Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Geometry	**Grade Level: Geometry**

G.G.CO.A Cluster: Experimenting with transformations in the plane

Learners formalize their transformation language by building precise definitions based on properties. They use formal notation and precise descriptions of transformations and sequences of transformations. Rotational and reflection symmetry are identified with a specific degree of rotation or line(s) of symmetry.

****This is a MAJOR cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

G.G.CO.A.1 State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane

G.G.CO.A.2 Represent transformations in the plane. (e.g., using transparencies and/or geometry software)

- **a.** Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
- **b.** Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).

G.G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that
map the figure onto itself.

G.G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, (e.g., using graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will map a given figure onto another.

Aspects of Rigor for Student Learning: (Conceptual, Procedural, and/or Application)

G.G.CO.A.1 State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane

Conceptual Understanding	***Procedural Fluency***	***Application***
Use precise definitions of transformations to formalize geometric language. For example: an angle is a rotation of a ray, or a translation of a line results in a parallel line.		Build precise definitions from transformation(s) based on the undefined terms point, line, and plane.

G.G.CO.A.2a Represent transformations in the plane. (e.g., using transparencies and/or geometry software). Describe transformations as functions that take points in the plane as inputs and give other points as outputs.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Write a function that maps a preimage to its image from an image or a description of a transformation.		

G.G.CO.A.2b Represent transformations in the plane. (e.g., using transparencies and/or geometry software); Compared transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).		
Conceptual Understanding	**Procedural Fluency**	**Application**
Understand which transformations result in figures with congruent sides and angles and which do not.		

G.G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Recognize rotations that map a figure onto itself. Learners will accurately describe the line(s) of reflection that maps a figure onto itself.	Calculate the degree of rotational symmetry.	

G.G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.		
Conceptual Understanding	**Procedural Fluency**	**Application**
Build definitions of rotations, reflections, and translations in terms of their properties. For example, rigid transformations preserve distance, preserve angle measures, map lines to lines, rays to rays, and segments to segments.		

G.G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, (e.g., using graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will map a given figure onto another.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Graph an image that results from a rotation, reflection, or translation. Give precise descriptions of sequences of rigid motions that map one figure onto another.		

Describe a sequence of rigid transformations using formal notation.		

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
    - Learners must be challenged to develop deep understanding of the ideas in the clusters through exploring tasks that require problem solving.
    - As learners reason and experiment with rigid motions, determining a correct sequence of transformations will require perseverance.
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
    - Learners should be encouraged to form arguments explaining why certain patterns hold in transformations and constructions, and to critique arguments that are presented to them.
    - In addition to creating proofs (constructing viable arguments), learners should have an opportunity to compare their proofs to those created by their classmates (critique the reasoning of others).
4. **Model with mathematics.**
    - Learners will apply ideas about transformations to model real-world contexts.
5. **Use appropriate tools strategically.**
    - Learners should use a variety of tools constructions and transformations, including compass and straightedge, graph paper, tracing paper, or geometry software.
    - The use of a variety of tools will be useful in making generalizations and proofs about when two triangles will be congruent.
6. **Attend to precision.**
    - While appearances might lead learners to certain conclusions, they need to precisely describe the transformations that they use and ensure that they work as they thought.
    - Learners need to use precise mathematical language to thoroughly explain the reasoning behind their work and when formalizing definitions.
    - Precision is of crucial importance in constructions, since even small errors in executing a construction may lead to results that don’t work.
    - Use precise geometric language within proofs.
7. **Look for and make use of structure.**
    - Learners should look for patterns in their explorations, leading them to making generalizations about the transformations and constructions.
    - Looking for structure is an important aspect of conjecturing.
8. **Look for and express regularity in repeated reasoning.**
    - Looking for patterns is an important aspect of conjecturing.
    - Learners will need to look for patterns that will help them see general methods for constructions that can be used.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	**Current Learning Connections**	**Future Learning Connections**
In 4th grade, learners work with lines of symmetry. In 8th grade, learners understand congruence and similarity using physical models, transparencies, or geometry software.They have described sequences of rigid motions informally and in terms of coordinates. Learners have verified experimentally the properties of transformations, and describe their effects on two-dimensional figures using coordinates.	Transformation definitions will serve as the basis for theorems that will be proven later in the course. Learners will use transformations as the foundation for studying congruence and similarity in future standards.	Learners will apply their knowledge of transformations to functions in later courses. They will use transformation language to compare a function to its parent function, identify lines of symmetry, and other characteristics of functions.

Vocabulary (key terms and definitions)		
● parallel ● perpendicular ● angle ● ray ● line segment ● circle ● point	● line ● plane ● transformation ● translation ● dilation ● reflection ● rotation	● image ● preimage ● line of symmetry ● center of rotation ● angle of rotation ● rigid motion ● degree of rotation

Relevance, Explanations, and Examples:

When developing conceptual understanding of transformations there are many tools that are helpful for exploration such as tracing paper, reflective devices such as Miras, and technology such as Geogebra, Sketchpad, Desmos, or graphing calculators.

Rotations should not be limited to multiples of 90 degrees. Other rotations can be performed with a compass and protractor or technology.

Emphasis should be placed on conceptual knowledge when writing rules for transformations. Providing formulas takes away from developing deeper understandings.

Achievement Level Descriptors

Cluster: Experiment with transformations in the plane

Concepts and Procedures	***Level 1:*** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2: Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.
	Level 3: *Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.*
	Level 4: Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.