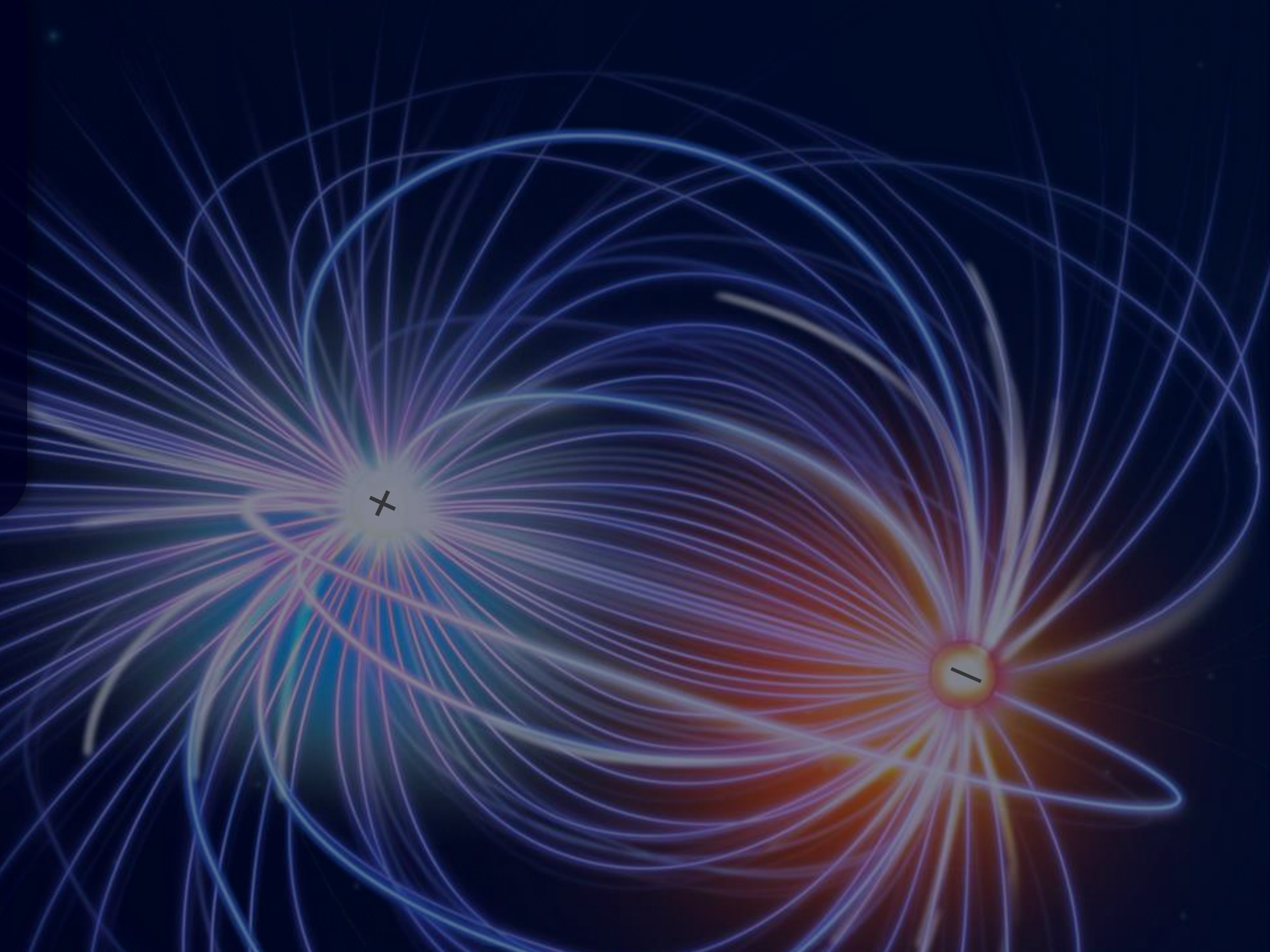
Welcome to

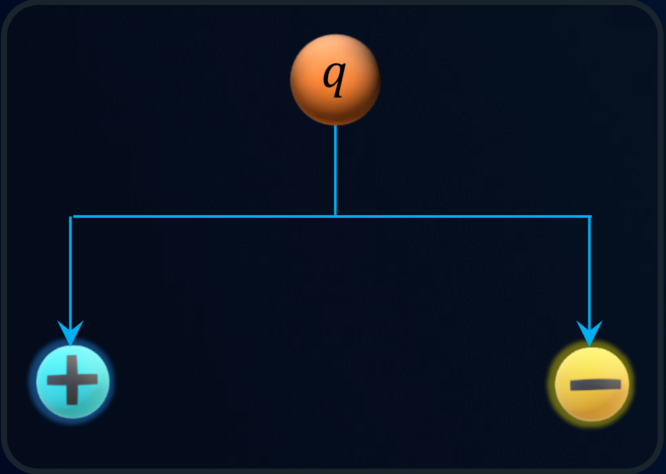
Aakash + BYJU'S NOTES

Electric Charges and Fields



Charge

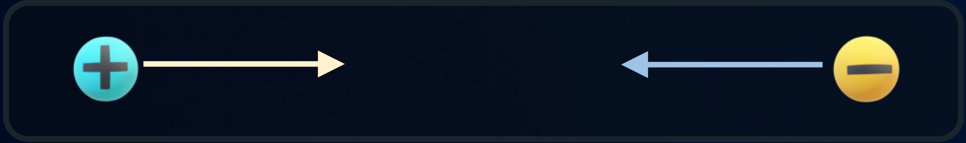
- An intrinsic property of matter.
- A charged body exerts a force on other charged bodies near it.
  - Attractive force
  - Repulsive force
- There are two types of charges:



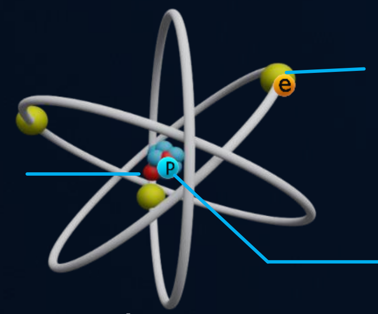
- Unit of Charge: $Coulomb(C)$,

  Dimensions of charge: $[M^0L^0T^1A^1]$

- Opposite charges attract one another



- Similar charges repel each other



Electron

Negative Charge:

$$e^- = -1e = -1.6 \times 10^{-19}\ C$$

Proton

Positive Charge:

$$p^+ = +1e = 1.6 \times 10^{-19}\ C$$

Neutron

No Charge

Note:

- Charge can neither be created nor be destroyed.
- Charge can only be transferred from one body to another.
- The charge on a proton/magnitude of charge on an electron is also known as Elementary charge or Fundamental charge.
- Charge on any object is an integral multiple of e $\quad q = \pm ne \quad (n = 0,1,2 \ldots.)$
- Charges on a body can be algebraically added (or subtracted) to get the net charge on that body

$$q_{net} = q_1 + q_2 - q_3 - q_4 + q_5$$

A body has acquired a charge of 80 C through a particular process. What is the difference between the number of protons and electrons in the body?

Given: $q = 80\ C$

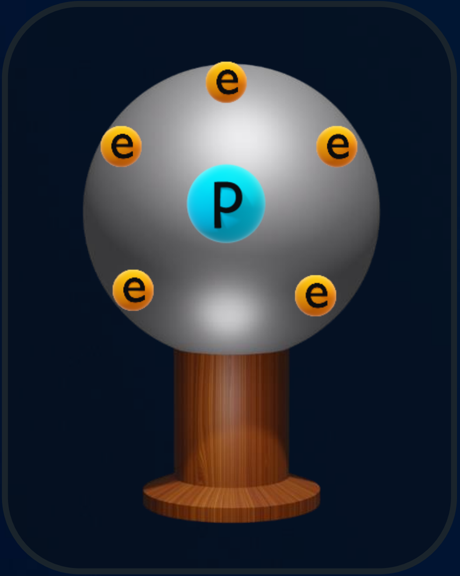
To find: $(n_p - n_e)$

Solution: Applying quantization of charge principle,

$$q = (n_p - n_e)e$$

$$\Rightarrow 80 = (n_p - n_e) \times 1.6 \times 10^{-19}$$

$$\therefore (n_p - n_e) = 5 \times 10^{20}$$

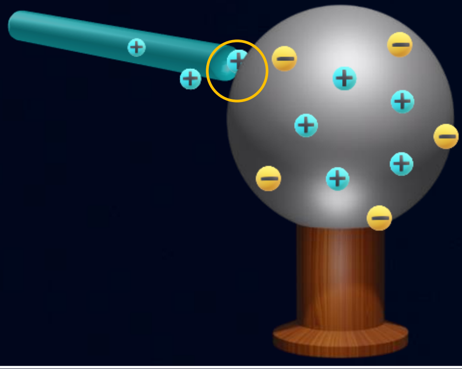


Note:

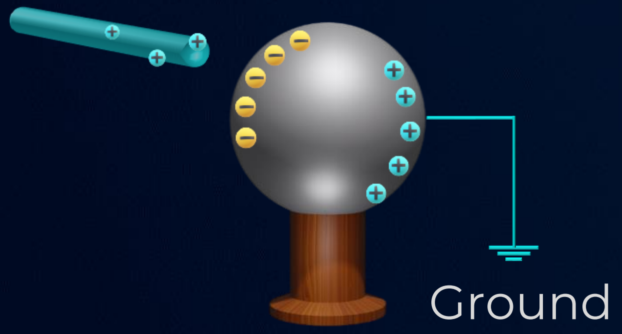
Charge observes relativistic invariance, i.e., its measured value is independent of the frame of reference.

Methods of Charging

Charging by Conduction



Charging by Induction



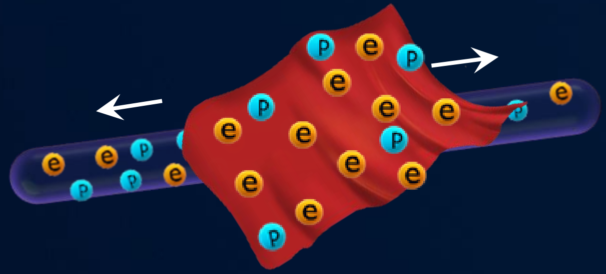
Charging by conduction and charging by induction: Can charge only conductors

- Actual contact is needed
- Free electrons are needed

Charging by induction:

- Actual contact not needed
- Neutral object needs to be grounded
- Initial charged body can be insulator

Charging by Friction



Can charge both conductors and insulators

- Used for charging insulators
- Heat energy generated by rubbing is used by the electrons to be freed and thus transferred.

Triboelectric Series

Triboelectric Series (Positive → Decreases → Negative)

- Air
- Human body
- Glass
- Nylon
- Wool
- Lead
- Aluminium
- Paper
- Cotton
- Steel
- Wood
- Gelatin

Triboelectric Series (Positive → Decreases → Negative)

- Nickel, Copper
- Gold, Platinum
- Natural rubber
- Sulfur
- Acetate
- Polyester
- Celluloid
- Polyurethane
- Polyethylene
- Vinyl
- Silicon
- Teflon

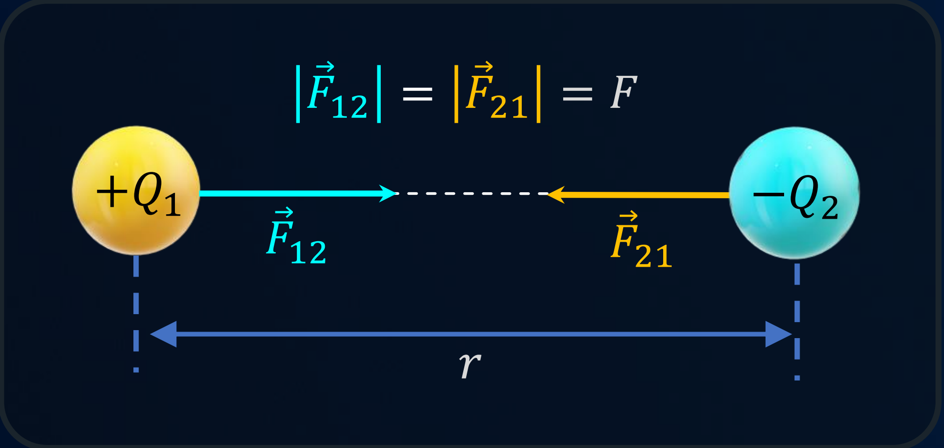
Note:

Two substances farther apart in the series makes a better charging pair

Coulomb's Law

"The electrostatic force of attraction or repulsion between two stationary point charges is directly proportional to the product of charges and inversely proportional to the square of the distance of separation between them."

