Mathematics I Overview

The Real Number System	Mathematical Practices
A. Extend the properties of exponents to rational exponents	1. Make sense of problems and persevere in solving them.
Quantities	
A. Reason quantitatively and use units to solve problems	2. Reason abstractly and quantitatively.
Seeing Structure in Expressions	3. Construct viable arguments and critique the reasoning of others.
A. Interpret the structure of expressions **B.** Write expressions in equivalent forms to solve problems	4. Model with mathematics.
Creating Equations	
A. Create equations that describe numbers or relationships	5. Use appropriate tools strategically.
Reasoning with Equations and Inequalities	6. Attend to precision.
A. Understand solving equations as a process of reasoning and explain the reasoning **B.** Solve equations and inequalities in one variable **C.** Solve systems of equations **D.** Represent and solve equations and inequalities graphically	7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Interpreting Functions	
A. Understand the concept of a function and use function notation **B.** Interpret functions that arise in applications in terms of the context **C.** Analyze functions using different representations	
Building Functions	
A. Build a function that models a relationship between two quantities **B.** Build new functions from existing functions	
Linear, Quadratic and Exponential Models	
A. Construct and compare linear and exponential models and solve problems. **B.** Interpret expressions for functions in terms of the situation they model.	
Congruence	
A. Experiment with transformations in the plane **B.** Understand congruence in terms of rigid motions **D.** Make geometric constructions	
Expressing Geometric Properties with Equations	
B. Use coordinates to prove simple geometric theorems algebraically	
Interpreting Categorical and Quantitative Data	
A. Summarize, represent and interpret data on a single	

count or measurement variable. **B.** Summarize, represent and interpret data on two categorical and quantitative variables. **C.** Interpret linear models.	

Integrated Mathematics I Introduction

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. These standards are the baseline expectations of students completing this course. Individual school districts or teachers are welcome to expand on these standards as they see fit to meet the needs of their students. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

This course ties together the algebraic, geometric, and statistical ideas studied. Students will deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades.

In Mathematics I, students use quantities to model and analyze situations, interpret expressions, and create equations to describe situations. Students will learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

This course builds on earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities, and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

This course builds upon prior students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

In this course, students establish triangle congruence criteria, based on analyses of rigid motions and constructions. They solve problems about triangles, quadrilaterals, and other polygons. Students use a for quadrant coordinate system to verify geometric relationships, including properties of special triangles, quadrilaterals, and slopes of parallel and perpendicular lines.

Integrated Mathematics I Standards

The Real Number System   N.RN

A. Extend the properties of exponents to rational exponents.
1. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Quantities *   N.Q

A. Reason quantitatively and use units to solve problems.
1. Use unit analysis to understand and guide the process of solving multi-step problems; choose and interpret units consistently in formulas; and choose and interpret the scale and origin in graphs and data displays. *
2. Define appropriate quantities for the purpose of descriptive modeling. *
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. *

Seeing Structure in Expression   A.SSE

A. Interpret the structure of expressions.
1. (i) Interpret expressions that represent a quantity in terms of its context. *
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context.
2. (i) Recognize and use the structure of an expression to identify ways to rewrite it.

B. Write expressions in equivalent forms to solve problems.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *
    c. Use the properties of exponents to write equivalent expressions for exponential functions.

Creating Equations *   A.CED

A. Create equations that describe numbers or relationships.
1. Create equations and inequalities in one variable arising from situations in which linear, quadratic, and exponential functions are appropriate and use them to solve problems. *

Instructional Note:
Integrated Math I requires students to create equations and inequalities from situations in which linear and exponential functions are appropriate and use them to solve problems. Students are required to create equations and inequalities from situations in which quadratic functions are appropriate and use them to solve problems in Integrated Math II.

1. (i) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.*
2. (i) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.*
3. (i) Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations.*

Reasoning With Equations and Inequalities   A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

B. Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

C. Solve systems of equations.

5. Understand the principles of the elimination method.
6. Solve systems of linear equations exactly and approximately by graphing, focusing on pairs of linear equations in two variables.

D. Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. (i) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, quadratic and exponential.

<mark>Integrated Math I requires students to include cases where $f(x)$ and/or $g(x)$ are linear and exponential. Students are required to include cases where $f(x)$ and/or $g(x)$ are quadratic in Integrated Math II.</mark>

12. Graph a linear inequality (strict or inclusive) in two variables; graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions   F.IF

A. Understand the concept of a function and use functions notation.

1. Understand that a function maps each element of the domain to exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph off is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions, and interpret statements that use function notation in terms of a context.
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

B. Interpret functions that arise in applications in terms of the context.

4. (i) For functions, including linear, ~~quadratic~~, and exponential, that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing or decreasing, including using interval notation; maximums and minimums; symmetries.* *

<mark>Instructional Note:</mark>
<mark>Integrated Math I requires students to interpret key features of only linear and exponential functions. Students are required to interpret these features of quadratic functions in Integrated Math II.</mark>

5. (i) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem.*

6.   Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph. *

C.   Analyze functions using different representations.

7.   (i) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *
     a.   Graph linear, exponential, and quadratic functions and show intercepts, maxima, and minima.

Instructional Note:
Integrated Math I requires students to graph parent functions and transformations of only linear and exponential functions. Students are required to graph parent functions and transformations of quadratic functions in Integrated Math II.

8.   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
9.   Interpret expressions for exponential growth and decay.
10. (i) Compare properties of two functions (linear, quadratic and exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Instructional Note:

Integrated Math I requires students to compare properties of linear and exponential functions. Students are required to compare properties of quadratic functions in Integrated Math II.

Building Functions   F.BF

A.   Build a function that models a relationship between two quantities.

1.   Write a function (linear, quadratic, and exponential) that describes a relationship between two quantities. *
     a.   Determine an explicit expression, a recursive process, or steps for calculation from a context. *
     b.   Determine an explicit expression from a graph. *
     c.   Combine standard function types using arithmetic operations. *

Instructional Note:
Integrated Math I requires students to write only linear and exponential functions. Students are required to write quadratic functions in Integrated Math II.

2.   Write arithmetic and geometric sequences both recursively and with an explicit formula and use them to model situations. *

B.   Build new functions from existing functions.

3.   (i) Identify the effect on the graph of $f(x)$ (linear, exponential, quadratic) replaced with $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.

Instructional Note:
Integrated Math I requires students to identify effects of $f(x)$ for linear and exponential functions. Students are required to identify effects of $f(x)$ for quadratic functions in Integrated Math II.

Linear, Quadratic and Exponential Models *   F.LE

A.   Construct and compare linear and exponential models to solve problems.

1.   Distinguish between situations that can be modeled with linear functions and with exponential functions.
     a.   Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
     b.   Recognize situations in which one quantity changes at constant rate per unit interval relative to another.

      c.   Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *
3. Recognize, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. *

B. Interpret expressions for functions in terms of the situation they model
5. Interpret the parameters in a linear or exponential function in terms of a context. *

Congruence   G-CO

A. Experiment with transformations in the plane.
1. State and apply precise definitions of angle, circle, perpendicular, parallel, ray, line segment, and distance based on the undefined notions of point, line, and plane.
2. Represent transformations in the plane. (e.g., using transparencies and/or geometry software);
      a.   Describe transformations as functions that take points in the plane as inputs and give other points as outputs.
      b.   Compare transformations that preserve distance and angle to those that do not (e.g., translation versus dilation).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and/or reflections that map the figure onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, (e.g., using graph paper, tracing paper, or geometry software). Specify a sequence of transformations that will map a given figure onto another.

