Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Ratios and Proportional Relationships	**Grade Level: 7th**

7.RP.A Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Students will continue their work with ratios to analyze proportions and proportional relationships. Students expand their knowledge of unit rates to include computations with complex fractions. They recognize and represent proportional relationships in equations, in tables, and on graphs. Students use proportional reasoning to solve multi-step ratio and percent problems involving real world scenarios (percent change, sales tax, simple interest, etc.)

****This is a MAJOR cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

7.RP.1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour.

7.RP.2. Recognize and represent proportional relationships between quantities.

- **a.** Decide whether two quantities are in a proportional relationship. For example, by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- **b.** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **c.** Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items. Purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.
- **d.** Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.

7.RP.3. Use proportional relationships to solve multistep ratio and percent problems. For example, simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Aspects of Rigor for Student Learning:(Conceptual, Procedural, and/or Application)

Conceptual Understanding	***Procedural Fluency***	***Application***
Understand unit rate as a comparison of an amount per one unit **(7.RP.1)** Recognize a complex fraction is a fraction where the numerator, denominator, or both contain a fraction. **(7.RP.1)**	Simplify complex fractions, including calculating unit rates from a given complex fraction or ratio with unlike units. **(7.RP.1)**	Compute unit rates that involve complex fractions or ratios in real world problems. **(7.RP.1)**

$\frac{4}{\frac{3}{6}}$, $\frac{\frac{2}{4}}{8}$, $\frac{\frac{3}{9}}{\frac{4}{12}}$ **Numerator, Denominator or both containing Fraction is called Complex Fraction** (image from mathtutorvista.com) The above complex fractions represent 4 ÷ 3/6, 2/4 ÷ 8, and 3/9 ÷ 4/12. Recognize that a complex fraction can be simplified by dividing the numerator by the denominator. **(7.RP.1)** Example: $\frac{\frac{1}{2}}{\frac{1}{4}}$ is solved $\frac{1}{2} \div \frac{1}{4}$		
In an equation that represents a proportional relationship, the coefficient represents the same quantity as the unit rate, as well as the constant of proportionality. **(7.RP.2)** Understand unit rate as a constant of proportionality. **(7.RP.2)** The graph of a proportional relationship is a line that passes through the origin. **(7.RP.2a)** Identify the constant of proportionality when provided a table, graph, equation, diagram, or verbal description of a proportional relationship. (**7. RP.2b)** On the graph of a proportional relationship, the point (1, r) is the unit rate.**(7.RP.2d)**	Graph relationships to determine if two quantities are a proportional relationship and interpret the ordered pairs. **(7.RP.2)** Model proportional relationships by creating tables and graphs. **(7.RP.2)** Write equations from context and identify the coefficient as the unit rate (which is also the constant of proportionality.) **(7.RP.2)**	Examine a scenario to determine the constant of proportionality. **(7.RP.2)** Explain what the points (0, 0) and (1, *r*) and other points on the line mean in the context of a real-world situation. **(7.RP.2)** Given a verbal description of a proportional relationship, students create an equation in the form y = mx. **(7.RP.2c)**
Understand that a percent is a ratio comparing a number to 100. **(7.RP.3)** Start by identifying the whole/original amount of the percent being defined to understand the meaning of the percent of increase/decrease. **(7.RP.3)**	Determine the percent of a number. **(7.RP.3)** Determine the percent change from one quantity to another, as well as identify that change as an increase or a decrease. **(7.RP.3)**	Apply procedural fluencies to a variety of real-world contexts (simple interest, sales tax, commissions, etc.) **(7.RP.3)** Note: Students are encouraged to explain or show their work using a variety of representations (numbers, words, pictures, physical objects, or equations). **(7.RP.3)**

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
   - Students solve multi-step ratio and real-world percent problems.
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
   - Students recognize proportional relationships from non-proportional ones and discuss their reasoning with others.
4. **Model with mathematics.**
   - Students learn to represent proportional relationships as tables, graphs, verbal descriptions, diagrams, and equations.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
   - Students use units in their ratios requiring them to attend to the units such as 8 miles in 4 hours is a rate of 2 miles per one hour.
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions

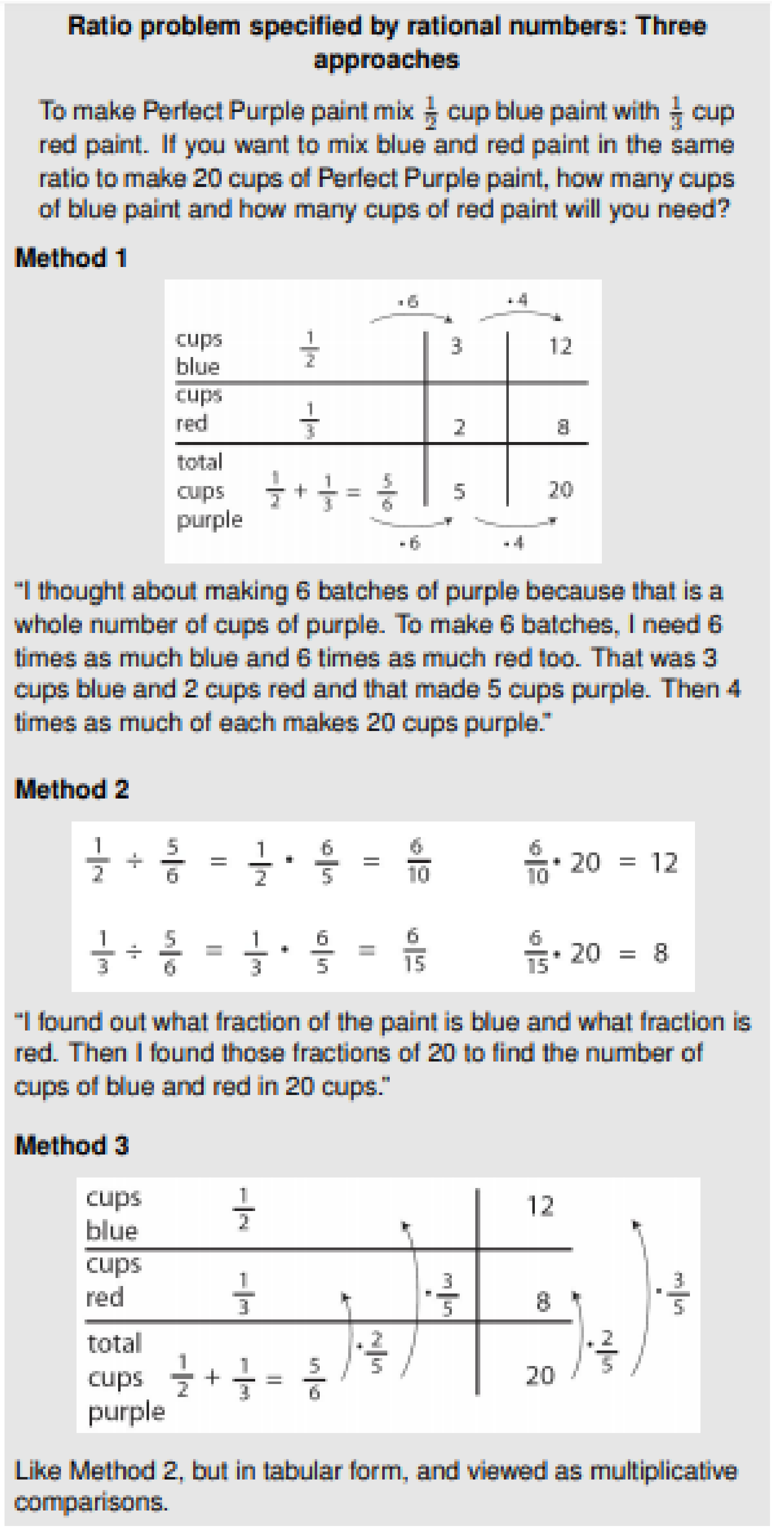
Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In grade 6, learners understand and represent ratios as part-to-part and part-to-whole. In grade 6, learners represent ratios with ratio tables, tape diagrams, and double line graphs, and use them to compare ratios and find missing values in tables. In grade 6, learners apply ratio reasoning to convert and compare different units of measure.	Learners connect their understanding of rational numbers to solve for unit rates, proportional reasoning and percent problems. Learners will also have the opportunity to compute percents using a decimal format when working in the number sense cluster.	Learners use units as a way to understand problems and find the solution in a multi-step problem. Students choose and interpret units consistently in formulas, choose and interpret the scale and origin in graphs and data displays.

