Unpacked South Dakota State Mathematics Standards

Purpose: *In order for students to have the best chance of success, standards, assessment, curriculum resources, and instruction must be aligned in focus, coherence, and rigor. Unpacked standards documents are intended to help align instruction to the focus, coherence, and rigor of the South Dakota State Mathematics Standards. The standards have been organized in clusters as they are not so much built from topics, but rather woven out of progressions. Not all content in a given grade is emphasized equally in the mathematics standards. Some clusters require greater emphasis than others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting standards will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade.*

Domain: Functions		**Grade Level: Algebra 2**
A2.F.BF.B Cluster: Build New Functions From Existing Functions *Students will describe relationships between two variables by creating new functions or combining existing functions.*		
****This is a SUPPORTING cluster.** *Students should spend the large majority of their time (65-85%) on the major work of the grade. Supporting work and, where appropriate, additional work should be connected to and engage students in the major work of the grade.* **A2.F.BF.A.3. (ii) Identify the effect on the graph of f(x) replaced with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.** **A2.F.BF.A.4. Find inverse functions.** **a. Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse.** **b. Verify by composition that one function is the inverse of another.** **c. Read values of an inverse function from a graph or a table, given that the function has an inverse.** **A2.F.BF.A.5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.**		
Aspects of Rigor for Students: (Conceptual, Procedural, and/or Application)		
A2.F.BF.A.3. (ii) Identify the effect on the graph of f(x) replaced with f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.		
Conceptual Understanding	***Procedural Fluency***	***Application***
Learners will understand and explain how the symbolic change to a function rule affects the graph of the original function. Learners will also be able to read and interpret a graph and write the corresponding function rule.	Learners will be able to describe transformation(s) resulting from a symbolic change to a function rule and from a graph.	
A2.F.BF.A.4. Find inverse functions. **a. Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse.** **b. Verify by composition that one function is the inverse of another.** **c. Read values of an inverse function from a graph or a table, given that the function has an inverse.**		

Conceptual Understanding	***Procedural Fluency***	***Application***
Learners will understand and explain how the function and the inverse function relate to each other both graphically and algebraically (tables and equations).	Learners will be able to: ● Determine inverse functions (both graphically and equations). ● Verify inverse functions using composition of functions $f(f^{-1}(x)) = x$.	

A2.F.BF.A.5. Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Conceptual Understanding	***Procedural Fluency***	***Application***
Learners will understand and explain how and why exponents and logarithms have an inverse relationship (both graphically and algebraically).	Learners will solve exponential and logarithmic equations algebraically (properties of logarithms and change of base rule are included with this cluster).	Learners will solve exponential and logarithmic equations in context Example: exponential growth/decay real world problems, Richter scale, etc.

Enacting the Mathematical Practices - Evidence of Students Engaging in the Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
   - Students will be able to see patterns and be able to create function rules from those patterns.
3. **Construct viable arguments and critique the reasoning of others.**
   - Students will explain the process they used to build a new function from existing functions.
4. **Model with mathematics.**
   - Students will be able to build functions using context.
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
   - When combining existing functions, students are using mathematical structures with which they are familiar.
8. **Look for and express regularity in repeated reasoning.**

Vertical and Horizontal Coherence and Learning Progressions

Previous Learning Connections	***Current Learning Connections***	***Future Learning Connections***
Students have performed operations (addition, subtraction, multiplication, and division) with expressions in previous math courses. Included, but not limited to: $(3x^2 + 7x - 1) + (7x - 6); (4x^2 - 3x) - (9x^2 - 2);$ $(8x - 7)(3x + 1);$ $\frac{15x^{7}y^{2}}{9y^{8}}$	*This cluster is an extension of transformations of parent functions. In addition, the mathematical process of simplifying is also extended to rational expressions.*	*Subsequent math courses build upon transformations of relations (both functions and non-functions) and inverse functions, both graphically and algebraically.*

In Algebra 1, students have learned how to compare functions to each other in terms of linear, quadratic and exponential. *Also, in previous math courses, students have learned how to substitute an expression into another (i.e. solving systems algebraically, substituting values in, etc)*		

Vocabulary (key terms and definitions)

- Composition
- Inverse Function

Relevance, Explanations, and Examples:

Notation for operations and simplifying functions:

$$f(x) + g(x) = (f + g)(x)$$
$$f(x) - g(x) = (f - g)(x)$$
$$f(x) \cdot g(x) = (fg)(x)$$
$$f(x) \div g(x) = \left(\frac{f}{g}\right)(x)$$
$$f(g(x)) = (f \circ g)(x)$$

Example of Using Composition of Functions to Verify Inverses Below.

Determine algebraically whether $f(x) = \frac{1}{2}x + \frac{3}{2}$ & $g(x) = 2x - 3$ are inverses

Find $f(g(x)) \Rightarrow$

$$f(x) = \frac{1}{2}x + \frac{3}{2}$$
$$f(g(x)) = \frac{1}{2}(g(x)) + \frac{3}{2}$$
$$f(2x-3) = \frac{1}{2}(2x-3) + \frac{3}{2}$$
$$f(2x-3) = x - \frac{3}{2} + \frac{3}{2}$$
$$f(2x-3) = x$$
$$f(g(x)) = x \quad \boxed{\checkmark}$$

Find $g(f(x)) \Rightarrow$

$$g(x) = 2x - 3$$
$$g(f(x)) = 2(f(x)) - 3$$
$$g(\tfrac{1}{2}x + \tfrac{3}{2}) = 2(\tfrac{1}{2}x + \tfrac{3}{2}) - 3$$
$$g(\tfrac{1}{2}x + \tfrac{3}{2}) = x + 3 - 3$$
$$g(\tfrac{1}{2}x + \tfrac{3}{2}) = x$$
$$g(f(x)) = x \quad \boxed{\checkmark}$$

Because $f(g(x)) = x = g(f(x))$, $f(x)$ and $g(x)$ are inverses.

Note that above, $f(f^{-1}(x)) = x$.was used in reference to verifying inverses instead of using f(g(x)) = x and g(f(x)) = x.

Achievement Level Descriptors	
Cluster: Building New Functions from Existing Functions	
Concepts and Procedures	***Level 1:*** Students should be able to base arguments on concrete referents such as objects, drawings, diagrams, and actions and identify obvious flawed arguments in familiar contexts.
	Level 2: Students should be able to find and identify the flaw in an argument by using examples or particular cases. Students should be able to break a familiar argument given in a highly scaffolded situation into cases to determine when the argument does or does not hold.

	Level 3: Students should be able to use stated assumptions, definitions, and previously established results and examples to test and support their reasoning or to identify, explain, and repair the flaw in an argument. Students should be able to break an argument into cases to determine when the argument does or does not hold.
	Level 4: Students should be able to use stated assumptions, definitions, and previously established results to support their reasoning or repair and explain the flaw in an argument. They should be able to construct a chain of logic to justify or refute a proposition or conjecture and to determine the conditions under which an argument does or does not apply.