending to increase θ:

$$\Theta = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta}\right) - \frac{\partial T}{\partial \theta} = B\,\frac{d^2\theta}{dt^2} + \dot\varphi^2\,(C - A)\sin\theta\cos\theta\,. \qquad (2)$$

Here we may remark once for all, that in this experiment the axis OY rotates with constant velocity in a horizontal plane about OZ', so that $\dot\varphi$ is a constant. In order that without an external couple Θ a stable state may be possible in which OZ and OZ' coincide, so that $\theta = 0$, C must be greater than A. Then the body can perform a vibration consisting in periodical changes of the angle θ, and for infinitely small amplitudes we find the frequency :

$$n = \dot{\varphi} \sqrt{\frac{C-A}{B}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If there are circulating electrons we shall have to introduce new parameters in addition to the angles φ and θ. For these parameters we choose the angles ψ, measured from a fixed point in the path of each electron towards its momentary position.

The kinetic energy will now contain a part corresponding to (1) which depends on $\dot{\theta}$ and $\dot{\varphi}$ and which we shall call T_1. But besides it will contain two parts T_2 and T_3, to which we shall soon refer.

As to T_1, it is the question, whether when there are moving electrons, the moments of inertia A, B, C will perhaps depend on ψ, so that they are no longer constants.

For the sake of simplicity we suppose the iron core to be magnetized to saturation, so that all the molecular axes are directed along the axis of the coil, while the circular currents are perpendicular to this line. We neglect the heat motion which would prevent this perfect orientation.

To calculate the moment of inertia with respect to one of the axes OX, OY, OZ for an electron, moving in a circle with the center M, we draw through M a line parallel to the axis considered. The moment of inertia with respect to this line can easily be calculated and from it we deduce the required moment of inertia by a well known rule. It is evident, that the part contributed to A by the circulating electrons will not depend on the positions in their paths, but will have a constant value.

On the contrary, the part contributed by one circulating electron to B and C (i. e. to the moments of inertia with respect to the axes in the plane of the circle) will change continually. But as soon as there are in each circle more than two electrons, at fixed distances from each other (so that one angle ψ determines the positions of them all) the terms in B and C due to them will have constant value. This is easily seen. Let n electrons circulate in the path. Their moment of inertia with respect to a diameter of the circle is proportional to

$$\sin^2 \psi + \sin^2\left(\psi + \frac{2\pi}{n}\right) + \sin^2\left(\psi + \frac{4\pi}{n}\right) \ldots . =$$

$$= \tfrac{1}{2}\left\{n - \cos 2\psi - \cos 2\left(\psi + \frac{2\pi}{n}\right) - \cos 2\left(\psi + \frac{4\pi}{n}\right) - \ldots .\right\}$$

and the value of this is $\frac{1}{2}n$ as soon as $n > 2$. Indeed, the points on the circle, determined by the angles 2ψ, $2\left(\psi + \frac{2\pi}{n}\right)$, $2\left(\psi + \frac{4\pi}{n}\right)$, and so on, are the vertices of a regular polygon and therefore the resultant of the vectors drawn from the center towards these points is zero. Thus the moments of inertia A, B, C will not depend on ψ. Though they will be increased to a certain extent by the presence of circulating electrons, we may continue to represent them by A, B, C. No ambiguity will arise from this.

The second part T_2 of the kinetic energy depends on ψ. It is evident, that one electron gives

$$\frac{1}{2} m r^2 \dot{\psi}^2,$$

if m is its mass and r the radius of its path. As we suppose $\dot{\psi}$, m and r to be the same for all the electrons, we may write

$$2T_2 = D \dot{\psi}^2.$$

where

$$D = \Sigma m r^2,$$

the summation being extended to all the circulating electrons.

The third part T_3 will depend on the products $\dot{\varphi}\dot{\psi}, \dot{\theta}\dot{\psi}$. It is the existence of this part, that MAXWELL wanted to test experimentally.

The calculation of T_3 requires somewhat more consideration. A material point with mass m and two velocities v_1 and v_2 (vectors) has the kinetic energy.