Vocabulary *(Key Terms Used by Teachers and Students in this Cluster):*

- Ratio
- Rate
- Unit rate
- Constant of proportionality
- Proportional relationships
- Proportions
- Complex fraction
- Origin
- Percent error
- Gratuity
- Commission
- Markup
- Markdown
- Simple interest
- Percent increase
- Percent decrease
- Fees
- Discount
- Tax
- Principal
- Percent
- Coefficient

Relevance, Explanations, and Examples:

7.RP.1:



Ratio problem specified by rational numbers: Three approaches

To make Perfect Purple paint mix $\frac{1}{2}$ cup blue paint with $\frac{1}{3}$ cup red paint. If you want to mix blue and red paint in the same ratio to make 20 cups of Perfect Purple paint, how many cups of blue paint and how many cups of red paint will you need?

Method 1

cups blue	$\frac{1}{2}$	3	12
cups red	$\frac{1}{3}$	2	8
total cups purple	$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$	5	20

"I thought about making 6 batches of purple because that is a whole number of cups of purple. To make 6 batches, I need 6 times as much blue and 6 times as much red too. That was 3 cups blue and 2 cups red and that made 5 cups purple. Then 4 times as much of each makes 20 cups purple."

Method 2

$$\frac{1}{2} \div \frac{5}{6} = \frac{1}{2} \cdot \frac{6}{5} = \frac{6}{10} \qquad \frac{6}{10} \cdot 20 = 12$$

$$\frac{1}{3} \div \frac{5}{6} = \frac{1}{3} \cdot \frac{6}{5} = \frac{6}{15} \qquad \frac{6}{15} \cdot 20 = 8$$

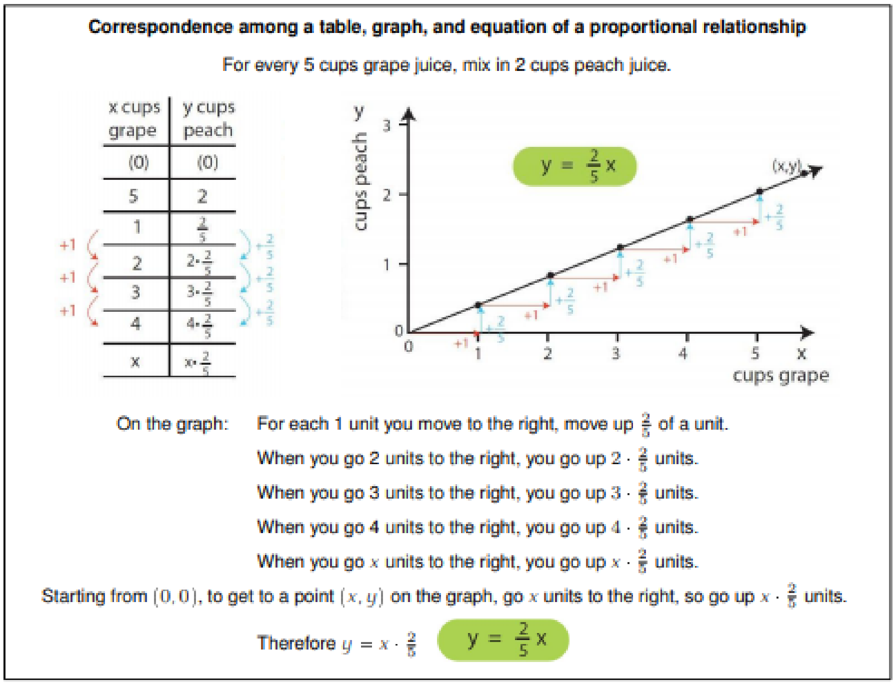
"I found out what fraction of the paint is blue and what fraction is red. Then I found those fractions of 20 to find the number of cups of blue and red in 20 cups."