Coulomb's law states that: $F \propto Q_1 Q_2$ $\quad F \propto 1/r^2$



Combining, we get, $F \propto \dfrac{Q_1 Q_2}{r^2}$

$$F = k\frac{Q_1 Q_2}{r^2}$$

	Electrostatic Force	Gravitational Force
Nature	Attractive or repulsive	Always Attractive
Mathematical Expression	$F = k\dfrac{Q_1 Q_2}{r^2}$	$F = G\dfrac{m_1 m_2}{r^2}$
Strength Comparison	Stronger	Weaker
Nature of Proportionality Constant	Depends on the medium	Universal

Coulomb's Constant (k)

$$F = k\frac{Q_1Q_2}{r^2}$$

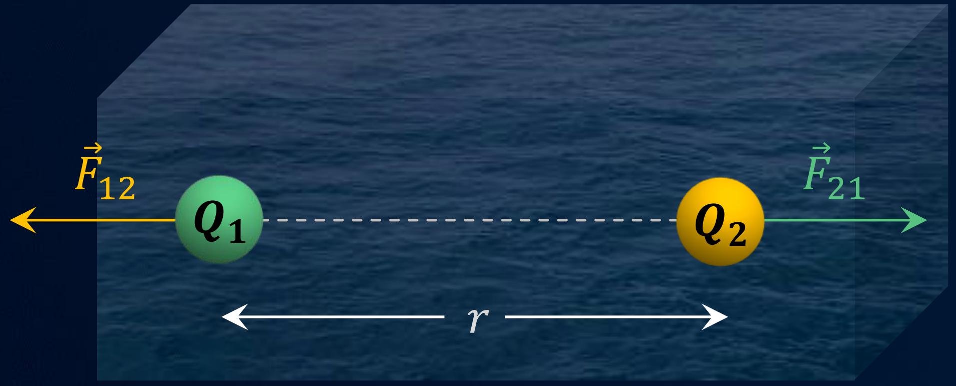
$$|\vec{F}_{12}| = |\vec{F}_{21}| = k\frac{|Q_1Q_2|}{r^2}$$

$k = \dfrac{1}{4\pi\varepsilon}$ = Coulomb's constant

ε = Permittivity of the medium

$= \varepsilon_0\varepsilon_r$

ε_r = Relative Permittivity of the medium



Ex : $\varepsilon_r = 81$ for water

The net electrostatic force in water reduces to 1/81 times as compared to vacuum

For vacuum ($\varepsilon_r = 1$)

$$\Rightarrow \varepsilon = \varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \frac{Nm^2}{C^2}$$

Coulomb's Law in Vector form

$$F = k\frac{Q_1Q_2}{r^2}$$

$$|\vec{F}_{21}| = \frac{kQ_1Q_2}{|\vec{r}|^2} \qquad \vec{r} = \vec{r}_2 - \vec{r}_1$$

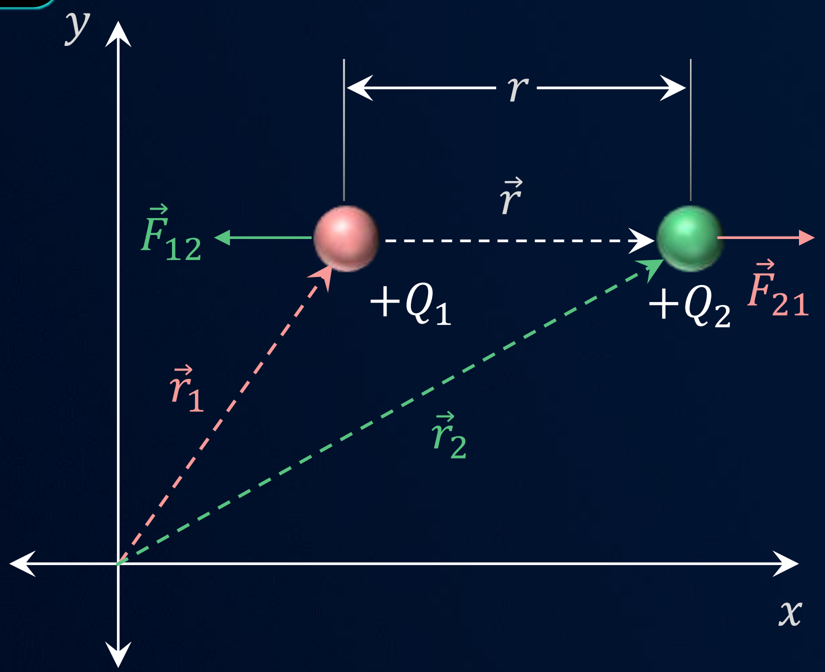
$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^2}(\hat{r})$$

Magnitude Direction

$$\vec{F}_{21} = \frac{kQ_1Q_2}{r^3}(\vec{r}) \quad \left(\because \hat{r} = \frac{\vec{r}}{r}\right)$$

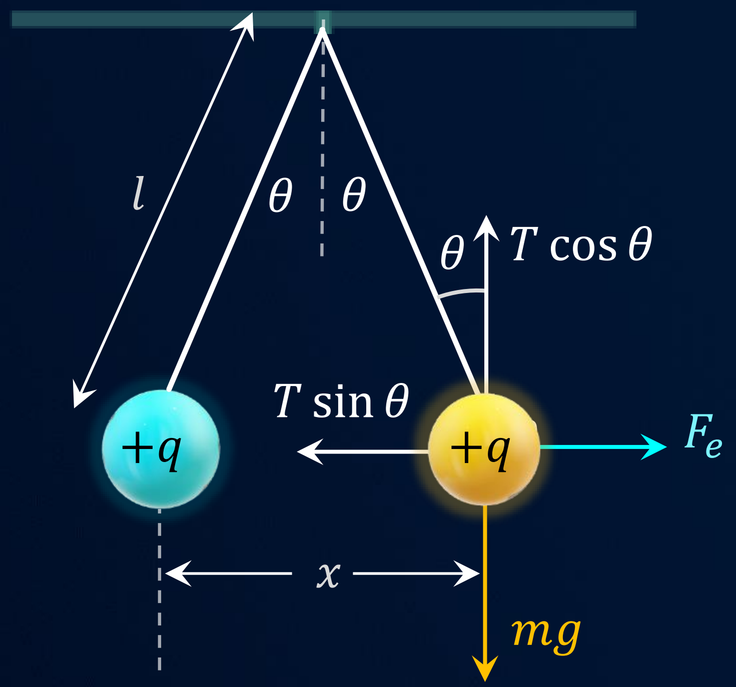
$$\vec{F}_{12} = -\frac{kQ_1Q_2}{r^3}(\vec{r})$$

$\vec{F}_{12} = -\vec{F}_{21}$ (Newton's third law)



Two balls of same mass m and carrying equal charge q are hung from a fixed support of length l. At electrostatic equilibrium, assuming that the angles made by each thread with the vertical are very small, the separation x between the balls is proportional to:

Solution :



at Equilibrium: $F_e = T\sin\theta$ $\quad mg = T\cos\theta$

$$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\varepsilon_0 x^2 \times mg}$$

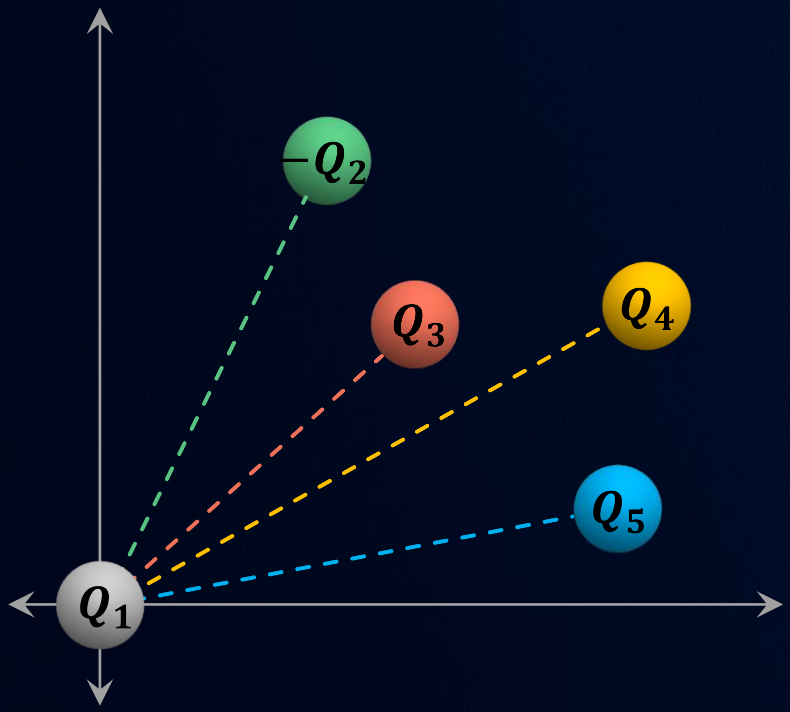
$$\tan\theta \approx \sin\theta = \frac{x/2}{l}$$

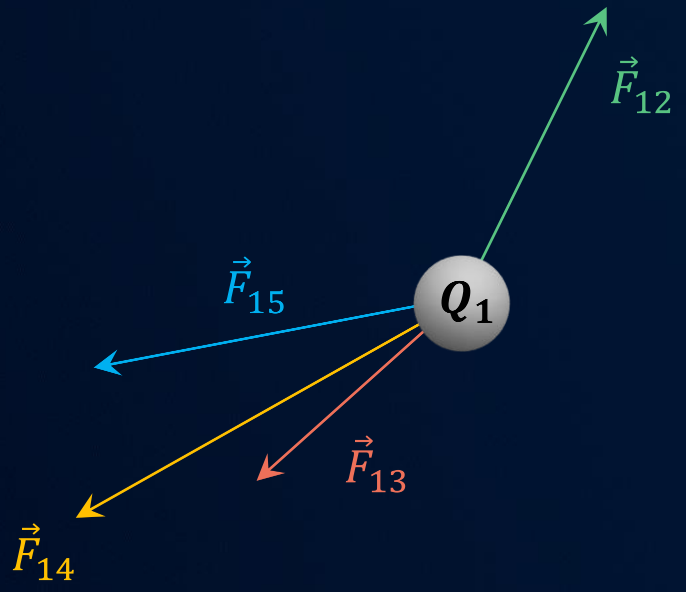
$$\frac{x/2}{l} = \frac{q^2}{4\pi\varepsilon_0 x^2 \times mg}$$

$$x^3 = \frac{lq^2}{2\pi\varepsilon_0 \times mg}$$

$$x \propto l^{1/3}$$

Superposition Principle





$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15}$$

The net electrostatic force acting on a given charge is equal to the vector sum of electrostatic forces exerted on it by all the other charges in its surroundings.

Four particles A, B, C and D having charge $+q, +q, -q$ and $+q$ respectively, are placed on the vertices of a square having sides of length a. Find the resultant force acting on particle C.