$$\frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2 + m (v_1 \cdot v_2) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We shall apply this to each circulating electron. It will possess firstly the velocity v_2 due to its motion in the circle and secondly the velocity v_1 due to the motion of the whole body. Now the first two parts of (4) are contained in T_1 and T_2. The velocity v_1 may be resolved into the velocities due to the rotations of the body about OX, OY, OZ, and each of these rotations may be replaced by a translation and a rotation about a parallel axis through the center M of the path of the electron. Let us call the velocities of the electron belonging to these rotations v_{1a}, v_{1b}, v_{1c}. It is clear that the two latter components are perpendicular to v_2, so that they do not contribute anything to the scalar product $(v_1 \cdot v_2)$. The same may be said of the component of the translatory velocity perpendicular to the plane of that circle. As to the translatory velocity in this plane, it may contribute a term to (v_1, v_2) in the case of one electron, but it is easily found that these contributions compensate each other, when n electrons are moving in the same circle, the reason being that the velocities v_2 of all these particles have the same value but different directions which succeed each other at equal angular intervals. The only component that contributes a part to the scalar product, is v_{1a}, which always has the same direction as v_2. As the component of the angular velocity about the axis through M parallel to OX is equal to $-\dot{\varphi} \sin\theta$, we find for $m(v_1 \cdot v_2)$

$$-mr\dot{\psi} \cdot r\dot{\varphi} \sin\theta.$$

Taking the sum for all the circulating electrons, we find

$$-\dot{\psi}\dot{\varphi} \sin\theta \,.\, \Sigma\, mr^2 = -D\,\dot{\psi}\dot{\varphi} \sin\theta.$$

Thus the whole kinetic energy becomes

$$T = \tfrac{1}{2}\{A\,\dot{\varphi}^2 \sin^2\theta + B\,\dot{\theta}^2 + C\,\dot{\varphi}^2 \cos^2\theta + D\,\dot{\psi}^2 - 2\,D\,\dot{\psi}\,\dot{\varphi} \sin\theta\}$$

Writing $A + D$ for A, we get

$$T = \tfrac{1}{2}\{A\,\dot{\varphi}^2 \sin^2\theta + B\,\dot{\theta}^2 + C\,\dot{\varphi}^2 \cos^2\theta + D\,(\dot{\psi} - \dot{\varphi} \sin\theta)^2\},$$

So that the force tending to increase ψ is given by

$$\Psi = \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}}\right) - \frac{\partial T}{\partial \psi} = \frac{d}{dt}\left\{D\,(\dot{\psi} - \dot{\varphi} \sin\theta)\right\}$$

We shall start from the supposition that there are no forces acting on the electrons, by which their velocities in their paths might be changed. We then have

$$\Psi = 0, \quad \text{or} \quad \dot{\psi} - \dot{\varphi} \sin\theta = const. = \gamma \quad . \quad . \quad . \quad . \quad (5)$$

and we find instead of (2)

$$\Theta = B\frac{d^2\theta}{dt^2} + (C - A)\,\dot{\varphi}^2 \sin\theta \cos\theta + D\,\dot{\varphi}\,\gamma \cos\theta \quad . \quad . \quad (5a)$$

We have seen already, that C must be greater than A.

In the stationary state, in which θ does not change, $\frac{d^2\theta}{dt^2}$ will be equal to zero; so that in the absence of a couple Θ, the angle θ will assume a constant value given by

$$\sin\theta = -\frac{D}{C-A}\frac{\gamma}{\dot{\varphi}}.$$

The same formula may be found in a somewhat simpler way by considering the moments of momentum. We then must apply the following principle. In any system the change of the resultant moment of momentum has the direction of the moment of the couple that gives rise to this change.

In our case the only couple acting on the system arises from the forces applied at the extremities of the axis OY, by which this axis is compelled to move in a horizontal plane with the constant angular velocity $\dot{\varphi}$. The axis of this couple lies in the plane XOZ. Independently of the magnetization the body possesses moments of momentum about its axes OX and OZ. In addition to these it has a magnetic moment of momentum about OZ. So at all events the total moment of momentum will fall along a certain line OL in the plane $Z'OX$. If now this resultant OL did not coincide with OZ', it would continually change its direction because of its rotation about OZ'. The corresponding change in the moment of momentum would be perpendicular to the plane $Z'OL$ and this ought also to be the direction of the couple acting on the body. We have seen, however, that this axis lies *in* the plane $Z'OX$. Hence OL and OZ' must coincide.

The condition for this is, that the moment of momentum about OX, divided by that about OZ must be equal to $-tg\,\theta$. Thus we have

$$\frac{D\dot{\psi} - \dot{\varphi}\sin\theta}{C\dot{\varphi}\cos\theta} = -tg\,\theta$$

or, writing again $A + D$ for A,

$$\frac{D(\dot{\psi} - \dot{\varphi}\sin\theta) - A\dot{\varphi}\sin\theta}{C\dot{\varphi}\cos\theta} = -tg\,\theta,$$

from which we infer

$$\sin\theta = -\frac{D}{C-A}\cdot\frac{\dot{\psi} - \dot{\varphi}\sin\theta}{\dot{\varphi}}$$

or if we introduce the value for $\dot{\psi} - \dot{\varphi}\sin\theta$, given by (5)

$$\sin\theta = -\frac{D}{C-A}\cdot\frac{\gamma}{\dot{\varphi}}. \quad \dots\dots\dots \quad (6)$$

To investigate, whether the experiment will give a perceptible value of θ, it is desirable to express $\sin\theta$ in quantities that can easily be estimated. As D and $\dot{\psi}$ are unknown, we shall introduce instead of them the magnetization I of the body.