Method 3

cups blue	$\frac{1}{2}$	12
cups red	$\frac{1}{3}$	8
total cups purple	$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$	20

Like Method 2, but in tabular form, and viewed as multiplicative comparisons.

7.RP.2:



Correspondence among a table, graph, and equation of a proportional relationship

For every 5 cups grape juice, mix in 2 cups peach juice.

x cups grape	y cups peach
(0)	(0)
5	2
1	$\frac{2}{5}$
2	$2 \cdot \frac{2}{5}$
3	$3 \cdot \frac{2}{5}$
4	$4 \cdot \frac{2}{5}$
x	$x \cdot \frac{2}{5}$

On the graph: For each 1 unit you move to the right, move up $\frac{2}{5}$ of a unit.

When you go 2 units to the right, you go up $2 \cdot \frac{2}{5}$ units.

When you go 3 units to the right, you go up $3 \cdot \frac{2}{5}$ units.

When you go 4 units to the right, you go up $4 \cdot \frac{2}{5}$ units.

When you go x units to the right, you go up $x \cdot \frac{2}{5}$ units.

Starting from $(0, 0)$, to get to a point (x, y) on the graph, go x units to the right, so go up $x \cdot \frac{2}{5}$ units.

Therefore $y = x \cdot \frac{2}{5}$ $y = \frac{2}{5}x$

7.RP.3

Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs $3.80.
projected cost of a gallon of gas for April 2015?

Solution:

Possible response: "The original cost of a gallon of gas is $3.80. An increase of 100% means that
double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas.
$3.80 is about $0.95, the projected cost of a gallon of gas should be around $8.15."

$3.80 + 3.80 + (0.24 • 3.80) = 2.24 x 3.80 = $8.15

100%	100%	24%

7.RP.3

A sweater is marked down 33% off the original price. The original price was $37.50. What is the sale price of the sweater before sales tax?

Solution:

The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

<table>
<tr><td colspan="2">37.50
Original Price of Sweater</td></tr>
<tr><td>33% of 37.50
Discount</td><td>67% of 37.50
Sale Price of Sweater</td></tr>
</table>