Solution :

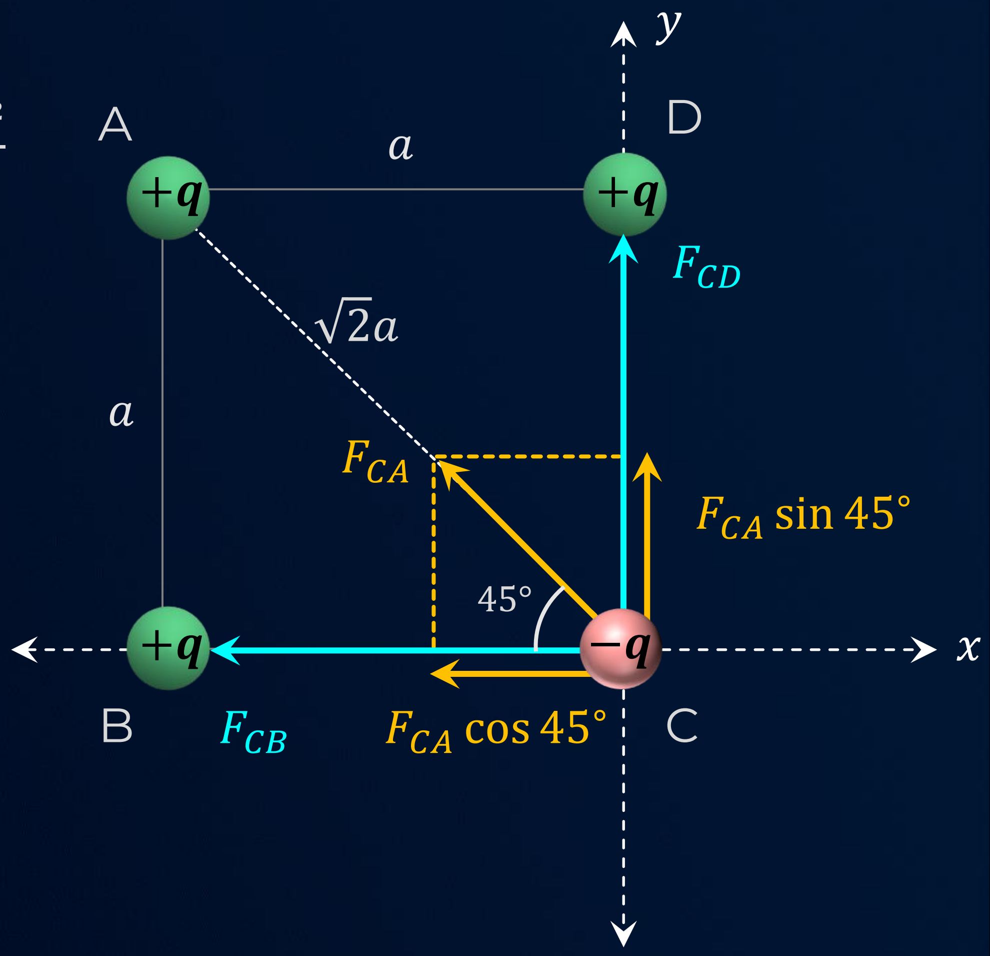
Force exerted by charge B and D on charge C : $|\vec{F}_{CB}| = |\vec{F}_{CD}| = \frac{kq^2}{a^2}$

Force exerted by charge A on charge C : $|\vec{F}_{CA}| = \frac{kq^2}{(\sqrt{2}a)^2}$

$$\Sigma\vec{F}_x = \left(-\frac{kq^2}{a^2} - \frac{kq^2}{2\sqrt{2}a^2}\right)\hat{\imath} \quad \Sigma\vec{F}_y = \left(\frac{kq^2}{a^2} + \frac{kq^2}{2\sqrt{2}a^2}\right)\hat{\jmath}$$

$$|\vec{F}_{net}| = |\vec{F}_x + \vec{F}_y| = \sqrt{|\vec{F}_x|^2 + |\vec{F}_y|^2} = \sqrt{2}.\left(\frac{kq^2}{a^2} + \frac{kq^2}{2\sqrt{2}a^2}\right)$$

$$F_{net} = \frac{kq^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$



Five balls, numbered 1 to 5, are suspended using separate threads. Pairs (1,2), (2,4) & (4,1) show electrostatic attraction, while pairs (2,3) and (4,5) show repulsion. Therefore charge on ball 1 must be -

Solution :

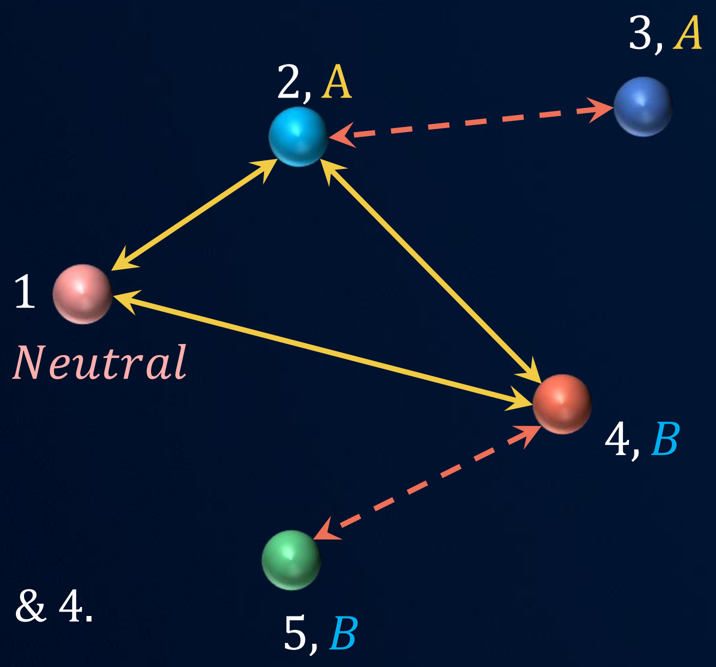
Like charges repel each other,

Balls (2,3) & (4,5) must be of same nature.

Unlike charges attract each other,

Balls (2,4) must have different types of charges.

Ball 1 has to be neutral to be attracted by both balls 2 & 4.



Two charge particles, each having charge q and mass m, are distance d apart from each other kept in vacuum. If two particles are in equilibrium under the gravitational and electrostatic force, then the ratio q/m is of the order

Solution :

Both the particles are in equilibrium under the gravitational and electrostatic forces.

$$F_{electrostatic} = F_{gravitational}$$

$$\Rightarrow \frac{kq^2}{d^2} = \frac{Gm^2}{d^2}$$

$$\Rightarrow \frac{q}{m} = \sqrt{\frac{G}{k}}$$

$$\frac{q}{m} \approx 10^{-10}$$

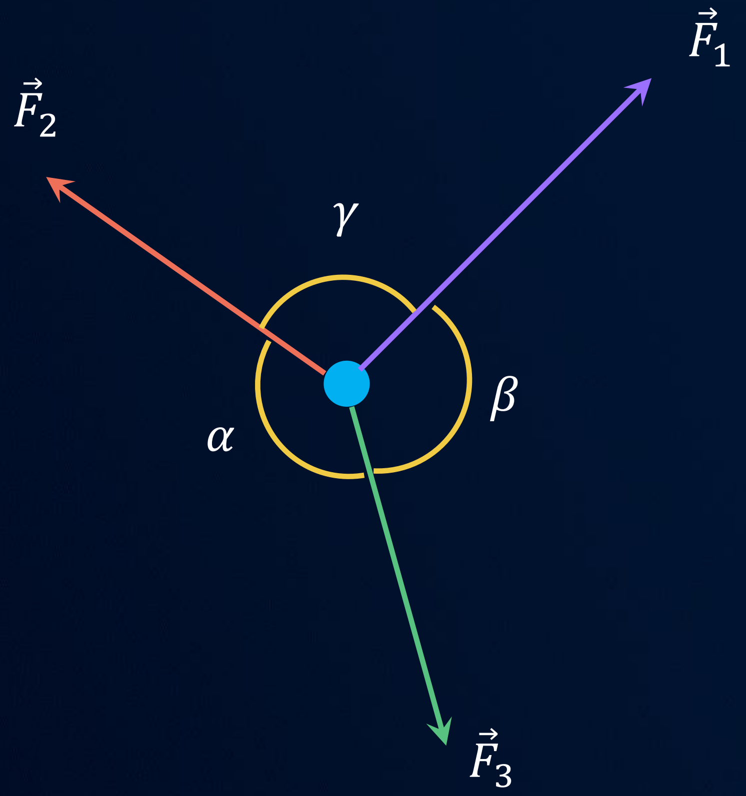
q/m is called specific charge.

Lami's Theorem

If three concurrent, coplanar and non-collinear forces $\vec{F}_1$, $\vec{F}_2$ & $\vec{F}_3$ are in equilibrium,

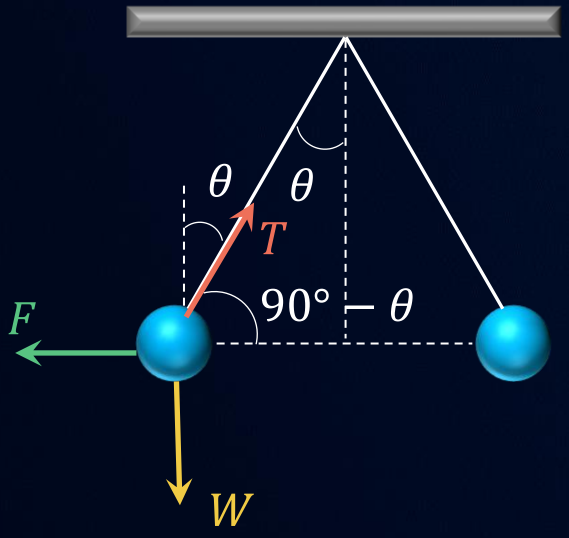
$\Rightarrow \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, then

$$\frac{|\vec{F}_1|}{\sin\alpha} = \frac{|\vec{F}_2|}{\sin\beta} = \frac{|\vec{F}_3|}{\sin\gamma}$$



Two identical balls, each having a density ρ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium each string makes an angle θ with vertical. Now, both the balls are immersed in a liquid. As a result of immersion in the liquid, the angle θ does not change. The density of the liquid is σ. The dielectric constant of the liquid is -

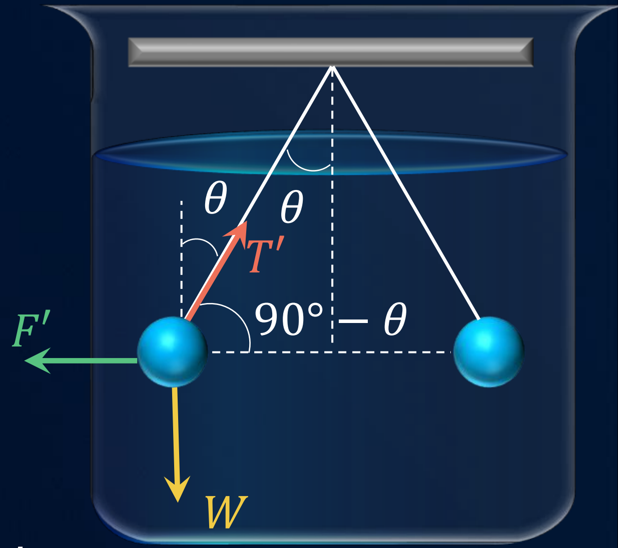
Solution :



Applying Lami's theorem, when in vacuum,

$$\frac{T}{\sin 90^\circ} = \frac{F}{\sin(180^\circ - \theta)} = \frac{W}{\sin(90^\circ + \theta)}$$

$$\Rightarrow \frac{W}{F} = \frac{\sin(90^\circ + \theta)}{\sin(180^\circ - \theta)}$$



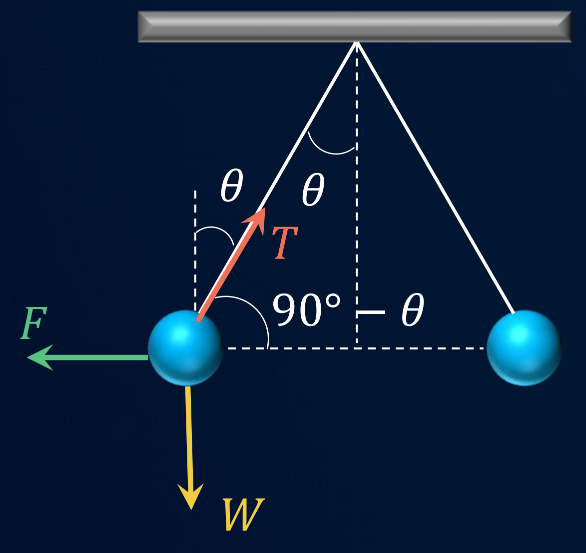
Applying Lami's theorem, when in liquid,

$$\frac{T'}{\sin 90^\circ} = \frac{F'}{\sin(180^\circ - \theta)} = \frac{W'}{\sin(90^\circ + \theta)}$$

$$\Rightarrow \frac{W'}{F'} = \frac{\sin(90^\circ + \theta)}{\sin(180^\circ - \theta)}$$

$$\Rightarrow \frac{W}{F} = \frac{\sin(90° + \theta)}{\sin(180° - \theta)}$$

$$\Rightarrow \frac{W'}{F'} = \frac{\sin(90° + \theta)}{\sin(180° - \theta)}$$



Weight of charge when immersed in liquid,

$$W' = W - F_b$$

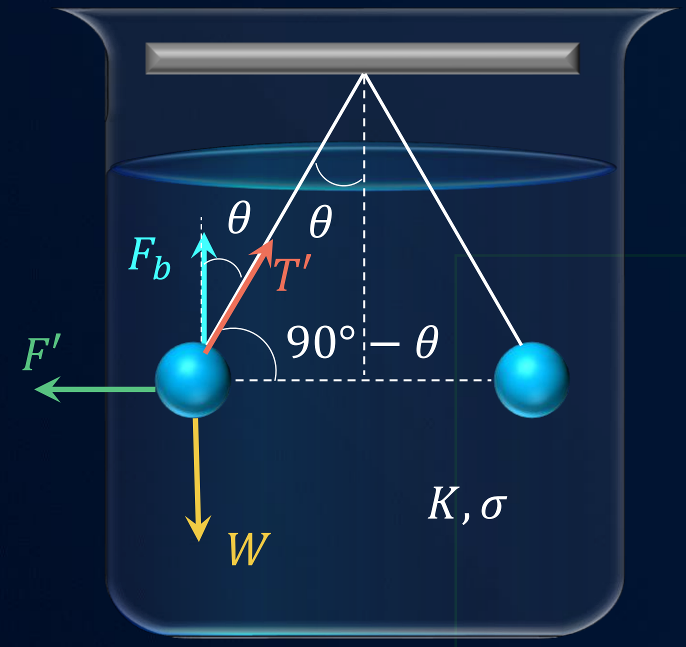
$$\Rightarrow W' = V\rho g - V\sigma g$$