B. Understand congruence in terms of rigid motions.
6. Use geometric descriptions of rigid motions to transform figures.
      a.   Predict the effect of a given rigid motion on a given figure.
      b.   Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

D. Make geometric constructions.
12. Perform geometric constructions with a compass and straightedge. including copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines/segments, constructing a line parallel to a given line through a point not on the line.
13. Construct an equilateral triangle, a square, and a regular hexagon.

Expressing Geometric Properties with Equations   G-GPE

B. Use coordinates to prove geometric relationships algebraically.

4. Use coordinates to prove geometric relationships algebraically. For example, determine whether a figure defined by four given points in the coordinate plane is a rectangle; determine whether the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

5. Define and use the slope criteria for parallel and perpendicular lines. (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. *

Interpreting Categorical and Quantitative Data   S.ID

A. **Summarize, represent and interpret data on a single count or measurement variable.**
   1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
   2. Use statistics appropriate to the shape and context of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
   3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
B. **Summarize, represent and interpret data on two categorical and quantitative variables.**
   5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
   6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
      a. Determine the function (linear, quadratic, or exponential model) that best fits a set of data and use that function fitted to data to solve problems within context.

Instructional Note:

==Integrated Math I requires students to determine the function for linear and exponential models. Students are required to determine the function of quadratic models in Integrated Math II.==
      b. Informally and using technology assess the fit of a function by plotting and analyzing residuals.
      c. Fit a linear function for a scatter plot that suggests a linear association.
C. **Interpret linear models.**
   7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
   8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
   9. Distinguish between correlation and causation

Integrated Mathematics II Overview

The Real Number System	Mathematical Practices
A. Extend the properties of exponents to rational exponents **B.** Use properties of rational and irrational numbers	1. Make sense of problems and persevere in solving them.
The Complex Number System	2. Reason abstractly and quantitatively.
A. Perform arithmetic operations with complex numbers **C.** Use complex numbers in polynomials identities and equations.	3. Construct viable arguments and critique the reasoning of others.
Seeing Structure in Expressions	4. Model with mathematics.
A. Interpret the structure of expressions **B.** Write expressions in equivalent forms to solve problems	5. Use appropriate tools strategically.
Arithmetic with Polynomials and Rational Expressions	6. Attend to precision.
A. Perform arithmetic operations on polynomials	7. Look for and make use of structure.
Creating Equations	8. Look for and express regularity in repeated reasoning.
A. Create equations that describe numbers or relationships	
Reasoning with Equations and Inequalities	
B. Solve equations and inequalities in one variable **C.** Solve systems of equations **D.** Represent and solve equations and inequalities graphically	
Interpreting Functions	
B. Interpret functions that arise in applications in terms of the context **C.** Analyze functions using different representations	
Building Functions	
A. Build a function that models a relationship between two quantities **B.** Build new functions from existing functions	
Linear, Quadratic and Exponential Models	
A. Construct and compare linear and exponential models and solve problems.	
Trigonometric Functions	
C. Prove and apply trigonometric identities.	
Congruence	
C. Prove geometric theorems	
Similarity, Right Triangles and Trigonometry	
A. Understand similarity in terms of similarity transformations **B.** Prove theorems involving similarity **C.** Define trigonometric ratios and solve problems involving right triangles	
Circles	
A. Understand and apply theorems about circles	

B. Find arc lengths and areas of sectors of circles
Expressing Geometric Properties with Equations
A. Translate between the geometric description and the equation for a circle
B. Use coordinates to prove simple geometric theorems algebraically
Geometric Measurement and Dimension
A. Explain volume and surface area formulas and use them to solve problems
Modeling with Geometry
A. Apply geometric concepts in modeling situations
Conditions Probability and the Rules of Probability
A. Understand independence and conditional probability and use them to interpret data
B. Use the rules of probability to compute probabilities of compound events in a uniform probability model
Interpreting Categorical and Quantitative Data
B. Summarize, represent and interpret data on two categorical and quantitative variables.