7.RP.3

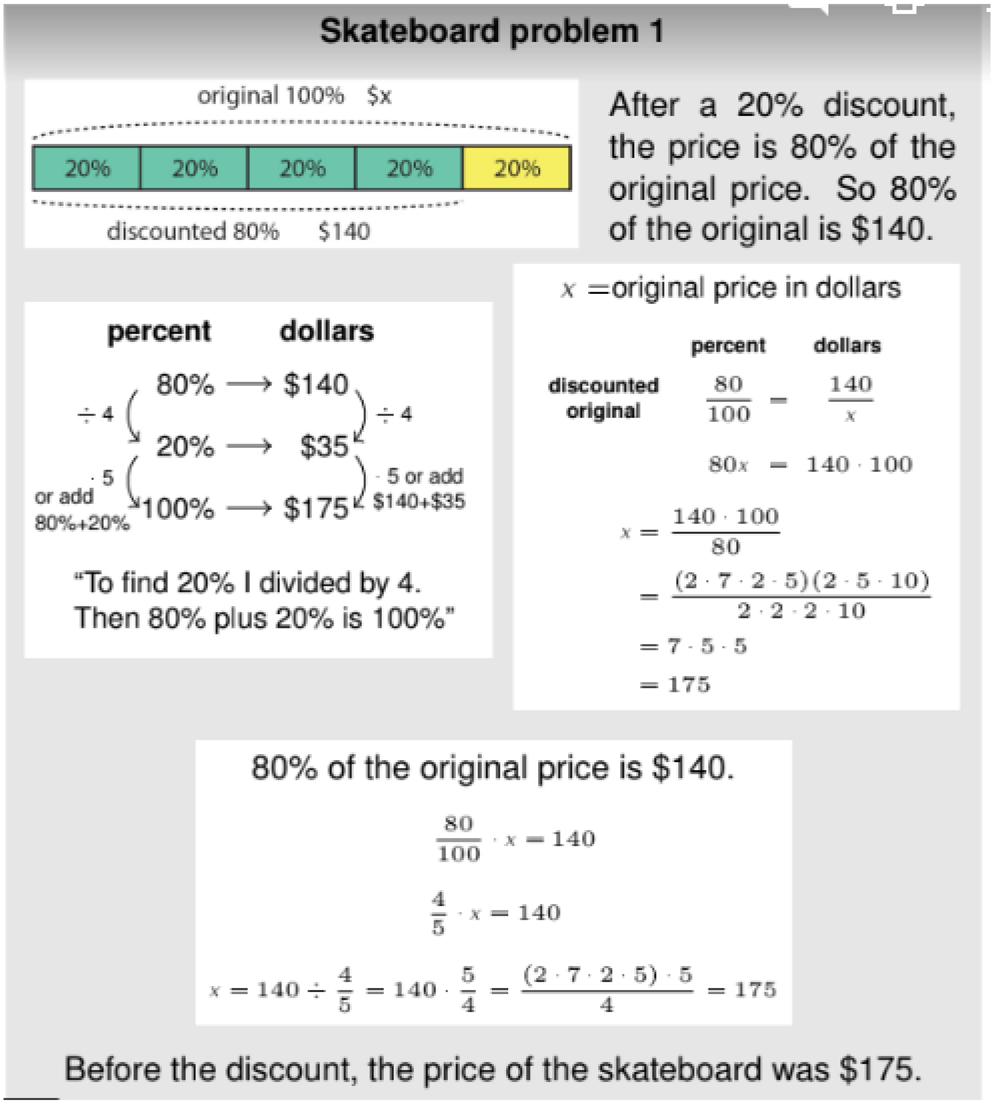
A shirt is on sale for 40% off. The sale price is \$12. What was the original price? What was the amount of the discount?

Solution:

Discount 40% of original	Sale Price → \$12 60% of original
Original Price (*p*)	

The sale price is 60% of the original price. This reasoning can be expressed as $12 = 0.60p$. Dividing both sides by 0.60 gives an original price of \$20.

7.RP.3



Skateboard problem 1

After a 20% discount, the price is 80% of the original price. So 80% of the original is \$140.

percent **dollars**

$$\begin{tikzcd}
80\% \arrow[r] \arrow[d, bend right, "\div 4"'] & \$140 \arrow[d, bend left, "\div 4"] \\
20\% \arrow[r] \arrow[d, bend right, "\cdot 5 \text{ or add } 80\%+20\%"'] & \$35 \arrow[d, bend left, "\cdot 5 \text{ or add } \$140+\$35"] \\
100\% \arrow[r] & \$175
\end{tikzcd}$$

