

Direct and Indirect Network Effects: Are They Equivalent?*

Matthew T. Clements[†]

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Abstract

Network effects may be either direct or indirect. While many analyses conflate the two, I show that the ways in which direct and indirect effects influence technological standardization are quite different. Some parameter changes have opposite effects in the two models, and some factors which are irrelevant under direct effects are central under indirect effects. Compatibility in particular has a different interpretation and more subtle implications for standardization in the indirect model.

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[†]University of Texas at Austin and University of British Columbia; clements@eco.utexas.edu.

1 Introduction

A telephone becomes more valuable to an individual as the total number of telephone users increases. This is a direct network effect. A DVD player becomes more valuable as the variety of available DVDs increases, and this variety increases as the total number of DVD users increases. This is an indirect network effect. Network effects have generally been modeled in a direct sense—individual utility increases with the total number of users—even when the effect is thought to operate in an indirect sense, through a complementary good. In this paper, I explicitly compare the implications of direct network effects with those of indirect effects, and thus explore to what extent a direct effects model can be viewed as a reduced form of an indirect effects model. I examine the propensity of a market to tip toward a single standard in some characteristic, and how this relates to model parameters and welfare. A greater mass of consumers makes standardization more likely under direct effects but less likely under indirect effects. Under direct effects, greater compatibility always hinders standardization; under indirect effects, compatibility can decrease the tendency toward standardization, but it can have the opposite effect. The only possible inefficiency under direct effects is underprovision of standardization; under indirect effects, the only possible inefficiency is overprovision of standardization. Thus, a model of direct network effects is inadequate in analyzing a market in which network effects are in fact indirect.

Models of direct network effects have been used to answer several kinds of questions. Katz and Shapiro (1985), Farrell and Saloner (1986b, 1988, 1992), and Economides and Flyer (1997) examine the effects of compatibility on competition and incentives for standardization. Farrell and Saloner (1985, 1986a) and Katz and Shapiro (1986) consider the implications of network effects and compatibility for technology adoption and R&D investment. Kende (1998) and others consider complementary

goods, but not as the source of a network effect: Direct network effects in one market affect firms' strategies in a complementary goods market. Matutes and Regibeau (1988) consider a market for systems of complementary goods, but the availability of variety does not depend on the number of consumers of either component; there is no network effect, in either a direct or an indirect sense.

There has been a very small body of work dealing specifically with indirect network effects. Chou and Shy (1990) find that consumers behave as if there are network externalities when there are increasing returns in the production of complementary products. Church and Gandal (1992) examine the incentives for standardization in a market for hardware and software, where software firms provide for one of two hardware technologies, and consumers buy one of these technologies. This paper contributes to the literature not only through comparison of direct and indirect effects, but also through enrichment of previous models of indirect effects. Whereas most of the literature has considered different technologies to be completely incompatible, I consider a continuum of compatibility. Farrell and Saloner (1992) consider partial compatibility, but only in a setting of direct network effects. Chou and Shy (1993) consider partial compatibility in a complementary goods context, but in a restricted fashion.¹ Apart from the inclusion of partial compatibility, the indirect portion of the present model is similar to that of Church and Gandal (1992), but with some different implications.²

The model, described in Section 2, incorporates both direct and indirect effects. I find the implications of direct effects in Section 3 and the implications of indirect effects in Section 4. I consider welfare issues in Section 5. Section 6 concludes.

¹In Chou and Shy (1993), partial compatibility between hardware and software means full compatibility between the hardware and some subset of all the software. In this formulation, increasing compatibility always leads to standardization.

²Software prices in Church and Gandal's model do not depend on the number of firms in the industry, in contrast to the present model. The two models imply opposite results regarding the relation of the number of software firms to standardization.

2 The model

I consider any two complementary goods to be “hardware” and “software.”³ Consumers value being part of a large network, i.e., using a technology that many other consumers also use. This is the direct network effect. Consumers also value a hardware technology for which there is a wide variety of software available, and more software firms associate with a hardware technology if more consumers use it. This is the indirect network effect. The hardware technologies are competitively supplied,⁴ whereas the software market is oligopolistic. The timing is as follows:

Stage 1: Hardware is introduced and competitively priced;

Stage 2: Software firms enter and choose a platform;

Stage 3: Consumers buy hardware;

Stage 4: Software firms set price and consumers buy software.

Although software is typically introduced more frequently than hardware, consumers generally have an idea of how much software will be available before buying hardware. Consumers would be particularly averse to buying hardware without knowing that some software will be provided for it. Thus software firms must commit to a platform before any hardware is sold.

The two hardware technologies are denoted X and Y . Hardware is horizontally differentiated in a product space of unit length: X is located at 0 and Y at 1. Each is produced (and priced) at constant marginal cost c^h . There are many potential firms that can provide software products, incurring a fixed cost F and marginal cost c^s . Each software product is designed primarily for one hardware platform; it may also

³For example, cars may be the hardware and service the software. Hardware is generally, but not necessarily, the more durable of the two.