Force between charges when immersed in liquid,

$$\Rightarrow F' = \frac{F}{K}$$

$$\Rightarrow \frac{W}{F} = \frac{W'}{F'}$$

$$\Rightarrow \frac{V\rho g}{F} = \frac{V\rho g - V\sigma g}{\left(\frac{F}{K}\right)} \Rightarrow K = \frac{\rho}{\rho - \sigma}$$



Three charges $q_1 = 1\ \mu C, q_2 = -2\ \mu C$ and $q_3 = 3\ \mu C$ are placed on the vertices of the equilateral triangle of side $1.0\ m$. Find the net electric force acting on charge q_1.

Solution : Magnitude of force between q_1 & q_2

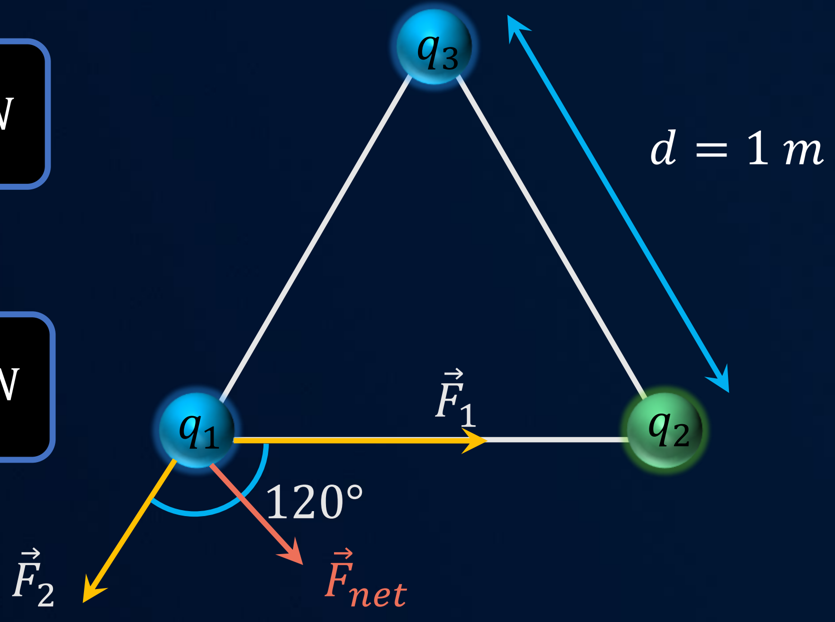
$$F_1 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{(1)^2}$$

$$\Rightarrow F_1 = 1.8 \times 10^{-2} N$$

Magnitude of force between q_1 & q_3

$$F_2 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 3 \times 10^{-6}}{(1)^2}$$

$$\Rightarrow F_2 = 2.7 \times 10^{-2} N$$



$$F_{net} = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 120°}$$

$$F_{net} = 2.38 \times 10^{-2} N$$

Two blocks each of charge $10^{-7}C$ and mass $5\ g$, stay in limiting equilibrium on a horizontal surface. The blocks have a separation of $10\ cm$ between them. Assume the coefficient of friction between each block and the table to be μ. Calculate μ.

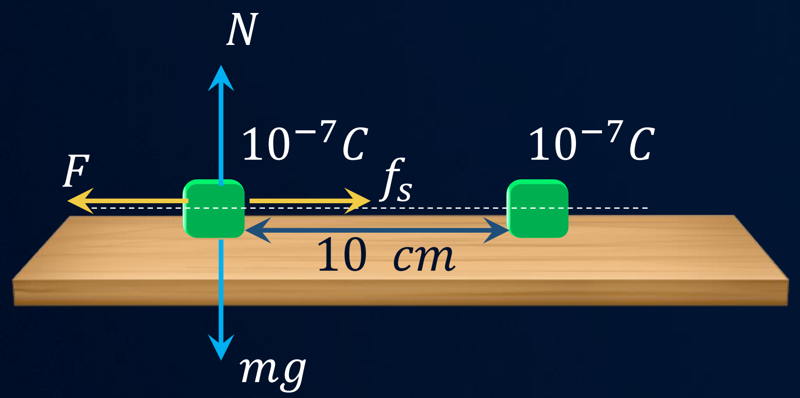
Solution : Net force on a block,

$$F - f_s = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_o}\frac{q \times q}{d^2} = \mu N \qquad [N = mg]$$

$$\Rightarrow \mu = \frac{q^2}{(4\pi\varepsilon_o)mgd^2}$$

$$\mu = 0.18$$



Two points charges Q_1 and Q_2 are $3\ m$ apart and their combined charge is $20\ \mu C$. If one attracts the other with a force of $0.525\ N$. Find the magnitude of the charges.

Solution : Force is attractive, So one these charges has to be negative,

$$-0.525 = \frac{1}{4\pi\varepsilon_o}\frac{Q_1 \times Q_2}{d^2} = \frac{9 \times 10^9 Q_1 Q_2}{9}$$

$$\Rightarrow Q_1 Q_2 = -525\ (\mu C)^2$$

$$\Rightarrow Q_1(20 - Q_1) = -525$$

$$\Rightarrow Q_1^2 - 20Q_1 - 525 = 0$$

$$Q_1 \& Q_2 = 35\ \mu C, -15\ \mu C$$

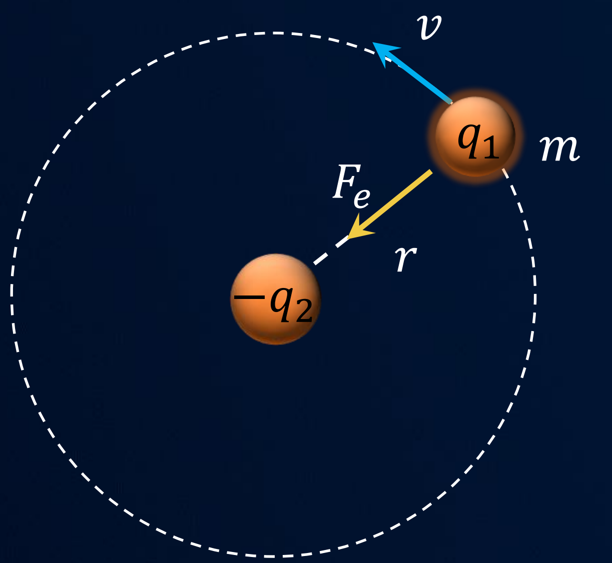
A particle of mass m carrying a charge q_1 is revolving with a uniform speed around a fixed charge $-q_2$ in gravity - free space along a circular path of radius r. Calculate the period of revolution and its speed.

Solution :

q_1 is revolving in a fixed orbit, hence

$$F_e = mr\omega^2$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} = \frac{4\pi^2 mr}{T^2} \Rightarrow T = 4\pi r\sqrt{\frac{\pi\varepsilon_o mr}{q_1 q_2}}$$



Speed of Charge,

$$\frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{q_1 q_2}{4\pi\varepsilon_o mr}}$$

Two identical point charges $+Q$ are fixed in a gravity-free space at points $(L, 0)$ and $(-L, 0)$. Another particle with mass m and charge $-q$ is placed at the origin. Now, this particle is displaced by a distance of y along the $Y-axis$ and then released. Show that this particle will execute SHM, if $y \ll L$.

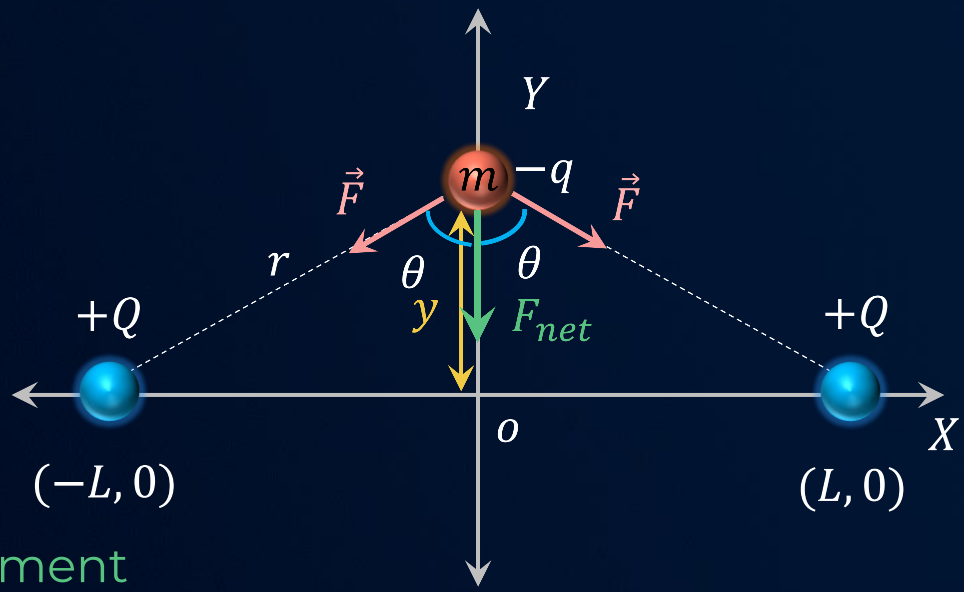
Solution :

Net restoring force on $-q$, $F_{net} = 2F\cos\theta$

$$\Rightarrow F_{net} = 2\left(\frac{KQq}{r^3}\right)y$$

The charge $-q$ is slightly displaced along the $y-axis, y \ll L$

$$\Rightarrow F_{net} = 2\left(\frac{KQq}{L^3}\right)y$$



F_{net} on the charge $-q$ is proportional to its displacement hence it will execute SHM with time period,

$$T = 2\pi\sqrt{\frac{mL^3}{2kQq}}$$

Electrostatic Equilibrium

Stable Equilibrium

- When a charge is displaced from its equilibrium position, it always comes back to its initial equilibrium position.

Unstable Equilibrium

- When a charge is displaced from its equilibrium position, it has no tendency to come back to its initial equilibrium position.

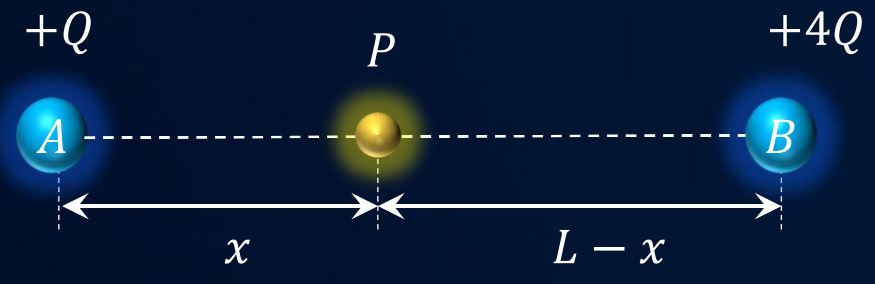
Neutral Equilibrium

- When a charge is displaced from its equilibrium position, it is still in equilibrium condition.