Integrated Math 2 Introduction

The focus of Mathematics II is to extend the mathematics that students learned in Integrated Mathematics I, with the understanding that this course teachers all mathematics domains, Numbers and Quantity, Algebra, Functions, Geometry and Statistics. These standards are the baseline expectation for students completing this course. Individual school districts or teachers are welcome to expand on these standards as they see fit to meet the needs of their students. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

In Mathematics II, students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions the number system must be extended so that solutions exist, analogous to the way in which extending the whole numbers to the negative numbers allows x+1 = 0 to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

In this course, Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from among these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. When quadratic equations do not have real solutions, students learn that that the graph of the related quadratic function does not cross the horizontal axis. They expand their experience with functions to include more specialized functions. In Mathematics II, Students focus on the structure of expressions, rewriting expressions to clarify and reveal aspects of the relationship they represent. They create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

This course builds on probability concepts that began in the middle grades, students use the languages of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions.

In this course Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. Students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They explore a variety of formats for writing proofs. Students also prove basic theorems about circles, such as a tangent line is perpendicular to a radius, inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center. Students develop informal arguments justifying common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Math II Standards

The Real Number System N.RN

A. Extend the properties of exponents to rational exponents.
1. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

B. Extend the properties of rational and irrational numbers.
3. Explain why the sum or product of two rational numbers is rational; the sum of a rational and an irrational number is irrational; and the product of a nonzero rational and an irrational number is irrational.

The Complex Number System N.CN

A. Perform arithmetic operations with complex numbers.
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ where a and b are real numbers.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

C. Use complex numbers in polynomials identities and equations.
7. Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions A.SSE

A. Interpret the structure of expressions.
1. (i) Interpret expressions that represent a quantity in terms of its context. *
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret underline complicated expressions by viewing one or more of their parts as a single entity in context.
2. (i) Recognize and use the structure of an expression to identify ways to rewrite it.

B. Write expressions in equivalent forms to solve problems.
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to write equivalent expressions for exponential functions.

Arithmetic with Polynomials and Rational Expressions A.APR

A. Perform arithmetic operations on polynomials.
1. Understand that polynomials form a system closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Creating Equations *   A.CED

A. Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable arising from situations in which linear, quadratic, and exponential functions are appropriate and use them to solve problems. *

Instructional Note:
Integrated Math II requires students to create equations and inequalities from situations in which quadratic functions are appropriate and use them to solve problems. Students are required to create equations and inequalities from situations in which linear and exponential functions are appropriate and use them to solve problems in Integrated Math I.

2. (ii) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *

3. (i) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *

4. (i) Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *

Reasoning with Equations and Inequalities   A.REI

B. Solve equations and inequalities in one variable.

4. (i) Solve quadratic equations in one variable.

   a.   Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)^2 = q$ that has the same solutions.

   b.   Derive the quadratic formula from this form completing the square.

   c.   Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.

   d.   Select, justify and apply appropriate methods to solve quadratic equations in one variable. Recognize complex solutions and write them as *a +/- bi* for real numbers *a* and *b*.

C. Solve systems of equations.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

D. Represent and solve equations and inequalities graphically.

11. (i) Explain why the *x*-coordinates of the points where the graphs of the equations *y = f(x)* and *y = g(x)* intersect are the solutions of the equation *f(x) = g(x)*; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f(x)* and/or *g(x)* are linear, quadratic and exponential.

Integrated Math II requires students to include cases where *f(x)* and/or *g(x)* are quadratic. Students are required to include cases where *f(x)* and/or *g(x)* are linear and exponential Integrated Math I.

Interpreting Functions   F.IF

B.   Interpret functions that arise in applications in terms of the context.

4.   (i) For functions, including linear, quadratic, and exponential, that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing or decreasing, including using interval notation; maximums and minimums; symmetries.* *

==**Instructional Note:**==

==**Integrated Math II requires students to interpret key features of only quadratic functions. Students are required to interpret these features of linear and exponential functions in Integrated Math I.**==

5.   (i) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem.*

6.   Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph. *

C.   Analyze functions using different representations.

7.   (i) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *

   a.   Graph linear, exponential, and quadratic functions and show intercepts, maxima, and minima.

