

Unpacked South Dakota State Mathematics Standards

Purpose: In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.

Domain: Geometry		Grade Level: Geometry
G.G.GMD.A Cluster: Explain volume and surface area formulas and use them to solve problems.		
Learners move from applying volume formulas to justifying them. Learners will be exposed to advanced concepts in an informal setting. Learners will deconstruct complex geometric shapes into basic three-dimensional shapes to calculate their surface areas and volumes.		
<p>**This is a SUPPORTING cluster. <i>Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.</i></p> <p>G.G.GMD.A.1 Give an informal argument for the formulas for the volume of a cylinder, pyramid, sphere, and cone. Use dissection arguments, and informal limit arguments.</p> <p>G.G.GMD.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.</p> <p>G.G.GMD.A.3 Know and apply volume and surface area formulas for cylinders, pyramids, cones, and spheres for composite figures to solve problems. *</p>		
Aspects of Rigor: (Conceptual, Procedural, and/or Application)		
G.G.GMD.A.1 Give an informal argument for the formulas for the volume of a cylinder, pyramid, sphere, and cone. Use dissection arguments, and informal limit arguments.		
Conceptual Understanding	Procedural Fluency	Application
<p>Understand volume as equivalent to the total of the area of stacked cross sections.</p> <p>Volume formulas can be justified using dissection arguments and limits.</p> <p>Learners relate the volumes among various solids with the same dimensions.</p>		
G.G.GMD.A.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.		
Conceptual Understanding	Procedural Fluency	Application
Understand the relationships between volumes of right and oblique solids.	Calculate the volumes of oblique solids.	

	Use volume formulas to explore relationship between spheres and cylinders.	
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G.G.GMD.A.3 Know and apply volume and surface area formulas for cylinders, pyramids, cones, and spheres for composite figures to solve problems. *

Conceptual Understanding	Procedural Fluency	Application
Composite solids can be deconstructed into component shapes to determine their volumes and surface areas.	Calculate volumes and surface areas of composite shapes composed of cylinders, pyramids, cones, and/or spheres.	<p>Calculate the surface area of a castle that needs to be painted.</p> <p>Calculate the volume of sand in a sand castle.</p> <p>Determine capacity of various containers.</p> <p>Determine an appropriate model satisfying desired dimensions or characteristics.</p>

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

- 1. Make sense of problems and persevere in solving them.**
 - Learners must be challenged to develop deep understanding through exploring a range of tasks that require problem solving.
 - Make sense of formulas and the relationships among them.
- 2. Reason abstractly and quantitatively.**
 - Justifying formulas will move learners from concrete to abstract thinking.
- 3. Construct viable arguments and critique the reasoning of others.**
 - A central focus is constructing viable arguments about formulas in order to avoid ambiguity.
- 4. Model with mathematics.**
 - There are multiple opportunities for modeling with geometric shapes representing real-world applications.
 - Develop generalizations using modeling.
- 5. Use appropriate tools strategically.**
 - Physical models and dynamic geometry software can be used to explore these concepts.
- 6. Attend to precision.**
 - Attend to units of measure.
 - Use precise mathematical language to avoid ambiguity.
- 7. Look for and make use of structure.**
 - Learners connect related formulas (i.e cylinder/cone and prism/pyramid) and deconstruct composite shapes.
- 8. Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions

<u>Previous Learning Connections</u>	<u>Current Learning Connections</u>	<u>Future Learning Connections</u>
In 7th grade, learners worked with area, and circumference. This will be extended to find components needed for surface area and volume. (7.G.4) In 6th, 7th and 8th grade, learners calculated the volumes and surface areas of prisms, cones, cylinders, and spheres (6.G.2, 7.G.6 and 8.G.9)	Learners expand their work to include composite figures. They justify (informally) volume formulas. Solids of rotation are related through constructions. (G.C.4)	In Calculus, learners will apply Cavalieri's principle to calculate volumes for solids of rotation.

Vocabulary (key terms and definitions)

- Cavalieri's Principle
- dissection
- informal limit arguments
- volume
- cylinder
- pyramid
- cone
- composite shape
- sphere
- oblique

Relevance, Explanations, and Examples:

Students should understand volume as more than just using a formula.

More general formulas for surface area and volume are preferable to shape-specific formula.. For example, for a cylinder, $V = \text{Base area} \times \text{height}$ is preferred over the formula $V = \pi r^2 h$.

The ideas in this cluster are quite advanced. While students should be encouraged to make sense of the formulas and relationships, justifying why the formulas work will require guidance by the teacher.

Since GMD.2(+) is an advanced standard, consider using as an extension. Cavalieri's principle can be modeled using a deck of cards with the same cross-section. (Stack vertically, and then stack at an oblique angle)

Achievement Level Descriptors

Cluster: Explain volume and surface area formulas and use them to solve problems.

Concepts and Procedures

Level 1: Students should be able to apply mathematics to solve familiar problems arising in everyday life, society, and the workplace by identifying important quantities and by beginning to develop a model.

Level 2: Students should be able to apply mathematics to propose solutions by identifying important quantities, locating missing information from relevant external resources, beginning to construct chains of reasoning to connect with a model.

Level 3: Students should be able to apply mathematics to solve unfamiliar problems arising in everyday life, society, and the workplace by identifying important quantities and mapping, displaying, explaining, or applying their relationship and by locating missing information from relevant external resources. They should be able to construct chains of reasoning to justify a model used, produce justification of interpretations, state logical assumptions, and compare and contrast multiple plausible solutions.

Level 4: Students should be able to apply mathematics to solve unfamiliar problems by constructing chains of reasoning to analyze a model, producing and analyzing justification of interpretations, stating logical assumptions, and constructing and comparing/contrasting multiple plausible solutions and approaches.