Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Numbers and Operations-Fractions	**Grade Level: 5**

5.NF.B Cluster: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Students use the meaning of fractions, of multiplication and division, and of the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense.

****This is a MAJOR cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

5.NF.3 Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- **a.** Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)
- **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.5 Interpret multiplication as scaling (resizing), by:

- **a.** Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- **b.** Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.

5.NF.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

- **a.** Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.
- **b.** Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between

multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

Aspects of Rigor for Student Learning: (Conceptual, Procedural, and/or Application)

Conceptual Understanding	***Procedural Fluency***	***Application***
Interpret a fraction as division of the numerator by the denominator. **(5.NF.3)**	Solve division problems involving whole numbers that lead to answers in the form of a fraction by using a visual fraction model or equations. [1] **(5.NF.3)**	Solve word problems involving whole numbers that lead to answers in the form of a fraction. [1] **(5.NF.3)**
Extend previous understandings of multiplication to multiply a fraction or a whole number by a fraction. [2] **(5.NF.4)** Understand that the product (a/b) x q is the same as a x q ÷ b. **(5.NF.4a)** [3]	Multiply a fraction or a whole number by a fraction. **(5.NF.4)**	Create a story context to multiply a fraction or a whole number by a fraction. **(5.NF.4a)**
Understand that finding the area of a rectangle with fractional side lengths by filling with tiles is the same as would be found by multiplying the side lengths. **(5.NF.4b)**	Find the area of a rectangle by tiling it with unit squares. **(5.NF.4b)** Multiply fractional side lengths to find the area of a rectangle. **(5.NF.4b)**	
Interpret multiplication by scaling, comparing the size of a product to the size of one factor based on the size of the other factor. **(5.NF.5.a)** Explain why multiplying a given number by a fraction greater than one results in a product greater than the given number and why multiplying a given number by a fraction less than one results in a product smaller than the given number. **(5.NF.5.b)** [4]		
	Solve problems involving the multiplication of fractions and mixed numbers using visual fraction models or equations. **(5.NF.6)**	Solve real world problems involving the multiplication of fractions and mixed numbers by using visual fraction models or equations. **(5.NF.6)**
Extend previous understanding of division to division of of unit fractions by whole numbers (non-zero) and whole numbers (non-zero) by unit fractions. **(5.NF.7.a,b)**	Solve division problems that include unit fractions divided by whole numbers (non-zero) and whole numbers (non-zero) divided by unit fractions. **(5.NF.7.a,b)**	Create a story context for and solve real world problems that include unit fractions divided by whole numbers (non-zero) and whole numbers (non-zero) divided by unit fractions. **(5.NF.7.a,b,c)**

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
    - Students use a variety of problem solving situations to develop understanding of multiplication of

fractions and mixed numbers.
    - Students persevere in dividing fractions, using both written explanations and models.
    - Students are making connections as they develop an understanding of the connection between whole number multiplication and division and multiplication and division using fractions.
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
    - Students use fraction models when solving both multiplication and division problems with fractions.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**
    - Students identify patterns that develop when solving multiplication and division problems involving fractions.
        - When multiplying a whole number greater than 1 the product greater and when multiplying a whole number by a fraction less than 1 the product will be smaller than the whole number.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In 3rd grade, students work to understand concepts of area and relate area to multiplication and to addition. **(3.MD.7)** In 4th grade, students use the four operations with whole numbers to solve problems. **(4.OA.1,2,3)** Students begin to understand the concept of equivalent fractions by using visual fraction models. **(4.NF.1)** They also begin to multiply a fraction by a whole number. **(4.NF.4)**	Students use their understanding of tenths and hundredths to perform operations with multi-digit whole numbers and with decimals to hundredths. **(5.NBT.5,6,7)** Students will use their knowledge of writing simple expressions to solve real problems with fraction. They will also interpret expressions without actually evaluating them. **(5.OA.2)** Students also use operations on fractions of a unit (1/2, 1/4, 1/8) to solve problems involving information presented in line plots. **(5.MD.2)**	Students in 6th grade will use ratios written as fractions and divide into decimal form 3 ÷ 4 = ¾ = 0.75. **(6.RP.1,3)** Students will solve multiplication equations that include non-negative rational numbers. **(6.EE.7)** Students in 6th grade will multiply and divide fractions by fractions. **(6.NS.1)**

