Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Functions	**Grade Level: Algebra 2**

A2.F.IF.A Cluster: Interpret Functions That Arise in Applications in Terms of the Context

Learners attain conceptual understanding of the key features of a function by focussing on graphs and tables of the related quantities.

****This is a MAJOR cluster.*** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.*

A2.F.IF.A.4(ii) For functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative;relative maximums and minimums; symmetries (including even, odd, or neither); end behavior; and periodicity. (Use Modeling)

A2.F.IF.A.5(ii) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. (Use Modeling)

A2.F.IF.A.6: Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph.

Aspects of Rigor for Students: (Conceptual, Procedural, and/or Application)

A2.F.IF.A.4(ii) ***For functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries (including even, odd, or neither); end behavior; and periodicity.*** **(Use Modeling)**

Conceptual Understanding	***Procedural Fluency***	***Application***
Learners understand and explain the key features of a graph. Students should be able to fluidly relate a function, a sketch of a graph (accurate key features) and a table.	Learners can identify key features of a graph (see key features listed in standard).	Learners can relate this cluster to real world situations. Examples: ● Cutting side lengths of a box to maximize volume ● Finding maximum profit for producing & selling a product ● Finding dimensions to maximize area

A2.F.IF.A.5 (ii) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem. (Use Modeling)

Conceptual Understanding	***Procedural Fluency***	***Application***
Learners understand what the domain of a function represents in terms of the function and what values are appropriate (i.e. discrete v. continuous values and other restrictions based on the type of function and the given context).	Learners are able to state the domain of a function from its graph and the given context.	Learners can relate this cluster to real world situations (with an emphasis on recognizing appropriate domain values). Examples: ● Cutting side lengths of a box to maximize volume ● Finding maximum profit for producing & selling a product ● Finding dimensions to maximize area

A2.F.IF.A.6: Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph.

Conceptual Understanding	***Procedural Fluency***	***Application***
Understand the average rate of change can be calculated for any function defined over a specified range. Understand the slope as an average rate of change. Understand the difference between absolute change and average rate of change for a nonlinear function.	Calculate the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a variety of types of functions. Interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.	Given a distance versus time graph or table, calculate the average rate of change and interpret it as the speed.

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
    - Students will be able to interchangeably use different representations of functions especially graphs, tables and symbols.
6. **Attend to precision.**
7. **Look for and make use of structure.**
    - Students will be able to find and interpret key features of a graph or table of a function. Key features include extreme values, end behavior, and intervals of increase and decrease. Students will be able to generalize key features of a given function and how those key features relate to other functions.
8. **Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
In Algebra 1, students learn key features (intercepts, min/max, rate of change, vertex, function open up/down) and domain of linear,	*This cluster transcends through the entire course, since it is discussed with every new function presented.*	*Subsequent math course build upon these key features and domain of relations (both functions and non-functions).*

quadratic, and exponential functions.		

Vocabulary (key terms and definitions)

- Absolute Maximum
- Absolute Minimum
- Decreasing
- Degree (even/odd)
- Domain
- End Behavior
- Increasing
- Period
- Relative Maximum
- Relative Minimum
- Symmetry (even, odd, or neither)
- X-intercept
- Y-intercept

Relevance, Explanations, and Examples:

Given the function:

$$f(x) = x^3 - 3x^2 - 4x + 12$$

Draw a sketch of the graph and describe and explain all key features of the graph without using graphing technology. Describe the domain of this function.