==**Instructional Note:**==

==**Integrated Math II requires students to graph parent functions and transformations of only quadratic functions. Students are required to graph parent functions and transformations of linear and exponential functions in Integrated Math I.**==

   b.   Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

8.   Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a.   Use the process of graphing, factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of context.

9.   (i) Compare properties of <u>two functions</u> (linear, quadratic and exponential) each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

==**Instructional Note:**==

==**Integrated Math II requires students to compare properties of only quadratic functions. Students are required to compare properties of linear and exponential functions in Integrated Math I**==

Building Functions   F.BF

A.   Build a function that models a relationship between two quantities.

1.   Write a function (linear, quadratic, and exponential) that describes a relationship between two quantities. *

   a.   Determine an explicit expression, a recursive process, or steps for calculation from a context. *

   b.   Determine an explicit expression from a graph. *

   c.   Combine standard function types using arithmetic operations. *

==**Instructional Note:**==

==**Integrated Math II requires students to write only quadratic functions. Students are required to write linear and exponential functions in Integrated Math I.**==

B. Build new functions from existing functions.

3. (i) Identify the effect on the graph of *f(x)* (linear, exponential, quadratic) replaced with *f(x) + k, k f(x), f(kx),* and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.

Instructional Note:
Integrated Math II requires students to identify effects of *f(x)* for quadratic functions. Students are required to identify effects of *f(x)* for linear and exponential functions in Integrated Math I.

Linear, Quadratic and Exponential Models * F.LE

A. Construct and compare linear and exponential models to solve problems.

3. Recognize, using graphs and tables, that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. *

Trigonometric Functions F.TF

C. Prove and apply trigonometric identities.
8. Prove the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$ and use it to calculate trigonometric ratios.

Congruence G-CO

C. Prove geometric theorems

11. Prove theorems about lines and angles. Theorems must include but not limited to: vertical angles are congruent; when a transversal intersects parallel lines, alternate interior angles are congruent and same side interior angles are supplementary (using corresponding angles postulate); points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints.

12. Prove congruence theorems about triangles. Theorems must include but not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the mid segment of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

13. Prove theorems about parallelograms. Theorems must include but not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Similarity, Right Triangles and Trigonometry G-SRT

A. Understand similarity in terms of similarity transformations.
1. Verify experimentally and apply the properties of dilations as determined by a center and a scale factor.
2. Determine whether figures are similar, using the definition of similarity and using similarity transformations.
3. Use the properties of similarity transformations to establish similarity theorems. Theorems must include AA, SAS, and SSS.

B. Prove theorems involving similarity.
4. Prove theorems about triangles involving similarity. Theorems must include but not limited to: a line parallel to one side of a triangle divides the other two proportionally, and its converse; the Pythagorean Theorem proved using triangle similarity.

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

C. Define trigonometric ratios and solve problems involving right triangles.

6. Define, using similarity, that side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios (sine, cosine, and tangent) for acute angles.

7. Explain and use the relationship between the sine and cosine of complementary angles.

8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. *

Circles   G-C

A. Understand and apply theorems about circles.

1. Prove that all circles are similar.

2. Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords.

3. Construct, using a compass and straight edge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

B. Find arc lengths and areas of sectors of circles.

4. Derive using similarity the length of the arc intercepted by an angle is proportional to the radius.

   a. Define the radian measure of the angle as the constant of proportionality;

   b. Derive and apply the formula for the area of a sector.

Expressing Geometric Properties with Equations   G-GPE

A. Translate between the geometric description and the equation for a conic section.

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

B. Use coordinates to prove geometric relationships algebraically.

4. Use coordinates to prove geometric relationships algebraically. For example, determine whether a figure defined by four given points in the coordinate plane is a rectangle; determine whether the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.