Two free charges $+Q$ and $+4Q$ are placed at a separation L. Find the magnitude, sign and the location of the third charge that can make the system stay in equilibrium.

Solution :

Let the net force be zero on a charge q at point P,



$$|F_{PA}| = |F_{PB}|$$

$$\frac{Qq}{4\pi\varepsilon_o x^2} = \frac{4Qq}{4\pi\varepsilon_o (L-x)^2}$$

$$x = \frac{L}{3}$$

q must be placed at a distance $L/3$ from Q or at $2L/3$ from $4Q$.

If the system is in equilibrium,

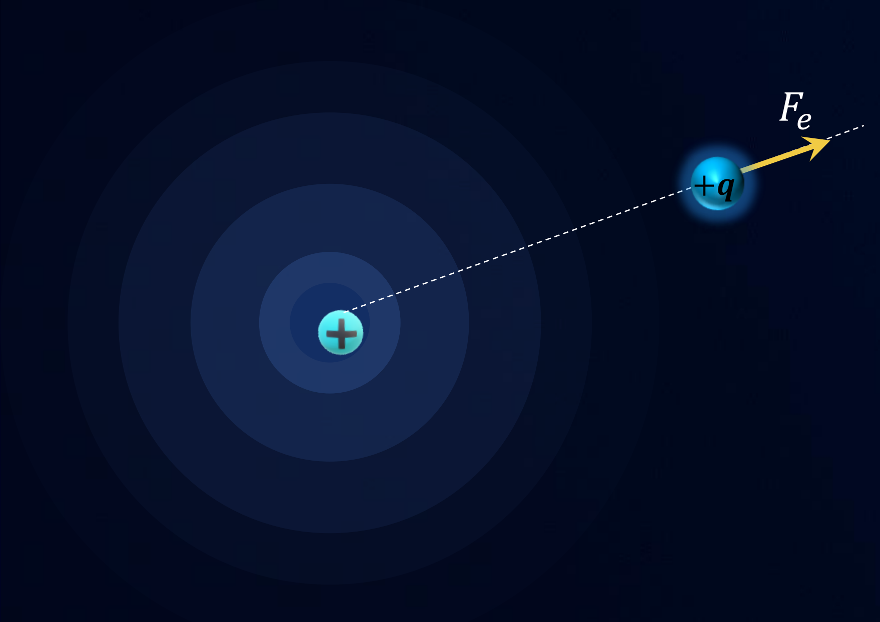
$\sum F_A = 0$ as well. $\quad F_{AP} + F_{AB} = 0$

$$\frac{Qq}{4\pi\varepsilon_o \left(\frac{L}{3}\right)^2} = \frac{-(4Q)Q}{4\pi\varepsilon_o L^2} \Rightarrow$$

$$q = -\frac{4Q}{9}\ C$$

Electric Field

Electric filed is the region surrounding a charge or a distribution of charge in which its electrical effects can be observed.



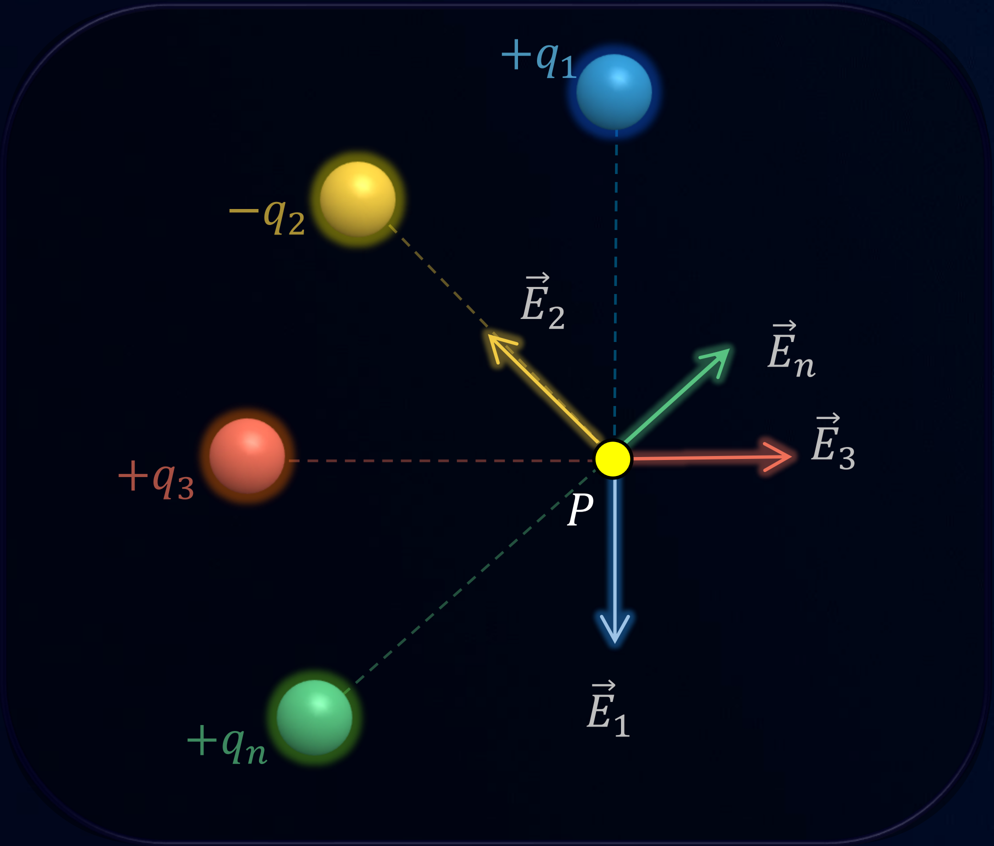
- The nature of the electric field produced by a point charge is non-uniform.
- The direction of the electric field is radially outwards for a positive source charge and radially inwards for a negative one.

The electric field strength (electric field) at a point is defined as the electrostatic force F_e per unit positive charge at that point.

- Unit : N/C
- Dimensional formula : $[MLT^{-3}A^{-1}]$

$$E = \frac{F_e}{q_o} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

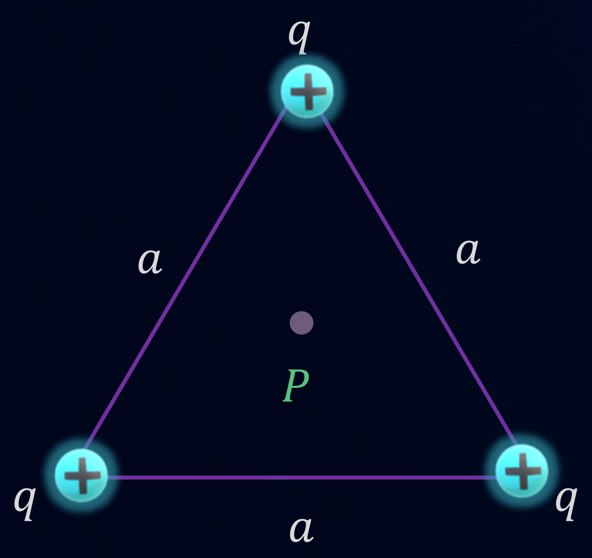
Superposition Principle



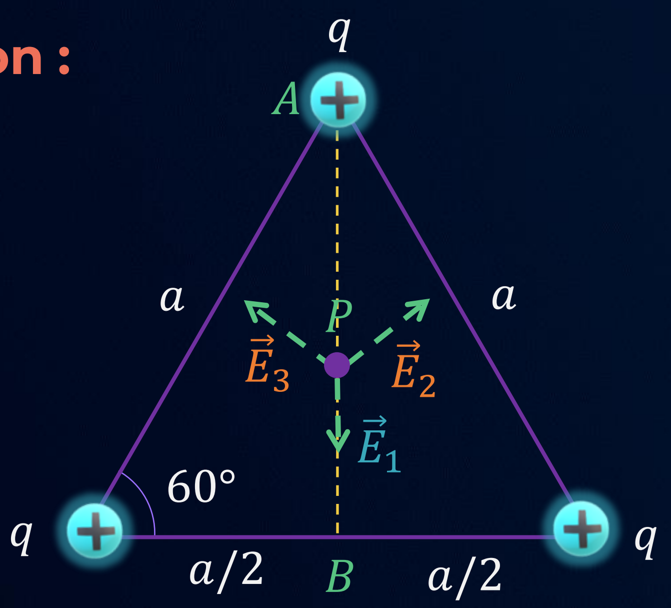
- If n number of charges are present in the space, then the net electric field due to them at a point P,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n$$

Find the net electric field at P (at centroid).

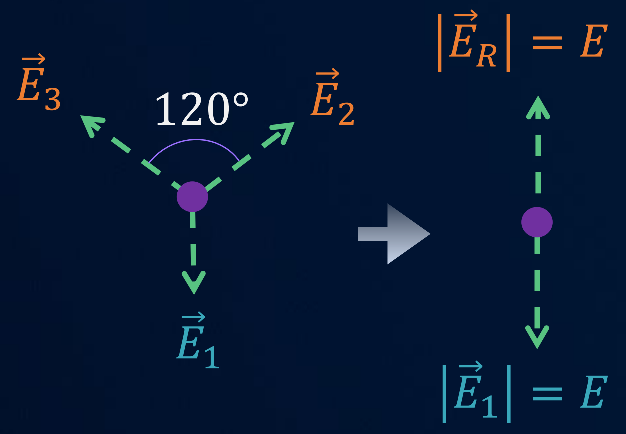


Solution :



$$AB = \frac{a}{2}\tan 60° = \frac{\sqrt{3}a}{2}$$

$$AP = \frac{2}{3} \times \frac{\sqrt{3}a}{2} = \frac{a}{\sqrt{3}}$$



$$E_R = \sqrt{E_2^2 + E_3^2 + 2E_2E_3\cos 120°} = E$$

The resultant $\vec{E}_R$ balances $\vec{E}_1$.

Net electric field at P is zero.

Electric Field for Symmetric Charge Distribution

- Symmetry check

  Applicable to:

  A geometrical configuration of n sides

  Point to check:

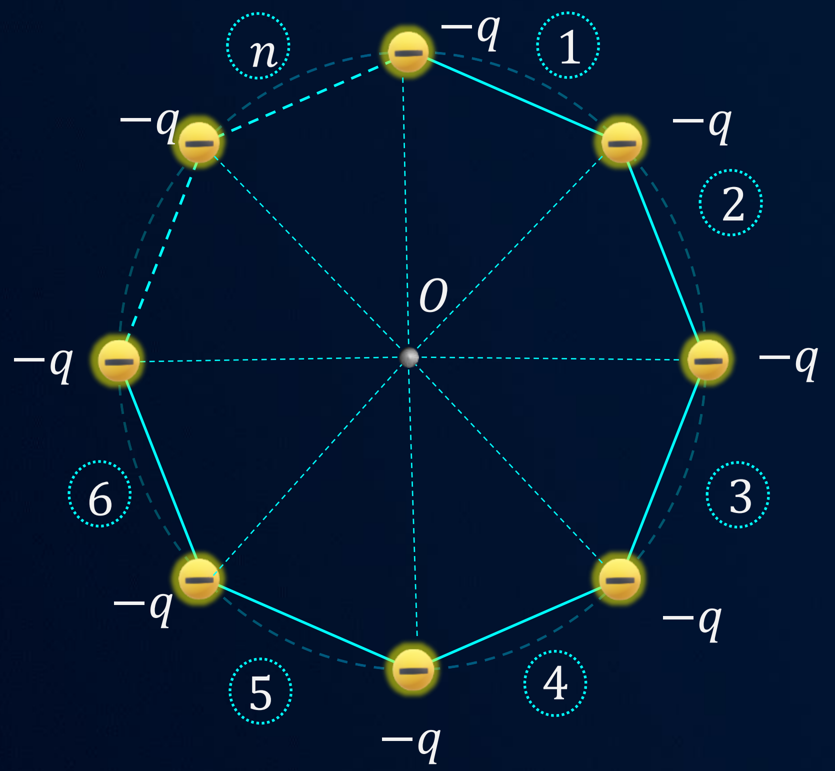
  Configuration remains same after rotation of