"To find 20% I divided by 4. Then 80% plus 20% is 100%"

x =original price in dollars

	percent		**dollars**
discounted **original**	$\frac{80}{100}$	=	$\frac{140}{x}$

$$80x = 140 \cdot 100$$

$$x = \frac{140 \cdot 100}{80}$$

$$= \frac{(2 \cdot 7 \cdot 2 \cdot 5)(2 \cdot 5 \cdot 10)}{2 \cdot 2 \cdot 2 \cdot 10}$$

$$= 7 \cdot 5 \cdot 5$$

$$= 175$$

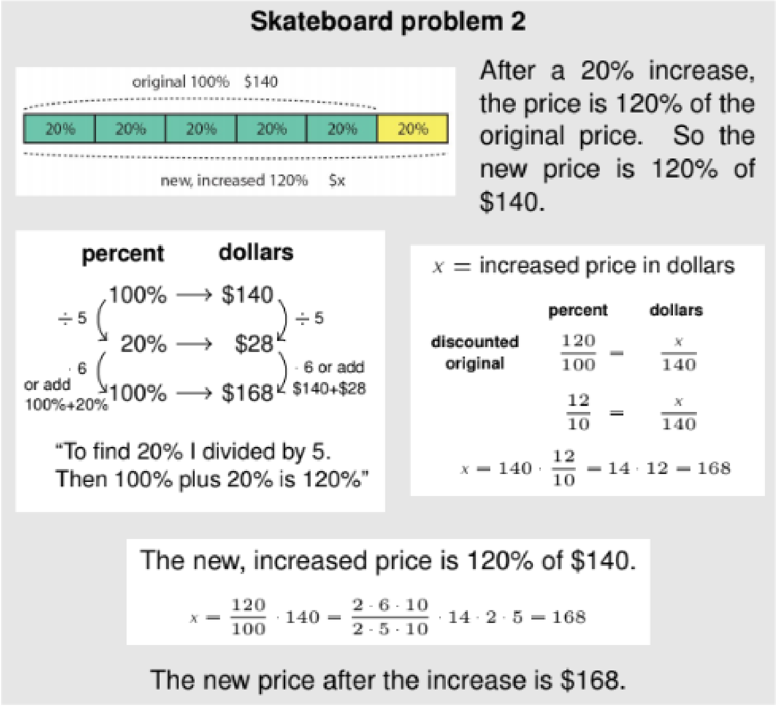
80% of the original price is \$140.

$$\frac{80}{100} \cdot x = 140$$

$$\frac{4}{5} \cdot x = 140$$

$$x = 140 \div \frac{4}{5} = 140 \cdot \frac{5}{4} = \frac{(2 \cdot 7 \cdot 2 \cdot 5) \cdot 5}{4} = 175$$

Before the discount, the price of the skateboard was \$175.



After a 20% increase, the price is 120% of the original price. So the new price is 120% of $140.

"To find 20% I divided by 5. Then 100% plus 20% is 120%"

$$\frac{120}{100} = \frac{x}{140}$$

$$\frac{12}{10} = \frac{x}{140}$$

$$x = 140 \cdot \frac{12}{10} = 14 \cdot 12 = 168$$

The new, increased price is 120% of $140.

$$x = \frac{120}{100} \cdot 140 = \frac{2 \cdot 6 \cdot 10}{2 \cdot 5 \cdot 10} \cdot 14 \cdot 2 \cdot 5 = 168$$

The new price after the increase is $168.

Achievement Level Descriptors

Cluster: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Concepts and Procedures	***Level 1:*** Level 1 students should be able to add, subtract, multiply, and divide nonnegative rational numbers. They should be able to add, subtract, multiply, and divide rational numbers with a number line or other manipulative.
	Level 2: Level 2 students should be able to find whole number proportionality constants in relationships presented in graphical, tabular, or verbal formats in familiar contexts. They should also be able to identify proportional relationships presented in equation formats and find unit rates involving whole numbers.
	Level 3: Level 3 students should be able to identify, represent, and analyze proportional relationships in various formats; find unit rates associated with ratios of fractions; and use unit rates to solve one-step problems involving rational numbers. They should be able to analyze a graph of a proportional relationship in order to explain what the points (x, y) and (1, r) represent, where r is the unit rate, and use this information to solve problems.
	Level 4: Level 4 students should be able to solve real-world problems involving proportional relationships and measurement conversions in various formats (e.g., verbally, tabularly, graphically) in a contextual scenario that involves identifying relationships between elements presented in various formats.