Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

<table>
<tr><td colspan="2">Domain: Functions</td><td>Grade Level: 4th Year</td></tr>
<tr><td colspan="3">HS4.F.TF.C Cluster: Prove and apply trigonometric identities.
Students will use the Fundamental Identities to develop additional identities that can be used to verify, evaluate and solve trigonometric problems.</td></tr>
<tr><td colspan="3">In a precalculus class this is a MAJOR cluster. Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.

F.TF.9 Prove the addition and subtraction, half-angle, and double-angle formulas for sine, cosine, and tangent and use them to solve problems.

F.TF.10. Use fundamental trigonometric identities.
a. Verify trigonometric identities
b. Evaluate trigonometric functions
c. Write equivalent trigonometric expressions
d. Solve trigonometric equations.</td></tr>
<tr><td colspan="3">Aspects of Rigor of Student Learning: (Conceptual, Procedural, and/or Application)</td></tr>
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<tr><td>Conceptual Understanding</td><td>Procedural Fluency</td><td>Application</td></tr>
<tr><td>Students will understand the proof of the sine or the cosine addition or subtraction identity and use that result to prove the other identities.</td><td>Students study equations involving sums and differences of angles and apply them when rewriting trigonometric expressions that contain functions of multiple or half angles.

Students solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.</td><td></td></tr>
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Students will understand that trigonometric identities can be used to rewrite trigonometric expressions and/or equations in order to simplify or solve.	Students rewrite trigonometric functions and verify trigonometric identities using fundamental trigonometric identities. Students solve trigonometric equations, including those written in quadratic form and equations containing more than one angle.	

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
    - Proving trigonometric identities allow students to practice perseverance.
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
    - Students will need to construct viable arguments when verifying trigonometric identities.
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
    - Students will demonstrate the use of appropriate tools as they choose identities to help them with their proofs.
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	*Current Learning Connections*	*Future Learning Connections*
Students will build upon the concept of proof and deductive reasoning studied in Geometry. Students have proved the Pythagorean identities in Algebra II.	Students will use the Fundamental Identities to develop additional identities that can be used to verify, evaluate and solve trigonometric problems.	The trigonometric identities developed in this cluster are critical for study in Calculus.

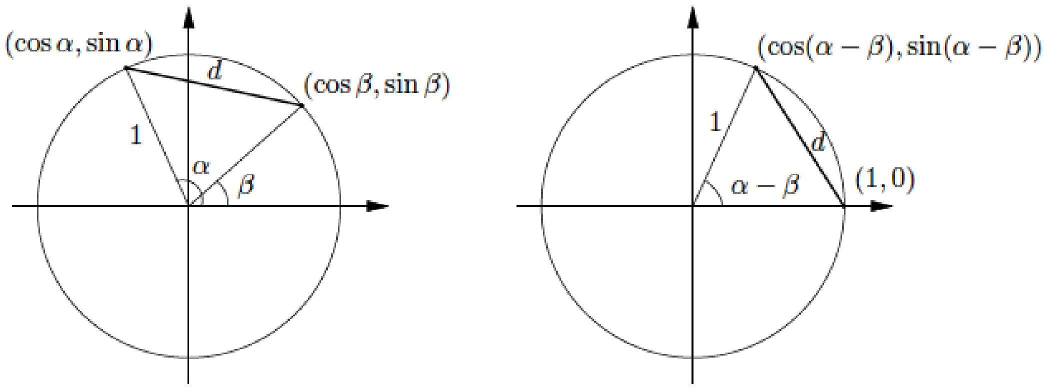
Vocabulary (key terms and definitions)

- Identity
- Fundamental trigonometric identities
- Addition and subtraction formulas (also referred to as sum and difference formulas)
- Double-angle formulas
- Half-angle formulas

Relevance, Explanations, and Examples:

One possible example of a proof of the cosine subtraction identity is shown below.

Proof. Consider two angles α and β. The distance d in the following two unit circles are equal.



From the first one we obtain

$$d = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2}.$$

From the second one we obtain

$$d = \sqrt{(\cos(\alpha-\beta) - 1)^2 + (\sin(\alpha-\beta) - 0)^2}.$$

From these two expressions for d, we can deduce

$$\begin{aligned}
d^2 &= d^2 \\
(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2 &= (\cos(\alpha-\beta) - 1)^2 + (\sin(\alpha-\beta) - 0)^2 \\
\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta + \sin^2\alpha - 2\sin\alpha\sin\beta + \sin^2\beta &= \cos^2(\alpha-\beta) - 2\cos(\alpha-\beta) + 1 + \sin^2(\alpha-\beta) \\
(\cos^2\alpha + \sin^2\alpha) - 2\,(\cos\alpha\cos\beta + \sin\alpha\sin\beta) + (\cos^2\beta + \sin^2\beta) &= (\cos^2(\alpha-\beta) + \sin^2(\alpha-\beta)) - 2\cos(\alpha-\beta) + 1 \\
2 - 2\,(\cos\alpha\cos\beta + \sin\alpha\sin\beta) &= 2 - 2\cos(\alpha-\beta).
\end{aligned}$$

Therefore,

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.}$$