⁴E.g., there may be a large number of firms supplying two non-proprietary hardware technologies. If there were one provider of each hardware technology, the qualitative results would be the same (but with higher hardware prices). Clements (2003) considers the incentives of hardware firms for location and compatibility in an oligopolistic hardware market.

have some value when used with the other platform. The price of a software variety intended for use with hardware X , produced by a “type- X ” firm, has price p^{sx} , and similarly for Y . Software firms are Bertrand competitors. In equilibrium, there are m type- X firms and n type- Y firms, where m and n are determined by zero-profit conditions. In the software pricing subgame, I consider only the symmetric Nash equilibrium, in which the prices of each variety of software for a given platform are the same; this equilibrium always exists and is unique. Software firms are equally distributed around the unit circle.⁵

Consumers are uniformly distributed on the unit line with respect to their preference for hardware, and uniformly distributed around the unit circle with respect to preference for software. The location of a consumer with respect to hardware is $a \in [0, 1]$. The total mass of consumers is A . The respective unit travel costs associated with hardware and software, reflecting the disutility of using a non-ideal product, are t^h and t^s .⁶ These can also be thought of as degrees of differentiation, or value of variety, in hardware and software.⁷

Consumers first decide which hardware they will buy, and then which variety of software. Each consumer buys one unit of each. Consumers consider the expected distance to the nearest software variety when making their hardware choice.⁸ A

⁵There is no correlation between the locations of firms of different types. Furthermore, the type- X firms do not know the locations of the type- Y firms, and vice versa. With this assumption, it is always possible to have symmetric pricing among software firms. The assumption of equal spacing between software firms is purely for analytical convenience. All that is necessary to obtain the results below is that adding a software firm to a platform increases price competition and decreases consumers’ expected distance to the nearest software variety.

⁶The results do not depend on the particular form of the travel costs; for example, linear or quadratic travel costs would lead to the same results.

⁷Differentiation is crucial for both hardware and software. Without software differentiation, there would be no indirect network effect. Without hardware differentiation, there would always be standardization upon a single hardware technology; non-standardization would never be privately or socially optimal.

⁸As in Katz and Shapiro (1985), I do not model the process through which consumers form expectations, but I assume that their expectations are fulfilled in equilibrium.

network with a greater variety of software is more valuable to an individual because the expected distance to that individual's most preferred variety is shorter.⁹ The total value of a hardware-software pair, disregarding prices and travel costs, is U_0 . Consumers have a reservation utility of zero, and $U_0 > c^h + t^h + c^s + t^s$; this guarantees that consumers always buy hardware and software.

For both hardware and software, the degree of compatibility is the extent to which consumers in one network benefit from the existence of the other network. Compatibility for hardware is denoted $\beta^h \in [0, 1]$, expressing the extent to which consumers of one hardware technology derive value from the existence of consumers using the other technology. If $\beta^h = 1$, a consumer using hardware X derives as much benefit from another consumer of X as from a consumer of Y . Compatibility for software is denoted $\beta^s \in [0, 1]$. For a consumer who has bought hardware X (a “type- X ” consumer), the travel cost for a variety of software X will be $d^x t^s$, where d^x is the distance to the nearest type- X software firm; but the travel cost for a variety of software Y will be $d^y t^s / \beta^s$, where d^y is the distance to the nearest type- Y software firm. No type- X consumer will buy a variety of type- Y software unless there is some compatibility, because the travel cost under incompatibility is infinite. When there is some degree of compatibility, a type- X consumer will buy a variety of type- Y software if, after buying hardware, the consumer discovers that some variety of software Y is much closer than the nearest variety of software X .

Let a^* be the location of the consumer indifferent between the two hardware types, and thus the proportion of consumers who buy hardware X . The strength of the direct network effect—the marginal value to a consumer of another user of

⁹Although the assumption of one unit of software per consumer is somewhat unrealistic for some markets, it captures the essential feature that consumers are better off when software variety is greater. This is operationally no different from a model in which consumers buy all available software varieties.

the same hardware technology—is given by η .¹⁰ A consumer located at a who buys hardware X and some variety of software X has utility of $U^{xx} = U_0 - c^h - at^h - p^{sx} - d^{xt^s} + A\eta [a^* + \beta^h (1 - a^*)]$. If the same consumer buys hardware X and some variety of software Y , utility is $U^{xy} = U_0 - c^h - at^h - p^{sy} - d^{yt^s}/\beta^s + A\eta [a^* + \beta^h (1 - a^*)]$. There are similar expressions for U^{yy} and U^{yx} .

Consumers can coordinate to a Pareto optimum. This assumption, which appears elsewhere in the network effects literature,¹¹ eliminates equilibria that are purely coordination problems.

In the following sections, I consider direct and indirect effects each separately. Although the model allows for both effects to be present, it is easiest to see the implications of each when they are isolated. For both kinds of effects, I consider the possibility that market participants will standardize on one technology. Consumer standardization occurs when all consumers buy the same hardware type; software standardization occurs when all software firms provide for the same hardware type. The two types of standardization occur together in some cases but not all.