6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. e.g. Determine the point(s) that divide the segment with endpoints of (-4, 7) and (6, 3) into the ratio 2:3

Geometric Measurement and Dimension   G-GMD

A. Explain volume and surface area formulas and use them to solve problems.

1. Give an informal argument for the formulas for the volume of a cylinder, pyramid, sphere, and cone. Use dissection arguments, and informal limit arguments.

3. Know and apply volume and surface area formulas for cylinders, pyramids, cones, and spheres for composite figures to solve problems. *

Modeling with Geometry   G-MG

A. Applying geometric concepts in modeling situations.

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *

2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *
3. Apply geometric concepts to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *

Statistics and Probability- Conditions Probability and Rules of Probability   S-CP

A. **Understand independence and conditional probability and use them to interpret data.**
   1. Describe events as subsets of a sample space or as unions, intersections, or complements of other events
   2. Determine whether two events *A* and *B* are independent.
   3. Determine conditional probabilities and interpret independence by analyzing conditional probability
   4. Construct and interpret two-way frequency tables of data. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
   5. Recognize and explain the concepts of conditional probability and independence in everyday language and situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

B. **Use the rules of probability to compute probabilities of compound events in a uniform probability model**
   6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the result.
   7. Apply the Addition Rule, P(A or B), and interpret the result.
   8. Apply the general Multiplication Rule, P(A and B), and interpret the result.

Interpreting Categorical and Quantitative Data   S.ID

B. **Summarize, represent and interpret data on two categorical and quantitative variables.**
   6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
      a. Determine the function (linear, quadratic, or exponential model) that best fits a set of data and use that function fitted to data to solve problems within context.

Instructional Note:
Integrated Math II requires students to determine the function quadratic models. Students are required to determine the function of linear and exponential models in Integrated Math I.
      b. Informally and using technology assess the fit of a function by plotting and analyzing residuals.

Math 3

The Real Number System	Mathematical Practices
A. Extend the properties of exponents to rational expressions	1. Make sense of problems and persevere in solving them.
The Complex Number System	
C. Use complex numbers in polynomials identities and equations.	2. Reason abstractly and quantitatively.
Seeing Structure in Expressions	3. Construct viable arguments and critique the reasoning of others.
A. Interpret the structure of expressions	
B. Write expressions in equivalent forms to solve problems	4. Model with mathematics.
Arithmetic with Polynomials and Rational Expressions	
A. Perform arithmetic operations on polynomials	5. Use appropriate tools strategically.
B. Understand the relationship between zeros and factors of polynomials	6. Attend to precision.
D. Rewrite rational expressions	
Creating Equations	7. Look for and make use of structure.
A. Create equations that describe numbers or relationships	Look for and express regularity in repeated reasoning.
Reasoning with Equations and Inequalities	
A. Understand solving equations as a process of reasoning and explain the reasoning	
B. Represent and solve equations and inequalities graphically	
Interpreting Functions	
B. Interpret functions that arise in applications in terms of the context.	
C. Analyze functions using different representations	
Building Functions	
A. Build a function that models a relationship between two quantities	
B. Build new functions from existing functions	
Linear, Quadratic and Exponential Models	
A. Construct and compare linear and exponential models and solve problems.	
Trigonometric Functions	
A. Extend the domain of trigonometric functions using the unit circle.	
B. Model periodic phenomena with trigonometric functions.	
C. Prove and apply trigonometric identities.	
Congruence	
C. Prove geometric theorems	
Similarity, Right Triangles and Trigonometry	
B. Prove theorems involving similarity	

Circles	
A. Understand and apply theorems about circles	
B. Find arc lengths and areas of sectors of circles	
Geometric Measurement and Dimension	
A. Explain volume and surface area formulas and use them to solve problems	
B. Visualize relationships between two-dimensional and three-dimensional objects	
Modeling with Geometry	
A. Apply geometric concepts in modeling situations	
Interpreting Categorical and Quantitative Data	
A. Summarize, represent and interpret data on a single count or measurement variable.	
B. Summarize, represent and interpret data on two categorical and quantitative variables.	
Making Inferences and Justifying Conclusions	
A. Understand and evaluate random processes underlying statistical experiments.	
B. Make inferences and justify conclusions from sample surveys, experiments and observations.	