  $$\theta = \frac{2\pi}{n}$$

- Regular polygon arrangement

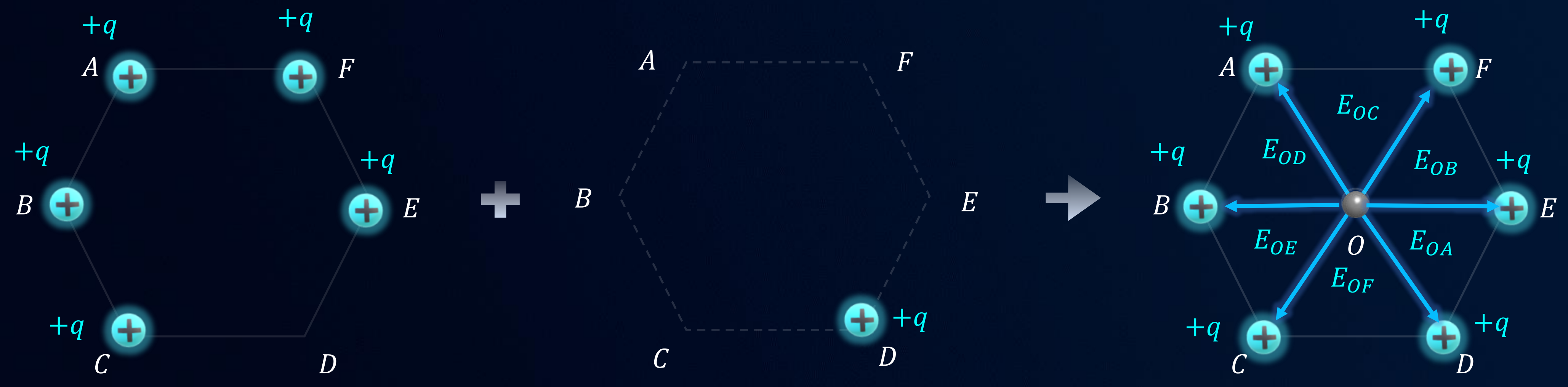
  Due to symmetry, electric field at centre of polygon

  $$\vec{E}_{net} = \vec{E}_o = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 \dots + \vec{E}_n = 0$$



Five charges each of magnitude $+q$ are placed at the corners of a regular hexagon of side a. Find the magnitude of electric field at centre O.

Solution :



Net electric field at centroid O

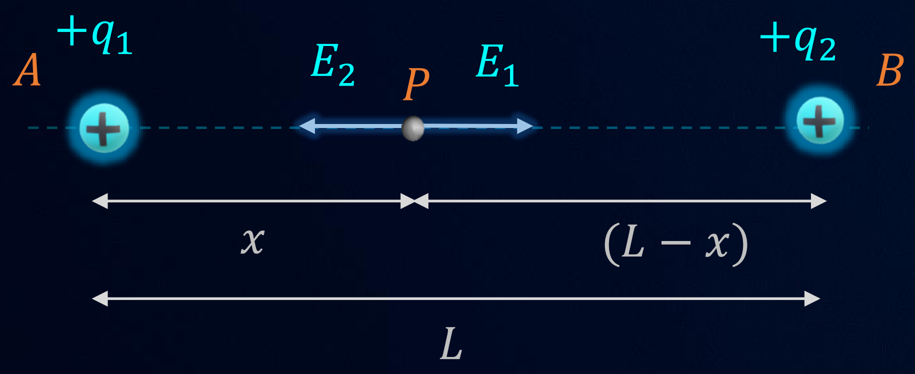
$$\vec{E}_{OA} + \vec{E}_{OB} + \vec{E}_{OC} + \vec{E}_{OD} + \vec{E}_{OE} + \vec{E}_{OF} = 0$$

$$\vec{E}_{OA} + \vec{E}_{OB} + \vec{E}_{OC} + \vec{E}_{OE} + \vec{E}_{OF} = -\vec{E}_{OD}$$

$$\left|\vec{E}_{OA} + \vec{E}_{OB} + \vec{E}_{OC} + \vec{E}_{OE} + \vec{E}_{OF}\right| = \left|\vec{E}_{OD}\right| = \frac{kq}{a^2}$$

Null Point

It is the position where net electrical field comes out to be zero as a vector sum.



Case-1:

For $|+q_1| = |+q_2|$:

$x = L/2$

Null point P on the line AB is equidistant from two charges.

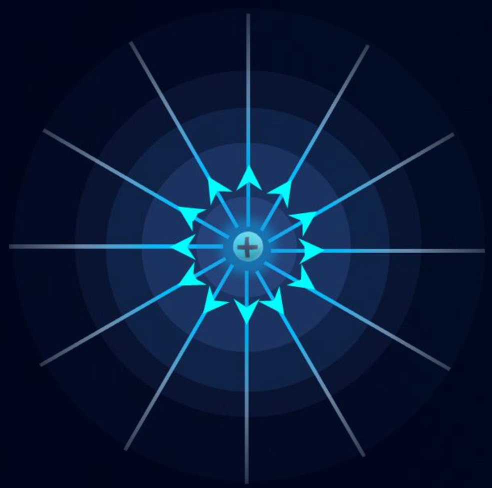
Case-2:

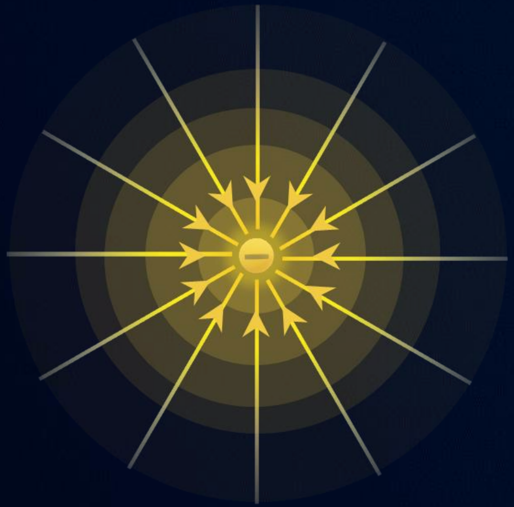
For $|+q_1| < |+q_2|$:

$x < L/2$

Null point is nearer to the charge of smaller magnitude.

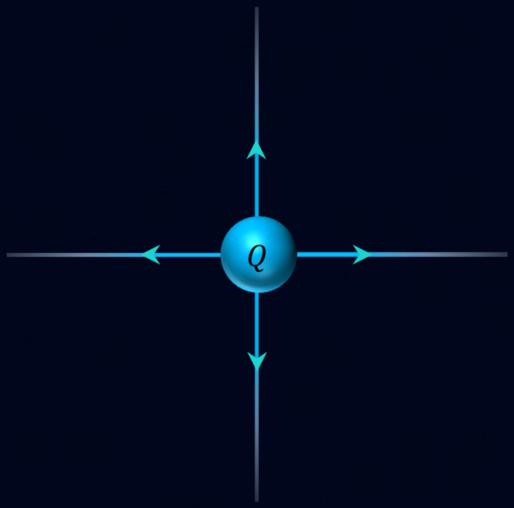
Electric Field lines





Field lines originate from and move away for $+q$

Field lines move toward and terminate for $-q$

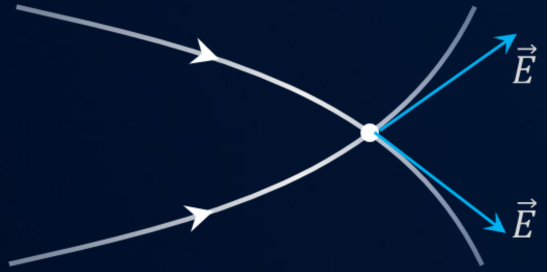


- Electric field lines are imaginary lines or curves drawn in such a way that the tangent to it at each point represents the direction of net electric field at that point. They are also called electric lines of force.

- Number of field lines should be proportional to the magnitude of charge.

- For reference, any number of field lines can be chosen, however proportionality must be maintained.

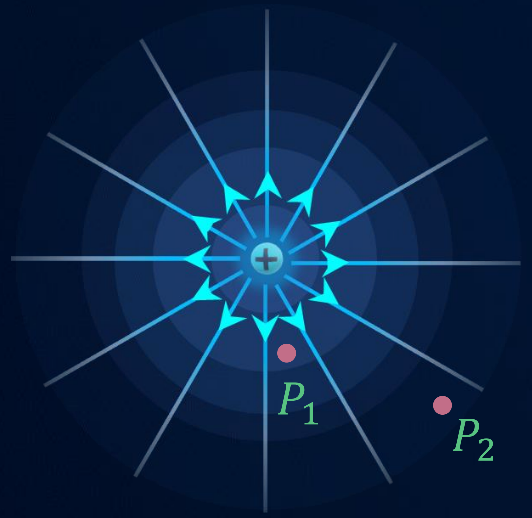
- Electric field lines do not cross each other as there cannot be two direction of $\vec{E}$ at a single point.



Properties of Electric Field Lines

- In a uniform field, the field lines are straight and uniformly spaced .

- The greater the field strength in a region, more denser the field lines will be.

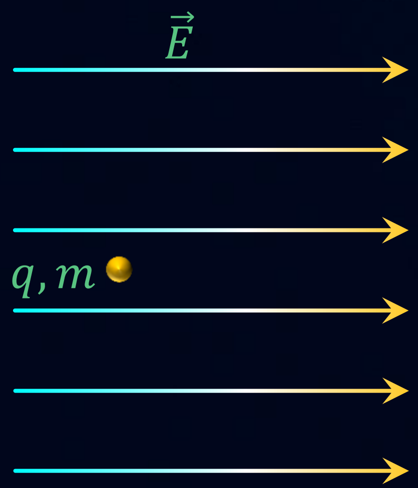


$$|\vec{E}_{P_1}| > |\vec{E}_{P_2}|$$

Motion of a Charged Particle in a Uniform Electric Field

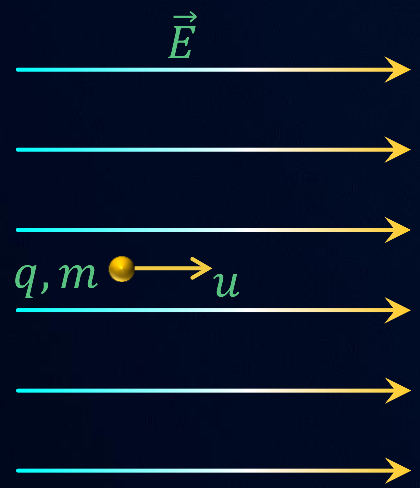
Case - 1

- Charge released in Field



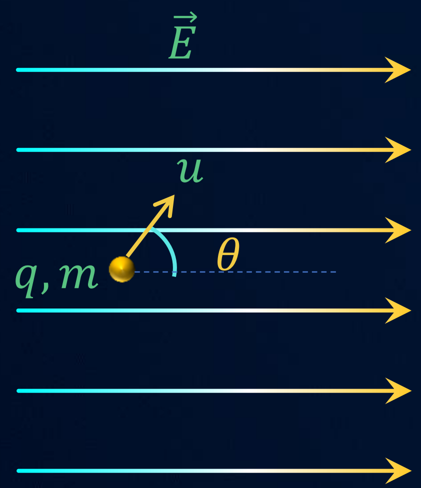
Case - 2

- Charge projected in Field



Case - 3

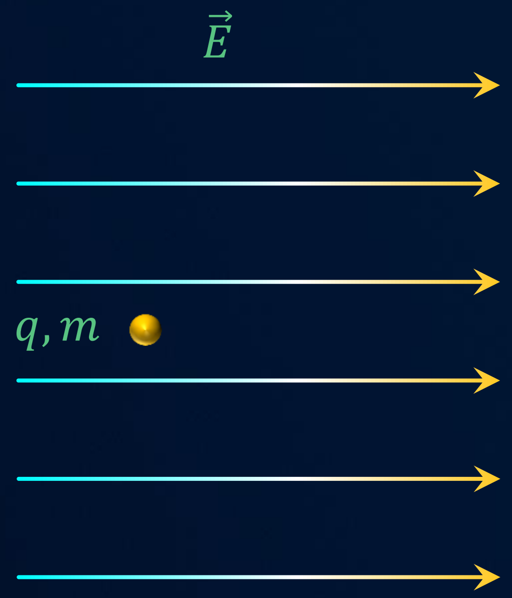
- Charge projected at an angle



Case-1

Assumptions:

- Electric field = uniform
- Initial velocity = Zero
- Gravity is neglected



Results:

Force on the charge $\vec{F} = q\vec{E}$

Acceleration $\vec{a} = \dfrac{q\vec{E}}{m}$

Speed at any instant $v = \dfrac{qE}{m}t$

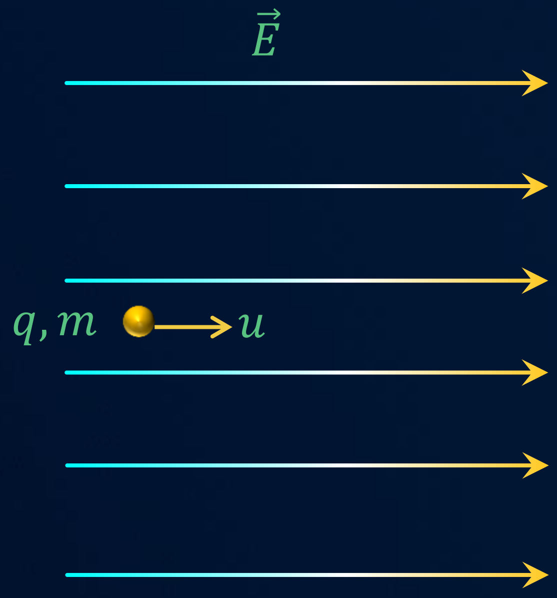
Distance travelled $S = \dfrac{1}{2}|\vec{a}|t^2 = \dfrac{1}{2}\dfrac{qE}{m}t^2$

Kinetic energy $K = \dfrac{1}{2}mv^2 = \dfrac{(qEt)^2}{2m}$

Case-2

Assumptions:

- Electric field = uniform
- Initial velocity = Non-zero
- Gravity is neglected



Results:

Force on the charge $\vec{F} = q\vec{E}$

Acceleration $\vec{a} = \frac{q\vec{E}}{m}$

Speed at any instant $v = u + \frac{qE}{m}t$

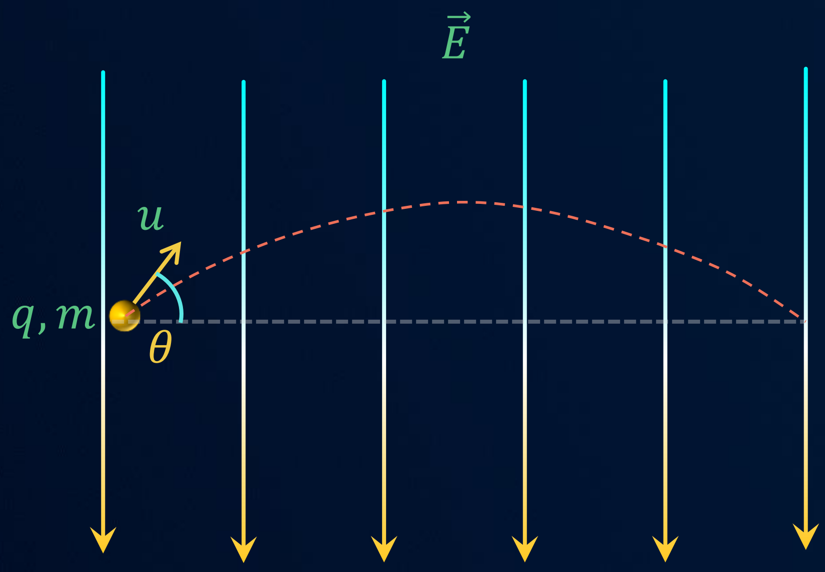
Distance travelled $S = ut + \frac{1}{2}|\vec{a}|t^2 = ut + \frac{1}{2}\frac{qE}{m}t^2$

Kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left[u + \frac{qE}{m}t\right]^2$

Case-3

Assumptions:

- Electric field = uniform
- Initial velocity = Non-zero
- Gravity is neglected



Results:

Force on the charge $\vec{F} = q\vec{E}$

Acceleration $a = \dfrac{qE}{m} = g'$

Time of flight $T = \dfrac{2u\sin\theta}{g'}$

Maximum height $H_{max} = \dfrac{u^2\sin^2\theta}{2g'}$

Horizontal Range $R = \dfrac{u^2\sin 2\theta}{g'}$

Electric Field Due to a Uniformly Charged Arc

Linear charge density: λ

Due to symmetry,

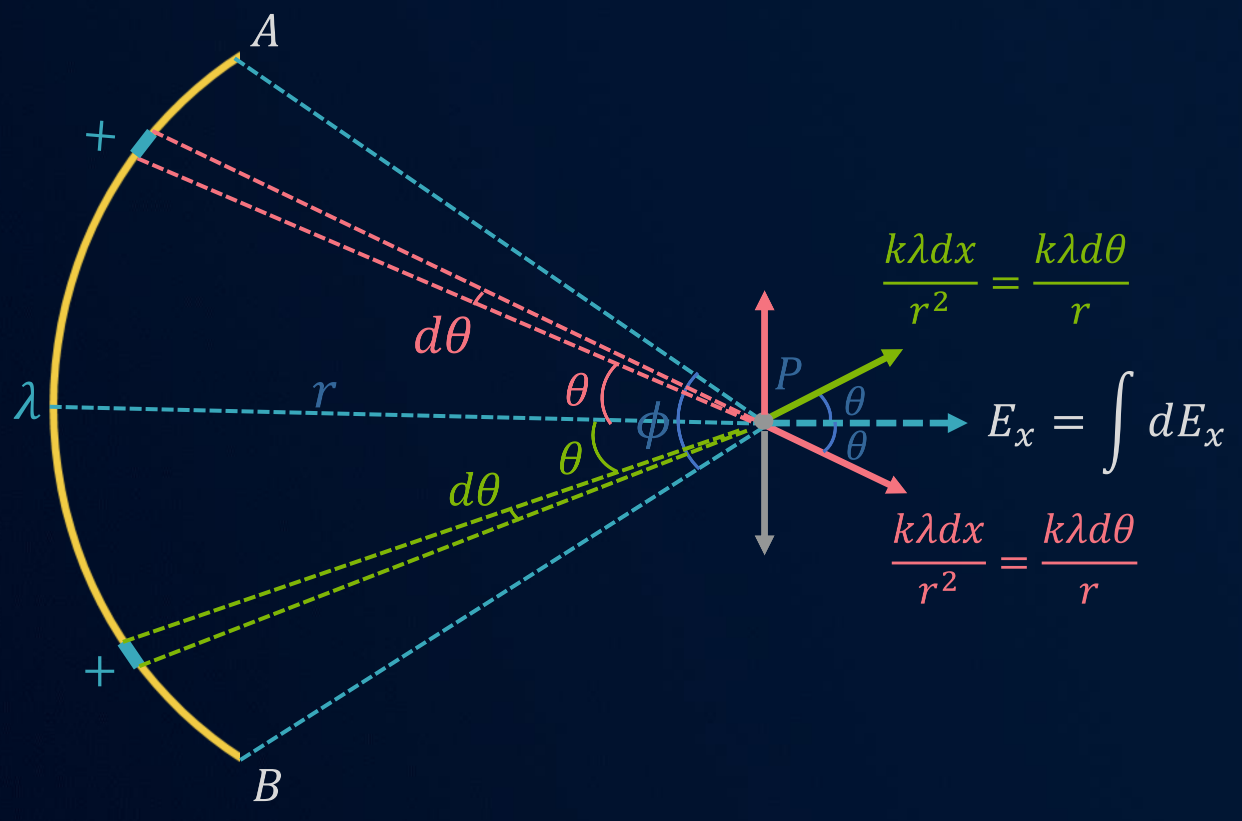
$$\int dE_y = 0$$

Net horizontal component:

$$E_x = \int_{-\frac{\phi}{2}}^{+\frac{\phi}{2}} dE_x = \int_{-\frac{\phi}{2}}^{+\frac{\phi}{2}} \frac{k\lambda}{r}\cos\theta\, d\theta$$

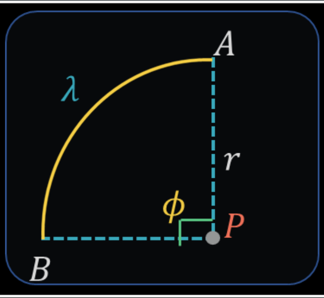
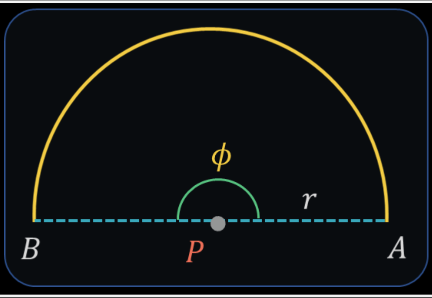
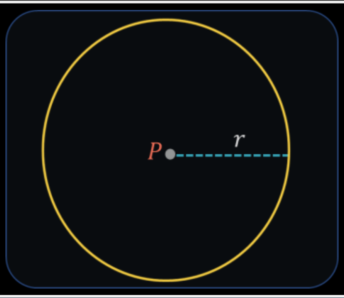
$$E_x = \frac{k\lambda}{r}\left[\sin\theta\right]_{-\frac{\phi}{2}}^{+\frac{\phi}{2}}$$

$$E_x = \frac{2k\lambda}{r}\left(\sin\frac{\phi}{2}\right)$$

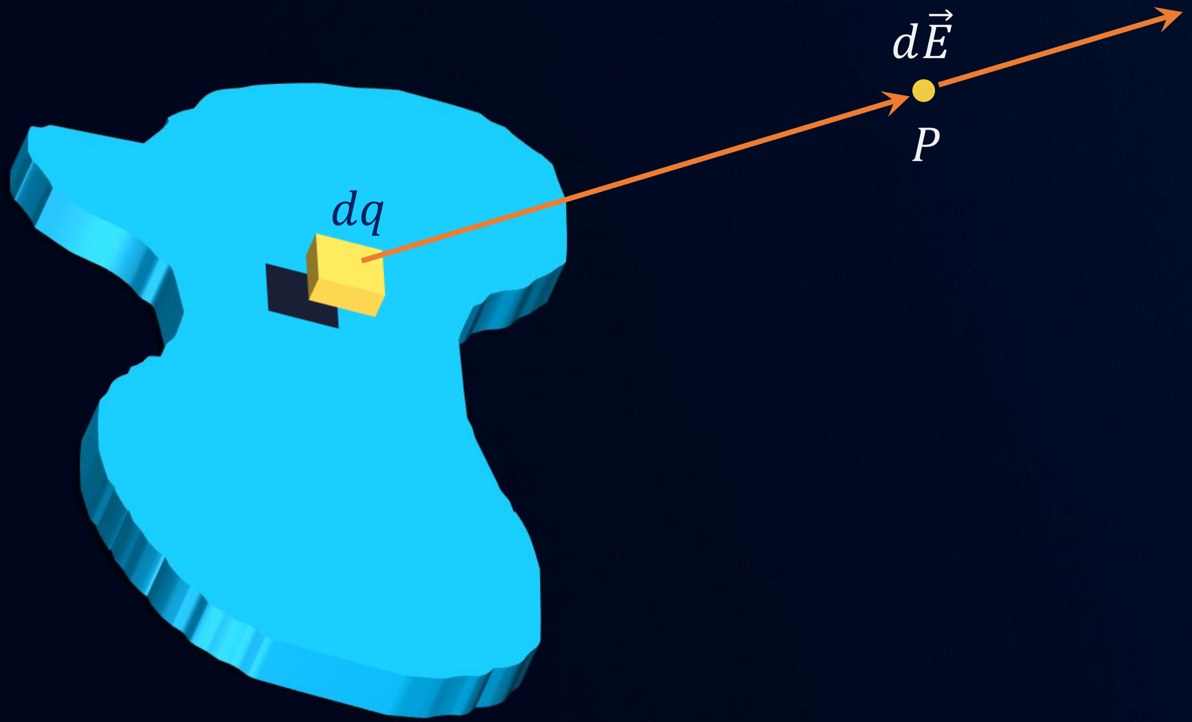


Special Cases: Summary

$$E_{net} = \frac{2k\lambda}{r}\left(\sin\frac{\phi}{2}\right)$$

Case	ϕ	E_{net}	Diagram
Quarter Ring	$90°$	$\frac{\sqrt{2}k\lambda}{R}$	
Semi Ring	$180°$	$\frac{2k\lambda}{R}$	
Complete Ring	$360°$	0	