Vocabulary *(Key Terms Used by Teachers and Students in this Cluster):*

- Equivalent
- mixed number

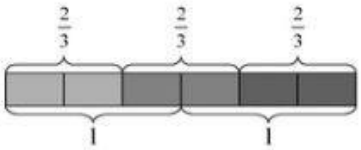
Relevance, Explanations, and Examples:

[1] 3 ÷ 4 = ¾

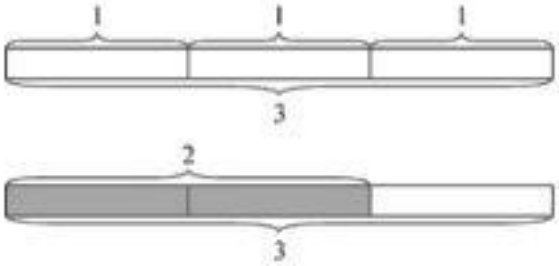
[2] Even though students have already learned the commutative property of multiplication, applying this to fractions extends their understanding. In 4th grade, students build understanding of multiplying a fraction by a whole number. For example, if I need ¼ cup of sugar for a recipe and want to make 3 batches, how many cups of sugar will I need. This is ¼ x 3 = ¾. In 5th grade, students develop understanding of multiplying a whole number by a fraction. For example, I have 10 feet of ribbon and use ⅕ to make a bow. How much ribbon did I use? This is 10 x ⅕ = 2. Though similar to the work done in 4th grade, the change in the order of the numbers can be challenging for students' conceptual understanding.

[3]

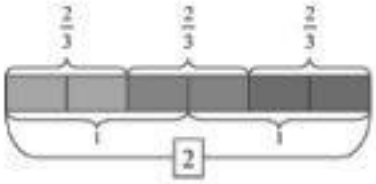
Makayla said, "I can represent $3 \times \frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$."



A rectangle of length 3 can be built from three rectangles of length 1.



Thus, $\frac{2}{3}$ of the large rectangle is 2 of the smaller rectangles of length 1.
We can see in Makayla's picture that there are 3 groups of 2 thirds, so there are 6 thirds all together. Since 3 thirds is 1 whole, 6 thirds is $6 \div 3 = 2$ wholes. So three rectangles each of length $\frac{2}{3}$ has a total length of 2.



In the picture for part (a), we can see that $\frac{2}{3}$ of 3 is also 2. So this shows that 3 groups of $\frac{2}{3}$ (an interpretation of $3 \times \frac{2}{3}$) is equal to $\frac{2}{3}$ of 3 (an interpretation of $\frac{2}{3} \times 3$), or

$$3 \times \frac{2}{3} = \frac{2}{3} \times 3$$

These pictures illustrate the commutative property of multiplication.

https://achievethecore.org/coherence-map/5/23/228/228

4

Decide which number is greater without multiplying.

a. 817 or 235×817

b. 99 or $\frac{1}{4} \times 99$

c. $\frac{51}{100}$ or $\frac{51}{100} \times 301$

d. $\frac{13}{90}$ or $\frac{2}{3} \times \frac{13}{90}$

e. $\frac{101}{102}$ or $\frac{101}{102} \times \frac{101}{102}$

f. $\frac{99}{5}$ or $\frac{99}{5} \times \frac{1}{2}$

g. $\frac{8}{21} \times 40$ or $\frac{28}{21} \times 40$

h. $\frac{8}{3} \times \frac{5}{7}$ or $\frac{8}{3} \times \frac{9}{4}$

Achievement Level Descriptors	
Cluster: **Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**	
Concepts and Procedures	***Level 1:*** Students should be able to apply their previous understandings of multiplication to multiply a fraction by a fraction; know the effect that whole number multiplication has on fractions; use or create visual models when multiplying a whole number by a fraction between 0 and 1; and interpret and perform division of a whole number by 1/2 or 1/3.
	Level 2: Students should be able to multiply a whole number by a mixed number; know the effect that a fraction greater than or less than 1 has on a whole number when multiplied; use or create visual models when multiplying two fractions between 0 and 1; extend their previous understandings of division to divide a unit fraction by a whole number; and understand that division of whole numbers can result in fractions.
	Level 3: Students should be able to multiply a mixed number by a mixed number; know the effect that a fraction has on another fraction when multiplied (proper and improper fractions); use or create visual models when multiplying two fractions, including when one fraction is larger than 1; and interpret and perform division of any unit fraction by a whole number.
	Level 4: Students should be able to understand and use the fact that a fraction multiplied by 1 in the form of a/a is equivalent to the original fraction.