Integrated Math 3 Introduction

It is in Mathematics III that students pull together and apply the accumulation of learning that they have from their previous courses. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational, and radical functions. They expand their study of right triangle trigonometry to include general triangles. And, finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems. These standards are the baseline expectation for students completing this course. Individual school districts or teachers are welcome to expand on these standard as they see fit to meet the needs of their students. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

In Mathematics 3, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

This course develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial.

In this course, students build on their understanding of right triangle trigonometry to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena. Students also build on their understanding of geometric proof. Students will formally prove theorems involving lines, angles, triangles, and other polygons.

In this course, students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" is at the heart of this unit. The narrative discussion and diagram of the modeling cycle should be considered when knowledge of functions, statistics, and geometry is applied in a modeling context.

Math 3 Standards

The Real Number System   N.RN

A. Extend the properties of exponents to rational exponents.
1. Explain how the definition of rational exponents follows from extending the properties of integer exponents, allowing for a notation for radicals in terms of rational exponents.
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

The Complex Number System   N.CN

C. Use complex numbers in polynomials identities and equations.
7. Solve quadratic equations with real coefficients that have complex solutions.

Seeing Structure in Expressions

A. Interpret the structure of expressions.
1. (ii) Interpret expressions that represent a quantity in terms of its context. *
    a. Interpret parts of an expression, such as terms, factors, and coefficients.
    b. Interpret complicated expressions by viewing one or more of their parts as a single entity in context.
2. (ii) Recognize and use the structure of an expression to identify ways to rewrite it.

B. Write expressions in equivalent forms to solve problems
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. *

Arithmetic with Polynomials and Rational Expressions   A.APR

A. Perform arithmetic operations on polynomials.
1. Understand that polynomials form a system closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

B. Understand the relationship between zeros and factors of polynomials
2. Know and apply the Remainder Theorem.
3. Identify zeros of polynomials by factoring.
    a. When suitable factorizations are available, use the zeros to construct a rough graph of the related function.
    b. When given a graph, use the zeros to construct a possible factorization of a polynomial.

D. Rewrite rational expressions.
6. Rewrite simple rational expressions in different forms; using inspection, synthetic division, long division, box method or, for the more complicated examples, a computer algebra system.

Creating Equations *   A.CED

A. Create equations that describe numbers or relationships.
1. Create equations and inequalities in one variable and use them to solve problems. *
2. (ii) Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *
3. (ii) Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *

4. (ii) Rewrite formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *

Reasoning with Equations and Inequalities — A.REI

A. Understand solving equations as a process of reasoning and explain the reasoning.

    2. Solve rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. *Rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2. Radical functions are limited to square roots or cube roots of at most quadratic polynomials.*

D. Represent and solve equations and inequalities graphically.

    11. (ii) Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, including but not limited to using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. *

Interpreting Functions — F.IF

B. Interpret functions that arise in applications in terms of the context.

    4. (ii) For functions that model a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries (including even, odd, or neither); end behavior; and periodicity.* *

    5. (ii) Relate the domain of a function to its graph and find an appropriate domain in the context of the problem.*

    6. Calculate and interpret the average rate of change of a function, both symbolically and from a table over a specified interval. Estimate the rate of change from a graph. *

C. Analyze functions using different representations.

    7. (ii) Graph parent functions and their transformations expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *

        b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

        c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

        d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.

        e. Graph logarithmic functions, showing intercepts and end behavior.

        f. Graph trigonometric functions (sine and cosine), showing period, midline, and amplitude.