Example of a Sketch of a Graph:

$f(x) = x^3 - 3x^2 - 4x + 12$

$0 = x^3 - 3x^2 - 4x + 12$

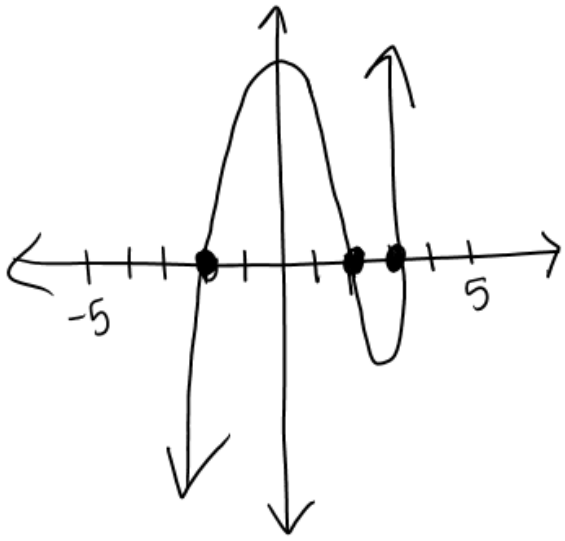
$0 = x^2(x-3) - 4(x-3)$

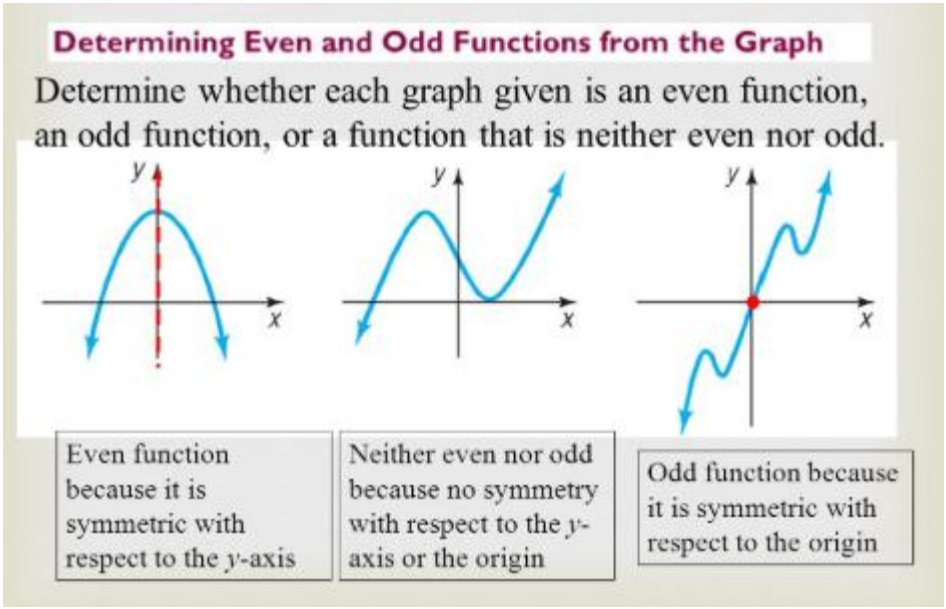
$0 = (x^2 - 4)(x-3)$

$0 = (x-2)(x+2)(x-3)$

$0 = x-2 \quad 0 = x+2 \quad 0 = x-3$

$2 = x \quad -2 = x \quad 3 = x$





Example of a contextual average rate of change:

The function given by $R(x) = 11.74x^{1/4}$ can be used to measure the maximum range $R(x)$, in miles, of a radar system with a peak power of x watts.

Find the rate at which the maximum range changes as peak power increases from 30,000 to 60,000 watts.

Achievement Level Descriptors	
Cluster: Interpret Functions That Arise in Applications in Terms of the Context	
Concepts and Procedures	***Level 1:*** Students should be able to interpret linear functions in context, and given the key features of a linear graph, they should be able to identify the appropriate graph.
	Level 2: Students should be able to interpret quadratic and other polynomial functions in two variables in context of the situation, and given the key features of a graph of a polynomial function, they should be able to identify the appropriate graph. They should be able to specify the average rate of change from an equation of a linear function and approximate it from a graph of a linear function.
	Level 3: Students should be able to graph various types of functions and interpret and relate key features, including range and domain, in familiar or scaffolded contexts. They should be able to specify the average rate of change of a function on a given domain from its equation or approximate the average rate of change of a function from its graph.
	Level 4: Students should be able to interpret complex key features such as holes, symmetries, and end behavior of graphs and functions in unfamiliar problems or contexts.