3 Direct effects

In this section, I assume that $\beta^s = 1$, which implies that there are effectively no indirect network effects and the software market can be ignored.¹² Whether one or both networks will exist in equilibrium depends on the strength of the network effect

¹⁰The network effect exhibits constant returns to scale. The results below would also hold if the network benefit function exhibits decreasing returns, as long as returns are not too sharply decreasing.

¹¹See, for example, Katz and Shapiro (1986).

¹²With the exclusion of the software market, the model is similar to that of Farrell and Saloner (1986b), and results (i) and (ii) in the corollary below correspond directly to results of theirs. However, Farrell and Saloner have two types of consumer rather than a continuum, and they have no parameters for compatibility or population size. They do discuss the possibility of a compatibility equilibrium, but their definition of compatibility is the same as my definition of standardization.

relative to the travel cost, the size of the market, and the degree of compatibility between the two hardware types.

Proposition 1 *Assume $\beta^s = 1$. When $t^h < \frac{A\eta(1-\beta^h)}{2}$, two equilibria exist: $a^* = 1$ (all consumers choose X) and $a^* = 0$ (all consumers choose Y); standardization is the unique outcome. When $t^h > \frac{A\eta(1-\beta^h)}{2}$, the unique equilibrium is $a^* = \frac{1}{2}$ (half of the consumers choose X and half choose Y); there is symmetric non-standardization.*

Corollary 1 *When network effects are direct, the propensity for the market to standardize increases as: (i) the strength of the network effect, η , increases; (ii) the hardware travel cost, t^h , decreases; (iii) the compatibility, β^h , decreases; and (iv) the population of consumers, A , increases.*

If the network effect is strong, consumers are more concerned with being in a large network than with how large a travel cost they will incur. Even those consumers that incur the maximum travel cost prefer being in one large network to being in a half-sized network and incurring no travel cost. Higher compatibility has the opposite effect: When there is a high degree of compatibility, there is not as much difference between the value of a large network and the value of a small network. Consumers prefer to incur smaller travel costs even if that means being part of a smaller network. Increasing the total size of the market magnifies the strength of the network effect.

4 Indirect effects

Here I assume $\eta = 0$, which implies that there are no direct network effects, and I focus on the software market.

4.1 No compatibility

When $\beta^s = 0$ (there is no software compatibility), non-standardization can hold in equilibrium only if no software firm would have incentive to change sides when software firms are equally divided between platforms. If a type- Y software firm switches to platform X , there are three effects: Each type- X firm's share of the market decreases, the proportion of consumers that buy hardware X increases, and the price of type- X software decreases. The first and last of these effects tend to make switching less profitable, while the second effect is in the opposite direction. Let N^S and N^{NS} denote the total number of software firms with free entry under standardization and non-standardization, respectively.¹³ Although N^S and N^{NS} are endogenous, it is convenient to think of the strength of the network effect in terms of the number of software firms. The following proposition gives conditions under which standardization and non-standardization equilibria exist.

Proposition 2 *Assume $\eta = 0$ and $\beta^s = 0$. (i) If $\sqrt{2}N^S < \frac{5t^s}{2t^h}$ (i.e. if the number of firms is relatively small), then two equilibria exist: $(m = N^S, n = 0)$ and $(m = 0, n = N^S)$. Standardization is the unique outcome. (ii) If $\frac{5t^s}{2t^h} < 2 \left(\frac{N^S - 1}{N^S - 2} \right) * \left(1 - \frac{2}{(N^S)^2} \right)$ (i.e. if the number of firms is relatively large), then, in the unique equilibrium, $m = n = N^{NS}/2$. Non-standardization is the unique outcome. (iii) If $2 \left(\frac{N^S - 1}{N^S - 2} \right) \left(1 - \frac{2}{(N^S)^2} \right) < \frac{5t^s}{2t^h} < \sqrt{2}N^S$ (i.e. if the number of firms is intermediate), then three equilibria exist: $(m = N^S, n = 0)$, $(m = 0, n = N^S)$, and $(m = n = N^{NS}/2)$. Both standardization and non-standardization are possible.*

Since $\left(\frac{N^S - 1}{N^S - 2} \right) \left(1 - \frac{2}{(N^S)^2} \right)$ is increasing in N^S , all of the factors that tend to make the condition in (i) true tend to make the condition in (ii) false. Using the free-entry condition to substitute for N^S leads to straightforward comparative statics:

¹³When there is non-standardization, the total number of firms the market can support is greater. This is because splitting into two networks mitigates the price competition among software firms.

Corollary 2 *When network effects are indirect, the propensity for the market to standardize increases as: (i) the hardware travel cost, t^h , decreases; (ii) the software travel cost, t^s , increases; (iii) the population of consumers, A , decreases; and (iv) software firms' fixed costs, F , increase.*

If the hardware travel cost is low relative to software travel cost, consumers are more concerned with having a large variety of software than having their preferred hardware. There is thus a large change in a network's market share if an additional software firm joins the network, which encourages standardization. Also, (iii) and (iv) imply that standardization is less likely for a greater number of software firms. This is the opposite of Church and Gandal's (1992) result.¹⁴

Under direct effects, a larger mass of consumers encourages standardization; the opposite is true under indirect effects. Under direct effects, the mass of consumers magnifies the strength of the network effect. Under indirect effects, the mass of consumers determines the total number of software firms, and a large number of firms hinders standardization.¹⁵

¹⁴The reason for the difference is that software prices in Church and Gandal's model do not depend on the number of firms in the industry. Here, equilibrium software prices decrease with the number of firms. This decreases firms' incentive to jump from a small network to a large one.