    9. (ii) Compare properties of <u>two functions</u> each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Building Functions — F.BF

A. Build a function that models a relationship between two quantities.

    1. (ii) Write a function that describes a relationship between two quantities. *

        a. Determine an explicit expression, a recursive process, or steps for calculation from a context. *

        b. Determine an explicit expression from a graph. *

        c. Combine standard function types using arithmetic operations. *

        d.   Compose functions *

B. **Build new functions from existing functions.**

    3.   (ii) Identify the effect on the graph of *f(x)* replaced with *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with contrasting cases and illustrate an explanation of the effects on the graph using technology.

    4.   Find inverse functions.

        a.   Solve an equation for the independent variable of a function f that has an inverse function and write an expression for the inverse.

        b.   Verify by composition that one function is the inverse of another.

        c.   Read values of an inverse function from a graph or a table, given that the function has an inverse.

    5.   (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic and Exponential Models *   F.LE

A. **Construct and compare linear and exponential models to solve problems.**

    4.   For exponential models, express as a logarithm the solution to ab(ct) = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. *

Trigonometric Functions   F.TF

A. **Extend the domain of trigonometric functions using the unit circle.**

    1.   Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

    2.   Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions (sine and cosine) to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

B. **Model periodic phenomena with trigonometric functions.**

    5.   Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline. *

C. **Prove and apply trigonometric identities.**

    8.   Prove the Pythagorean identity $\sin^2(A) + \cos^2(A) = 1$ and use it to calculate trigonometric ratios.

Congruence   G-CO

C. **Prove geometric theorems**

    11. Prove theorems about lines and angles. Theorems must include but not limited to: vertical angles are congruent; when a transversal intersects parallel lines, alternate interior angles are congruent and same side interior angles are supplementary (using corresponding angles postulate); points on a perpendicular bisector of a line segment are equidistant from the segment's endpoints.

    12. Prove congruence theorems about triangles. Theorems must include but not limited to: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the mid segment of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

    13. Prove theorems about parallelograms. Theorems must include but not limited to: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

Similarity, Right Triangles and Trigonometry   G-SRT

B. **Prove theorems involving similarity.**
4. Prove theorems about triangles involving similarity. Theorems must include but not limited to: a line parallel to one side of a triangle divides the other two proportionally, and its converse; the Pythagorean Theorem proved using triangle similarity.
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Circles   G-C

A. **Understand and apply theorems about circles.**
2. Identify and describe relationships among central angles, inscribed angles, circumscribed angles, radii, and chords.
3. Construct, using a compass and straight edge, the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.
B. **Find arc lengths and areas of sectors of circles.**
5. Derive using similarity the length of the arc intercepted by an angle is proportional to the radius.
    c. Define the radian measure of the angle as the constant of proportionality;
    d. Derive and apply the formula for the area of a sector.

Geometric Measurement and Dimension   G-GMD

A. **Explain volume and surface area formulas and use them to solve problems.**
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures
B. **Visualize relationships between two-dimensional and three- dimensional objects.**
4. Identify two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry   G-MG

A. **Applying geometric concepts in modeling situations.**
1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). *
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). *
3. Apply geometric concepts to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). *

Interpreting Categorical and Quantitative Data   S.ID

A. **Summarize, represent and interpret data on a single count or measurement variable.**
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

B. Summarize, represent and interpret data on two categorical and quantitative variables.
6.   Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
a.   Determine the function (linear, quadratic, or exponential model) that best fits a set of data and use that function fitted to data to solve problems within context.
b.   Informally and using technology assess the fit of a function by plotting and analyzing residuals.

Making Inferences and Justifying Conclusions   S.IC

A. Understand and evaluate random processes underlying statistical experiments.
1.   Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2.   Determine whether a specified model is consistent with results from a given data-generating process.

B. Make interferences and justify conclusions from sample surveys, experiments and observational studies.
3.   Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4.   Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5.   Use data from a randomized experiment to compare two treatment groups; use simulations to decide if differences between parameters are significant.
6.   Evaluate reports based on data.