Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Geometry	**Grade Level: Geometry**

G.G.C.A Cluster: Understand and apply theorems about circles

Learners will apply concepts of similarity to circles and their related components, explore inscribed and circumscribed circles and their associated polygons through constructions.This cluster builds many of the basic properties for angles, lines, and segments related to circles. Learners explore those properties and form conjectures. They should then be encouraged to create justifications regarding why their conjectures are correct. Learners refer back to their work with transformations to better understand these relationships.

****This is a ADDITIONAL cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

G.G.C.A.1 Prove that all circles are similar.

G.G.C.A.2 Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords.

G.G.C.A.3 Construct, using a compass and straightedge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.G.C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle.

Aspects of Rigor: (Conceptual, Procedural, and/or Application)

G.G.C.A.1 Prove that all circles are similar.

Conceptual Understanding	***Procedural Fluency***	***Application***
All circles are similar due to dilation of the length of radius.	Learners continue to work on proof processes and procedures. Compare ratios of radii and circumference to establish that circles are similar.	

G.G.C.A.2 Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords.

Conceptual Understanding	***Procedural Fluency***	***Application***
Know the components related to circles (arcs, chords, segments, lines, and angles).	Calculate measures of central angles, inscribed angles, circumscribed angles, and other related angles	

Understand the relationships between the components of a circle: angles and segments created from intersecting chords, secants, tangents, and diameters.	formed by intersecting secants from given arc measures. Calculate the length of segments created by intersecting chords, secants, and tangents.	

G.G.C.A.3 Construct, using a compass and straightedge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Conceptual Understanding	***Procedural Fluency***	***Application***
Polygons have inscribed and circumscribed circles that can be constructed using compass and straightedge. Inscribed quadrilaterals have characteristic angle relationships.	Construct inscribed circles of a triangle. Construct circumscribed circles of a triangle. Prove opposite angles of an inscribed quadrilateral are supplementary. Prove that the sum of the angles of an inscribed quadrilateral equals 360 degrees.	

G.G.C.A.4(+) Construct a tangent line from a point outside a given circle to the circle.

Conceptual Understanding	***Procedural Fluency***	***Application***
A line tangent to a circle intersects the circle at one point.	Construct (compass and straightedge) a line tangent to a circle.	

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
   - Perseverance and making sense of problems are fundamental to writing proofs.
   - Learners must be challenged to develop deep understanding of concepts through exploring tasks that require problem solving with a variety of tools and techniques.
2. **Reason abstractly and quantitatively.**
   - Use similarity to discover all circles are similar. Show similarity of circles with length of radii.
3. **Construct viable arguments and critique the reasoning of others.**
   - Write proofs and analyze proofs written by others.
   - Develop arguments to support conjectures and construct formal arguments when appropriate.
4. **Model with mathematics.**
   - There a multiple opportunities for modeling with circles, given their prevalence in the real-world.
   - Develop generalizations using modeling.
5. **Use appropriate tools strategically.**
   - Use compass and straightedge to construct figures.
   - Explore the parts of a circle and their relations using dynamic geometry software/tools.
6. **Attend to precision.**
   - Precision is of crucial importance in constructions, since even small errors in executing a construction may lead to results that don't work.
   - Use precise mathematical language to describe conjectures they develop.

7. **Look for and make use of structure.**
    - Many conjectures will be based on examination of underlying relationships within a problem, or determining a solution method that can be generalized into a theorem.
8. **Look for and express regularity in repeated reasoning.**
    - Look for patterns across a range of examples to better understand the behaviors of various parts of circles, triangles, and quadrilaterals.
    - Find various strategies for writing proofs and gain fluency selecting an effective argument strategy.

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In 8th grade, learners have worked with two-dimensional figures and verified their properties. (**8.G.A**)	Previous work with similarity will be applied to circles. Construction adds to learning from previous clusters by increasing skills with formal construction, building on angle congruence, perpendicular lines/segments, and properties of polygons. This will lead to work with arcs and areas of sectors, as well as prepare learners for future work with 3-dimensional geometry and cross-sections.	Unit circles, their central angles, and reference angles will build on foundational skills learned in this cluster.

Vocabulary (key terms and definitions)

- chord
- inscribed angle
- inscribed circle (incircle)
- circumscribed angle
- circumscribed circle (circumcircle)
- tangent line
- central angle
- incenter
- circumcenter
- opposite angles
- tangent line
- center of a circle
- arc
- major arc
- minor arc
- semi-circle

Relevance, Explanations, and Examples:

Note that circles/polygons can each be circumscribed or inscribed.

Include items in which learners must use properties of circles to find arc measures needed for calculating angle measures.

G.C.4 (+) Since this is a (+) standard, this material could be used as an extension.

Achievement Level Descriptors

Cluster: Understand and apply theorems about circles.

Concepts and Procedures	***Level 1:*** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2: Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.

	Level 3: Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4: Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.