¹⁵The market-share effect is sharply decreasing in the total number of software firms. By switching, a software firm decreases the size of one network and increases the size of the other. The effect on the market share of the larger network depends on the number of firms in each network. The negative effects—that in the larger network there will be more firms dividing up that network's share of consumers and there will be greater price competition—only depend on the number of firms in the larger network. Thus the negative effects dominate as the total number of firms increases.

4.2 Partial compatibility

When there is some degree of compatibility ($0 < \beta^s < 1$)¹⁶, some type- X consumers will buy type- Y software, and vice versa. Even if all software firms provide for platform X , some consumers will have such a strong preference for hardware Y that they will buy it and incur the inconvenience of using software that is not perfectly compatible. The following proposition establishes that increasing the compatibility always makes the division of consumers between platforms closer to even:

Proposition 3 *Assume $\eta = 0$ and $0 < \beta^s < 1$. Let a_1^* be the location of the indifferent consumer when compatibility is β^s , and a_2^* the location of the indifferent consumer when compatibility is $\beta^s + \varepsilon$, where $\varepsilon > 0$. If $a_1^* < \frac{1}{2}$, then $a_1^* < a_2^* < \frac{1}{2}$. If $a_1^* > \frac{1}{2}$, then $a_1^* > a_2^* > \frac{1}{2}$. If $a_1^* = \frac{1}{2}$, then $a_2^* = \frac{1}{2}$.*

This leads to another difference between the direct and indirect models:

Corollary 3 *Assume network effects are indirect, and compatibility is partial. When there is standardization of software firms on one hardware platform, consumers are asymmetrically split between hardware platforms.*

In the direct model, there can never be an asymmetric split of consumers unless there is an exogenous source of asymmetry.

To see whether a non-standardization equilibrium exists, we looked at an individual software firm's incentive to switch networks when the market is split. To examine the effect of compatibility on the possibility of non-standardization, we need to see how compatibility changes this incentive. Recall the three effects of a software firm

¹⁶The results in this section do not hold for $\beta^s = 1$. When there is full compatibility between platforms, software firms' choice of platform is trivial; a type- X consumer can use software Y just as easily as X . It is as if all software firms are part of the same network, and standardization is meaningless.

switching from one network to another: The new, larger network will get a larger share of the consumer market, but the profits from this larger share will be divided among a larger number of firms, and prices will be lower. Higher compatibility mitigates the first and last effects but does not change the second one. That is, higher compatibility decreases the gain from the larger share of the consumer market and decreases the loss to software firms from the lower prices. If t^s is large relative to t^h , compatibility has a relatively greater impact on the market-share effect of one firm switching networks than on the price-competition effect,¹⁷ and so higher compatibility increases the propensity toward standardization. The opposite is true if t^h is large relative to t^s . Thus, compatibility works against the effects of the travel costs. A high software travel cost (or low hardware travel cost) encourages standardization; but in the presence of a high software travel cost, higher compatibility discourages standardization.

Proposition 4 *Assume $\eta = 0$ and $0 < \beta^s < 1$. There exist $T_1 > 0$ and $T_2 > 0$ such that: (i) if $\frac{t^s}{t^h} > T_1$ ($\frac{t^s}{t^h} < T_1$), increasing compatibility expands (contracts) the region in which non-standardization is the unique equilibrium; (ii) if $\frac{t^s}{t^h} > T_2$ ($\frac{t^s}{t^h} < T_2$), increasing compatibility contracts (expands) the region in which standardization is the unique equilibrium.*

Generally, if compatibility expands the region in which standardization is the unique equilibrium, it contracts the region in which non-standardization is the unique equilibrium. This does not happen if, for example, $T_1 < \frac{t^s}{t^h} < T_2$. In that case, increasing compatibility expands both the region in which standardization is the unique

¹⁷The market share depends on the hardware travel cost but the price does not, because consumers have already bought their hardware when software firms set prices. Changes in software prices are directly proportional to t^s , but changes in market share are directly proportional to t^s and inversely proportional to t^h .

equilibrium and the region in which non-standardization is the unique equilibrium, thus contracting the region in which both kinds of equilibria exist.

5 Welfare

Since all firms earn zero profit, I take social welfare to be total consumer surplus. For analytical convenience, I consider only the no-compatibility case for indirect effects. Furthermore, I consider only the potential efficiency of standardization; there may be other inefficiencies, such as the amount of entry into the software market.¹⁸

When network effects are direct, anything that tends toward standardization tends to make standardization socially optimal as well.