If O is the area of a circular current, its magnetic moment in the case of only one electron circulating in it, is

$$e\,O\frac{\dot{\psi}}{2\pi} = \tfrac{1}{2}\,e\dot{\psi}\,r^2;$$

If we take the sum for all the circulating electrons this expression becomes

$$I = \Sigma\,\tfrac{1}{2}\,e\dot{\psi}\,r^2 = \frac{e}{2m}\,\dot{\psi}\,\Sigma\,mr^2 = \frac{e}{2m}\,\dot{\psi}\,D.$$

Hence

$$D\dot{\psi} = \frac{2m}{e} I$$

I is the magnetization which the body would have in the case $\dot{\varphi} = 0$ and $\theta = 0$. We may safely assume, that this value scarcely differs from the magnetization that would exist when θ and $\dot{\varphi}$ were different from zero. Thus

$$D\gamma = \frac{2m}{e} I.$$

Substituting this in (6), we find

$$\sin\theta = -\frac{2m}{e} I \quad \frac{1}{C-A} \cdot \frac{1}{\dot{\varphi}} \quad . \quad . \quad . \qquad . \quad . \quad (7)$$

According to this formula it would be possible to reach an infinite value of $\sin\theta$ by making C and A equal to each other. It is true that a strong magnetization can only be obtained by using a rather long rod of iron, but notwithstanding this the difference $C—A$ may be made as small as we like by adjusting non-magnetizable masses in a proper way. In reality however this ideal case can never be realised. The principal reason for this is, that the point of suspension will never coincide with the centre of gravity, but will always lie at a small distance say q from the latter.

In this case, contrary to what we have supposed, the force of gravity will produce a couple acting on the body. Taking $C—A=0$ and writing P for the weight of the body we now get instead of (5a)

$$-Pq \sin\theta = B \frac{d^2\theta}{dt^2} + D\dot{\varphi}\gamma \cos\theta \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

so that the condition for the stationary state becomes

$$tg\,\theta = -\frac{\dot{\varphi}}{Pq} D\gamma\cdot$$

Introducing

$$D\gamma = \frac{2m}{e} I$$

and the time of oscillation t for vibrations about OY under influence of gravity, a time that is given by

$$t^2 = 4\pi^2 \frac{B}{Pq},$$

we find finally

$$tg\,\theta = -\frac{2m}{e} I \cdot \frac{t^2\dot{\varphi}}{4\pi^2 B}.$$

We shall make an estimate of the value of θ by means of the following values, which have all been chosen very favourably. Density of magnetisation 1.000

$$I = 150, \dot{\varphi} = 100, t^2 = 10, B = 2, \frac{2m}{e} = 1,1 \ . \ 10^{-7}.$$

We then find

$$tg\,\theta = -\,0,00013,$$

from which we may conclude, that the deviation will be hardly perceptible.

Mathematics. — *"The circles that cut a plane curve perpendicularly"*. II. By Prof. HENDRIK DE VRIES.

(Communicated in the meeting of February 26, 1916).

§ **7.** We found in the preceding § that through the point Z_∞ pass three different kinds of branches of the rest nodal curve, and in particular the branches of the first kind arose in groups of 4 at a time. If $S_{1\infty}$, $S_{2\infty}$ are two simple points of intersection of k^ν with l_∞, then the lines of connection of these points with Z_∞ are double torsal lines of Ω (§ 2), and the 4 sheets passing through these torsal lines cut each other in 4 branches of the rest nodal curve, which of course all pass through Z_∞, and have only *one* tangent here, viz. the line of intersection of the two tangent planes along the torsal lines, i.e. the line of connection of Z_∞ with the intersection of the asymptotes of k^ν in $S_{1\infty}$ and $S_{2\infty}$.

The branches of this first kind behave again differently according to their going towards the foci and the vertices, or to other points of k^ν; the branches going to the foci and the vertices are their own images with regard to β, those to other points, as the nodes, the cusps, the intersections of the isotropical tangents, are e a c h o t h e r's images. This difference has an influence on the nature of the tangents in Z_∞; for a branch that is its own image Z_∞ must be a point of inflexion, as on a straight line passing through this point and cutting the curve twice, the two points of intersection approach to Z_∞ from different sides; two branches on the contrary that are each other's images and pass through Z_∞, simply have the same tangent in this point. But whichever of the two cases may arise the projection of 4 branches belonging together produces a node in the intersection of the associated asymptotes of k^ν. If namely a twisted curve is projected out of one of its points of inflection, the projection possesses