Proposition 5 *If $\beta^s = 1$, standardization is socially optimal if and only if $t^h < 2A\eta(1 - \beta^h)$.*

Comparing the above to the conditions for standardization from Proposition 1, we have the following:

Corollary 4 *When network effects are direct, standardization may be underprovided.*

Even if total surplus is maximized under standardization, some consumers may defect and create non-standardization (recall that consumers only coordinate to a Pareto optimum). There is no equilibrium in which there is inefficient standardization.¹⁹

When network effects are indirect, the condition for standardization to be socially optimal is again similar to the condition for a standardization equilibrium to exist:

¹⁸The proofs in this section are very straightforward and are available from the author.

¹⁹This is the opposite of the welfare result from Farrell and Saloner (1986b). The difference is due to the modeling of consumer preferences as a continuum rather than a discrete set: There is an unraveling of some equilibria.

Proposition 6 *Assume $\eta = 0$ and $\beta^s = 0$. Standardization is socially optimal if and only if $(t^h)^2 A < (3 - 2\sqrt{2}) Ft^s$.*

Comparing this to the conditions for standardization from Proposition 2, we have the following:

Corollary 5 *When network effects are indirect, standardization may be overprovided.*

Software firms may have too much incentive to standardize. If a software firm switches networks, it makes one network larger and the other smaller. The market share of a network depends on the number of firms in *each* network, whereas the price competition and profit shares within a network only depend on the number of firms in that network. An individual software firm has more influence over the market-share effect than the price-competition effect, and it is the market-share effect that makes standardization attractive to software firms. Thus software firms may standardize even when it is socially inefficient.

For both direct and indirect effects, the equilibrium coincides with the social optimum if the network effect is either too strong or too weak. However, direct and indirect effects can potentially lead to different inefficiencies: underprovision of standardization in the case of direct effects, and overprovision of standardization in the case of indirect effects.

6 Conclusion

To a certain extent, it is reasonable to model indirect network effects as if they were direct. Generally, stronger network effects, whether direct or indirect, increase the propensity toward standardization. However, there are some differences in the implications of the two kinds of effects, and these differences can only be seen by explicitly comparing direct and indirect effects. I have offered an explicit comparison

that makes the differences clear. With respect to the size of the market, the results of the indirect model contradict those of the direct model. The effect of partial compatibility is much less straightforward under indirect effects than it is under direct effects. Finally, the two models imply different potential inefficiencies. Consequently, considering the direct effects model to be a reduced form of the indirect effects model is problematic. Even if we disregard the contradictory aspects of the two kinds of effects, the direct effects model does not illustrate the elements of the software market that play a crucial role. A richer model is more useful in describing markets in which indirect network effects prevail.

This paper has considered the particular question of standardization. An explicit model of indirect network effects could be used to examine other issues the direct effects literature has considered: for example, the incentives of hardware firms for location choice, compatibility, or R&D investment; asymmetries that arise from a first-mover advantage; or the effects of an existing installed base of consumers.

7 Appendix

7.1 Proof of Proposition 1

The location of the consumer indifferent between hardware types is $a^* = \frac{t^h + A\eta(\beta^h - 1)}{2[t^h + A\eta(\beta^h - 1)]}$. Assume $a^* = \frac{1}{2}$. The consumer located at $a = 0$ would clearly prefer standardization on X . The consumer located at $a = 1$ obtains utility of $U^y = U_0 + A\eta(1 - \beta^h) / 2 - c^h$. If we had $a^* = 1$ instead, the consumer at $a = 1$ would get utility of $U^x = U_0 + A\eta - c^h - t^h$, which is greater if $t^h < \frac{A\eta(1 - \beta^h)}{2}$. This is the most extreme case; all consumers would prefer $a^* = 1$ (or, by similar reasoning, $a^* = 0$) to $a^* = \frac{1}{2}$. Furthermore, there cannot be an equilibrium with any a^* such that $0 < a^* < \frac{1}{2}$ or $\frac{1}{2} < a^* < 1$, because there cannot be indifference in these regions.

Now consider $a^* = 1$. The consumer located at $a = 1$ prefers switching to Y to staying in the X network: $U^x = U_0 + \eta - c^h - t^h$, $U^y = U_0 + \beta^h \eta - c^h$, and so $U^x < U^y$ if $t^h > \frac{A\eta(1-\beta^h)}{2}$. The same is true for every consumer in the interval $(1 - \delta, 1]$, where $\delta = t^h - \frac{A\eta(1-\beta^h)}{2}$. Each of these consumers will prefer to switch unilaterally to Y . Given that this subset of consumers will switch to Y , the Y network becomes more attractive, and more consumers prefer to switch. Because the network benefit function exhibits constant returns to scale, this effect will continue until $a^* = \frac{1}{2}$. Similar reasoning shows that $a^* = 0$ cannot hold in equilibrium if $t^h > \frac{A\eta(1-\beta^h)}{2}$.

7.2 Proof of Proposition 2

As in Salop's (1979) circular city model, in the software pricing subgame, software prices are $p^{sx} = c^s + \frac{t^s}{m}$ and $p^{sy} = c^s + \frac{t^s}{n}$. The location of the indifferent consumer is $a^* = \frac{1}{2} + \frac{1}{2t^h} [(Ed^y - Ed^x)t^s + p^{sy} - p^{sx}] = \frac{1}{2} + \frac{5t^s}{8t^h} \left(\frac{1}{N-m} - \frac{1}{m} \right)$, where $N = m + n$ is the free-entry number of software firms. The free-entry condition implies $N^S = \sqrt{\frac{At^s}{F}}$ and $N^{NS} = \sqrt{\frac{2At^s}{F}}$. I assume that $A > \frac{4F}{t^s}$, which implies $N^S > 2$.

The profit for a type- X firm when m firms are type- X is $\pi^{sx} = \frac{Aa^*}{m} (p^{sx} - c^s) - F$. The change in profit if another firm joins the X network is $\frac{\partial \pi^{sx}}{\partial m} = \frac{A}{m} [(p^{sx} - c^s) \left(\frac{-a^*}{m} + \frac{\partial a^*}{\partial m} \right) + a^* \frac{\partial p^{sx}}{\partial m}] + \frac{\partial a^*}{\partial m}$.

(i) Assume that, in the short run, N is fixed. Using the expressions for software price and indifferent consumer, $\frac{\partial p^{sx}}{\partial m} = \frac{-t^s}{m^2}$ and $\frac{\partial a^*}{\partial m} = \frac{5t^s}{8t^h} \left[\frac{1}{(N-m)^2} + \frac{1}{m^2} \right]$. Then $\frac{\partial \pi^{sx}}{\partial m} = \frac{A}{m} [(p^{sx} - c^s) \left(\frac{-a^*}{m} + \frac{\partial a^*}{\partial m} \right) + a^* \frac{\partial p^{sx}}{\partial m}] = \frac{A}{m} \left[\frac{t^s}{m} \left(\frac{-a^*}{m} + \frac{5t^s}{8t^h} \left[\frac{1}{(N-m)^2} + \frac{1}{m^2} \right] \right) + a^* \left(\frac{-t^s}{m^2} \right) \right]$. Symmetric non-standardization is not an equilibrium if one firm would want to switch to the other network: $\frac{\partial \pi^{sx}}{\partial m} > 0 \Leftrightarrow \frac{5t^s}{8t^h} \left[\frac{1}{(N-m)^2} + \frac{1}{m^2} \right] > \frac{2a^*}{m}$. For symmetric non-standardization, $m = N/2$ and $a^* = \frac{1}{2}$, and the condition is $N < \frac{5t^s}{2t^h}$. Since $N^S < N^{NS}$, if this condition is true for $N = N^{NS}$, it is also true for $N = N^S$. Furthermore, if $N < \frac{5t^s}{2t^h}$, we can show that $\frac{\partial \pi^{sx}}{\partial m} > 0$ for every $m > N/2$. The only equilibria

are standardization on one of the two platforms.

(ii) To see if standardization is an equilibrium, I assume that all software firms and consumers choose the same platform, and check whether any software firms have incentive to break off.²⁰ Without loss of generality, I assume that all software firms provide for X , and $a^* = 1$. As above, N is fixed in the short run. If one firm switches to Y , then $a^* = \frac{1}{2} + \frac{1}{2t^h} [(Ed^y - Ed^x) t^s + p^{sy} - p^{sx}] = \frac{1}{2} + \frac{5t^s}{8t^h} \left(1 - \frac{1}{N-1}\right)$ and the single type Y software firm will have profit $\pi^{sy} = A(1 - a^*)(p^{sy} - c^s) - F$. This is greater than the profit of each firm under standardization if

$$\frac{5t^s}{4t^h} < \frac{(N-1)}{(N-2)} \left(1 - \frac{2}{N^2}\right). \quad (1)$$

The profit for each remaining type X firm is $\frac{At^s}{(N-1)^2} \left[\frac{1}{2} + \frac{5t^s}{8t^h} \left(1 - \frac{1}{N-1}\right)\right] - F$. An additional firm will have incentive to switch to Y if the new profit of each type Y firm would be greater than this: if

$$\frac{1}{4} \left[\frac{1}{2} - \frac{5t^s}{8t^h} \left(\frac{1}{2} - \frac{1}{N-2} \right) \right] > \frac{1}{(N-1)^2} \left[\frac{1}{2} + \frac{5t^s}{8t^h} \left(\frac{1}{2} - \frac{1}{N-1} \right) \right]. \quad (2)$$

But if (1) is true, (2) must also be true; i.e., if one firm has incentive to switch to Y , then another firm will have incentive to follow suit. This will continue until $m = n$ and $a^* = \frac{1}{2}$. It is not possible for the market to settle on an asymmetric equilibrium.

(iii) The region in this case is non-degenerate because it is impossible for the conditions in (i) and (ii) both to be true at the same time. Note that $\left(\frac{N^S-1}{N^S-2}\right) \left(1 - \frac{2}{(N^S)^2}\right) < 1$. A necessary condition for (ii) is $\frac{5t^s}{4t^h} < 1 \Rightarrow \frac{5t^s}{2t^h} < 2$. Since $N^{NS} > N^S > 2$, $\frac{5t^s}{2t^h} < 2 \Rightarrow N^{NS} > \frac{5t^s}{2t^h}$. So if the condition in (ii) is true, the condition in (i) neces-

²⁰I am assuming that, if a single software firm breaks off and forms a smaller network, it follows the same pricing pattern that multiple firms follow: We always have $p^{sx} = c^s + \frac{t^s}{m}$, even if $m = 1$. The precise condition for standardization being an equilibrium depends on what is assumed about how software firms would behave in the (out-of-equilibrium) case that software firms are standardized but this is not sustainable. Qualitatively, the condition does not depend on such an assumption.

sarily is false.

7.3 Proof of Proposition 3

Consider the case where all software firms provide for hardware X . This corresponds to $a^* > \frac{1}{2}$. A consumer located near $a = 1$ is indifferent between hardware X and hardware Y if $U_0 - c^h - (1 - a)t^h - p^{sx} - E(d^x)t^s/\beta^s = U_0 - c^h - at^h - p^{sx} - E(d^x)t^s$. The location of the indifferent consumer, a^* , satisfies $a^* = \frac{1}{2} + \frac{E(d^x)t^s(1/\beta^s - 1)}{2t^h}$. Clearly $\frac{\partial a^*}{\partial \beta^s} < 0$, which implies $a_1^* > a_2^*$. Since $\frac{E(d^x)t^s(1/\beta^s - 1)}{2t^h} > 0$ for $\beta^s > 0$, a_1^* and a_2^* are both greater than $\frac{1}{2}$. In the case where all software firms provide for Y and $a^* < \frac{1}{2}$, we have $a^* = \frac{1}{2} - \frac{E(d^y)t^s(1/\beta^s - 1)}{2t^h}$ and $\frac{\partial a^*}{\partial \beta^s} > 0$, and so $a_1^* < a_2^*$. Given $\beta^s > 0$, a_1^* and a_2^* are both less than $\frac{1}{2}$. When $a^* = \frac{1}{2}$, a^* is not a function of β^s , and so $\frac{\partial a^*}{\partial \beta^s} = 0$, and $a_1^* = a_2^*$.

7.4 Proof of Proposition 4

The closed-form solutions for software price and the location of the consumer indifferent between the two hardware platforms are very complicated.²¹ However, some properties of these solutions are easily described. The equilibrium price of software is of the form $p^{sx} = c^s + t^s * f(m, n, \beta^s)$, where $\frac{\partial f}{\partial \beta^s} < 0$ and $\frac{\partial^2 f}{\partial m \partial \beta^s} > 0$. The location of the indifferent consumer is of the form $a^* = \frac{1}{2} + \frac{t^s}{t^h} * g(m, n, \beta^s)$, where $\frac{\partial g}{\partial m} > 0$ and $\frac{\partial^2 g}{\partial m \partial \beta^s} < 0$.

As in Proposition 2, when considering the possibility of a non-standardization equilibrium, it is convenient to consider the number of software firms to be a continuous variable. This does not alter the qualitative implications of changes in compatibility. When considering the possibility of a standardization equilibrium, we cannot consider the number of firms to be continuous because of the non-existence of deriv-

²¹Details are available from the author.

atives at $m = 0$ and $n = 0$. I therefore look at the effect of compatibility on the discrete change in a firm's profit when it leaves a standardized network.

(i) **Non-standardization** Assume $m = n$ and $a^* = \frac{1}{2}$. If one of the type- X firms switches to the Y network, the change in profit is $\frac{\partial \pi^{sx}}{\partial m} = \frac{A}{m}[(p^{sx} - c^s) \left(\frac{-a^*}{m} + \frac{\partial a^*}{\partial m}\right) + a^* \left(\frac{\partial p^{sx}}{\partial m}\right)]$. The change in this expression induced by a change in compatibility is $\frac{\partial}{\partial \beta^s} \left(\frac{\partial \pi^{sx}}{\partial m}\right) = \frac{A}{m} \left[(p^{sx} - c^s) \left(-\frac{1}{m} \frac{\partial a^*}{\partial \beta^s} + \frac{\partial^2 a^*}{\partial m \partial \beta^s}\right) + \frac{\partial p^{sx}}{\partial \beta^s} \left(\frac{-a^*}{m} + \frac{\partial a^*}{\partial m}\right) \right] + \frac{A}{m} \left[a^* \left(\frac{\partial^2 p^{sx}}{\partial m \partial \beta^s}\right) + \frac{\partial a^*}{\partial \beta^s} \left(\frac{\partial p^{sx}}{\partial m}\right) \right]$. When $a^* = \frac{1}{2}$ and $m = n$, by Proposition 3, $\frac{\partial a^*}{\partial \beta^s} = 0$, which is equivalent to $\frac{\partial g}{\partial \beta^s} = 0$. Therefore $\frac{\partial^2 \pi^{sx}}{\partial \beta^s \partial m} = \frac{At^s}{m} \left\{ \frac{t^s}{t^h} \left(f \frac{\partial^2 g}{\partial m \partial \beta^s} + \frac{\partial f}{\partial \beta^s} \frac{\partial g}{\partial m} \right) + a^* \left(\frac{\partial^2 f}{\partial m \partial \beta^s} - \frac{1}{m} \frac{\partial f}{\partial \beta^s} \right) \right\}$. Recalling the properties of f and g , this expression is of the form $\frac{At^s}{m} \left\{ \frac{t^s}{t^h} [(+) (-) + (-) (+)] + a^* [(+) - \frac{1}{m} (-)] \right\}$. If $\frac{t^s}{t^h} > T_1$, where $T_1 = \frac{a^* \left(\frac{1}{m} \frac{\partial f}{\partial \beta^s} - \frac{\partial^2 f}{\partial m \partial \beta^s} \right)}{\left(f \frac{\partial^2 g}{\partial m \partial \beta^s} + \frac{\partial f}{\partial \beta^s} \frac{\partial g}{\partial m} \right)}$, then $\frac{\partial^2 \pi^{sx}}{\partial \beta^s \partial m} < 0$. This means firms have less incentive to switch networks, and so non-standardization is more likely. If $\frac{t^s}{t^h} < T_1$, the second term inside the brackets dominates, and we will have $\frac{\partial^2 \pi^{sx}}{\partial \beta^s \partial m} > 0$, and thus non-standardization is less likely.

(ii) **Standardization** Assume there is standardization on X , and that the short-run number of software firms is N . The profit of each software firm is $\pi_1^{sx} = \frac{Aa^*}{N} (p_1^{sx} - c^s) - F$. If one firm defects to Y , that firm's profit will be $\pi_2^{sy} = A(1 - a_2^*) (p_2^{sy} - c^s) - F$. In both cases, the majority of consumers will choose platform X : $a_1^* > \frac{1}{2}$ and $a_2^* > \frac{1}{2}$. Standardization cannot be an equilibrium if $\pi_2^{sy} > \pi_1^{sx}$. Let $f_1 = f(N, 0, \beta^s)$, $f_2 = f(N - 1, 1, \beta^s)$, $g_1 = g(N, 0, \beta^s)$, and $g_2 = g(N - 1, 1, \beta^s)$. Then $\pi_2^{sy} > \pi_1^{sx} \Leftrightarrow \left(\frac{1}{2} - \frac{t^s}{t^h} g_2\right) f_2 - \frac{1}{N} \left(\frac{1}{2} + \frac{t^s}{t^h} g_1\right) f_1 > 0$. Let $\Delta \pi^{sy} = \left(\frac{1}{2} - \frac{t^s}{t^h} g_2\right) f_2 - \frac{1}{N} \left(\frac{1}{2} + \frac{t^s}{t^h} g_1\right) f_1$. Differentiating with respect to β^s yields

$$\frac{\partial}{\partial \beta^s} (\Delta \pi^{sy}) = \frac{t^s}{t^h} \left(-g_2 \frac{\partial f_2}{\partial \beta^s} - f_2 \frac{\partial g_2}{\partial \beta^s} - \frac{g_1}{N} \frac{\partial f_1}{\partial \beta^s} - \frac{f_1}{N} \frac{\partial g_1}{\partial \beta^s} \right) + \frac{1}{2} \frac{\partial f_2}{\partial \beta^s} - \frac{1}{2N} \frac{\partial f_1}{\partial \beta^s}. \quad (3)$$

Now, the properties of f imply $\frac{\partial f_2}{\partial \beta^s} < \frac{\partial f_1}{\partial \beta^s} < 0$. Furthermore, since $a_1^* > \frac{1}{2}$ and $a_2^* > \frac{1}{2}$, $g_1 > 0$ and $g_2 > 0$ (and $f > 0$ for its entire domain). Also, Proposition 3 implies that $\frac{\partial g_1}{\partial \beta^s} < 0$ and $\frac{\partial g_2}{\partial \beta^s} < 0$: when $a^* > \frac{1}{2}$, increases in β^s decrease a^* . This is equivalent to g decreasing. Therefore (3) is of the form $\frac{\partial}{\partial \beta^s} (\Delta \pi^{sy}) = \frac{t^s}{t^h} (+) + (-)$, where the terms inside the parentheses do not depend on either t^s or t^h . Therefore, if $\frac{t^s}{t^h} > T_2$, where $T_2 = \frac{\frac{1}{2N} \frac{\partial f_1}{\partial \beta^s} - \frac{1}{2} \frac{\partial f_2}{\partial \beta^s}}{-g_2 \frac{\partial f_2}{\partial \beta^s} - f_2 \frac{\partial g_2}{\partial \beta^s} - \frac{g_1}{N} \frac{\partial f_1}{\partial \beta^s} - \frac{f_1}{N} \frac{\partial g_1}{\partial \beta^s}}$, then $\frac{\partial}{\partial \beta^s} (\Delta \pi^{sy}) > 0$. This means that standardization is less likely. The opposite is true if t^h is large relative to t^s .

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