Continuous Charged Body



- Electric field at point P due to an infinitesimally small, charged element dq of the continuous charged body is,

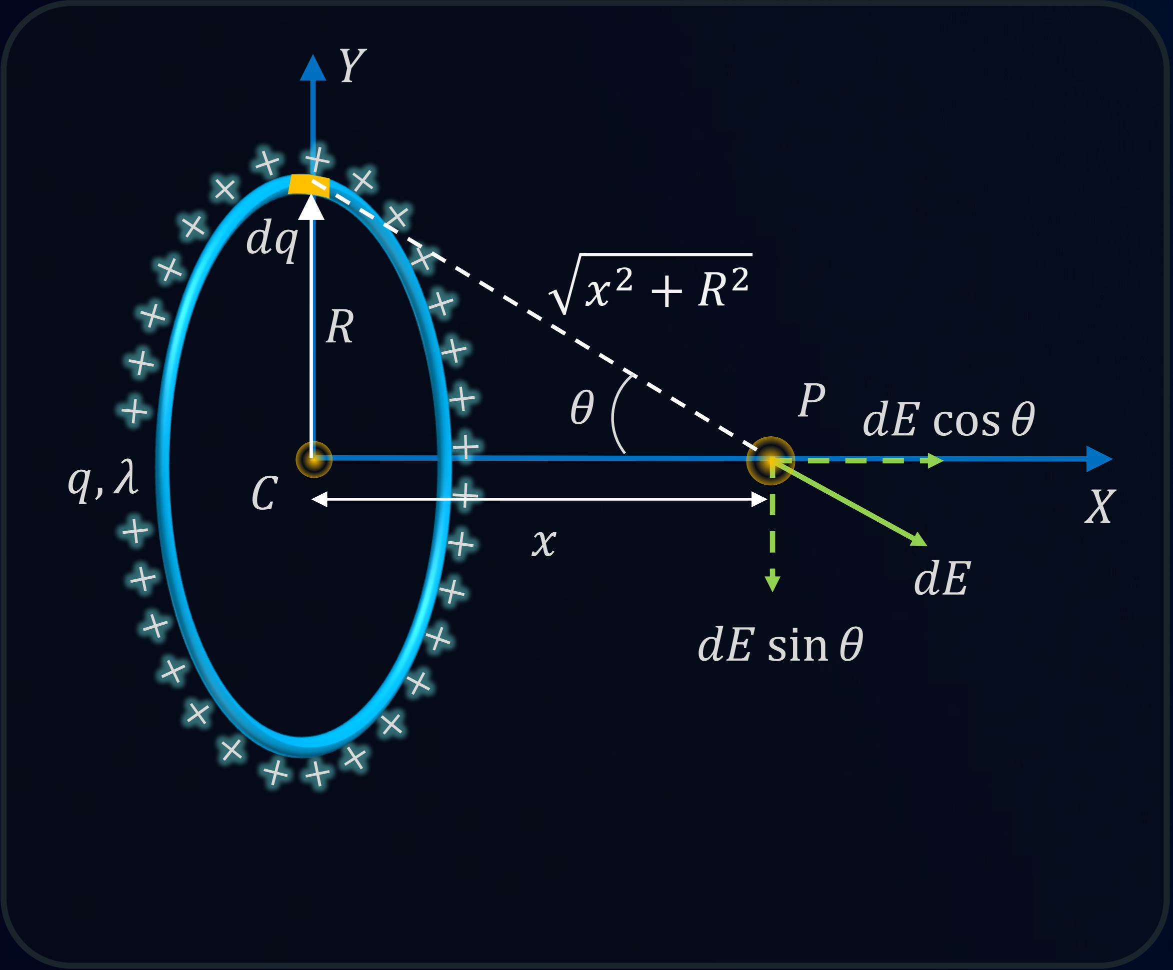
$$d\vec{E} = d\vec{E}_x + d\vec{E}_y + d\vec{E}_z$$

- Net electric field at point P due to continuous charged body is,

$$\int d\vec{E} = \int d\vec{E}_x + \int d\vec{E}_y + \int d\vec{E}_z$$

$$\vec{E}_{net} = \vec{E}_x + \vec{E}_y + \vec{E}_z$$

Axial Electric Field: Uniformly Charged Ring



$$\vec{E}_{net} = \int d\vec{E}$$

$$E_y = \int dE \ \sin\theta$$

(Due to symmetry)

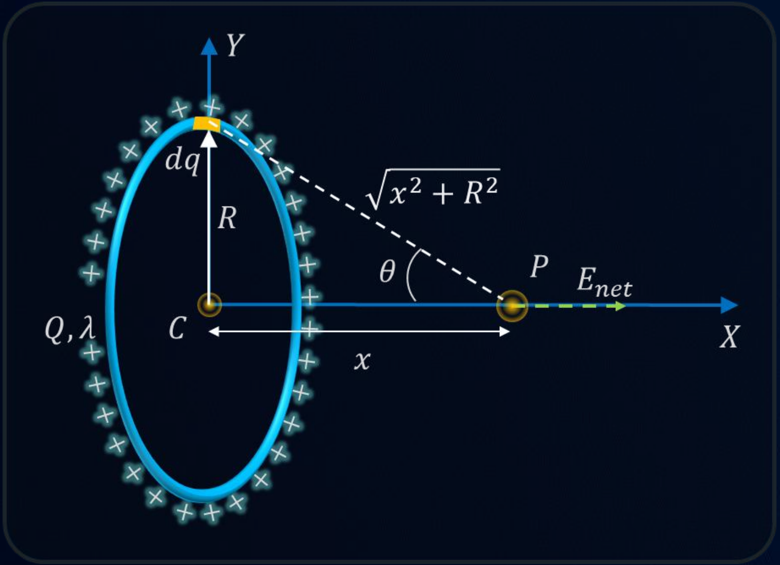
$$E_y = 0$$

$$E_x = \int dE \ \cos\theta$$

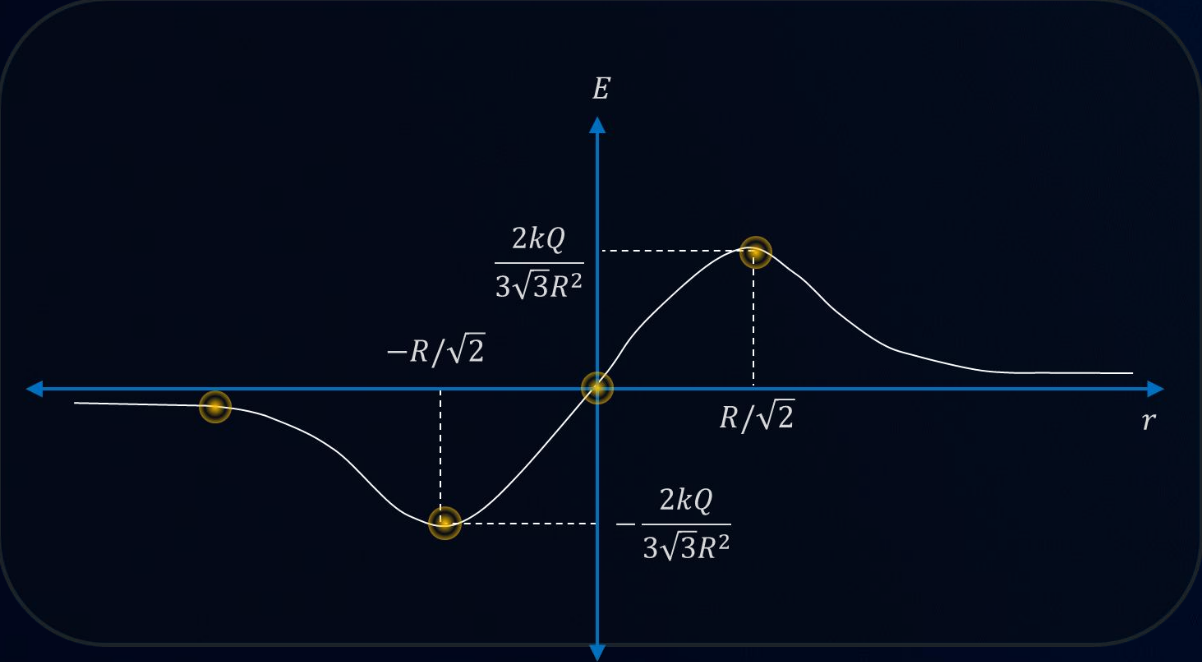
$$E_x = \int \frac{kxdq}{(x^2 + R^2)^{3/2}}$$

$$E_{net} = \frac{kqx}{(x^2 + R^2)^{\frac{3}{2}}}$$

Graph of Electric Field vs Distance



$$E_{net} = \frac{kqx}{(x^2 + R^2)^{\frac{3}{2}}}$$



- At $x = 0$: $E_{net} = 0$
- At $x \gg R$: $E_{net} = \frac{kQ}{x^2}$
- At $x \to \infty$: $E_{net} \to 0$
- At $x = \frac{R}{\sqrt{2}}$: $E_{net} = \frac{2kQ}{3\sqrt{3}R^2}$
- At $x = -\frac{R}{\sqrt{2}}$: $E_{net} = \frac{2kQ}{3\sqrt{3}R^2}$

Total charge $-Q$ is uniformly spread along the length of a ring of radius R. A small test charge $+q$ of mass m is kept at the center of the ring and is given a gentle push along the axis of the ring. Prove that the small charge will oscillate performing SHM.

Solution :

- For $|x| \ll R$, Electric field due to a uniformly charged ring is,

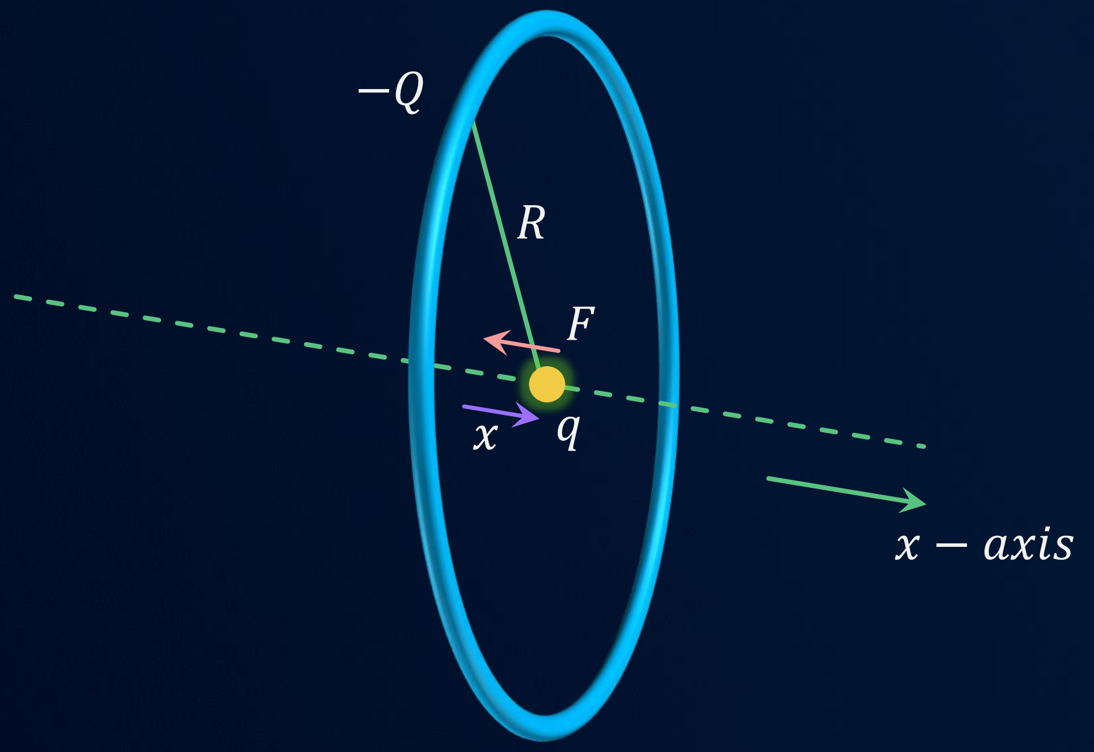
$$|E_{net}| = \frac{1}{4\pi\epsilon_0}\frac{Qx}{R^3}$$

- Electrostatic force on a charged particle q at $x(\ll R)$,

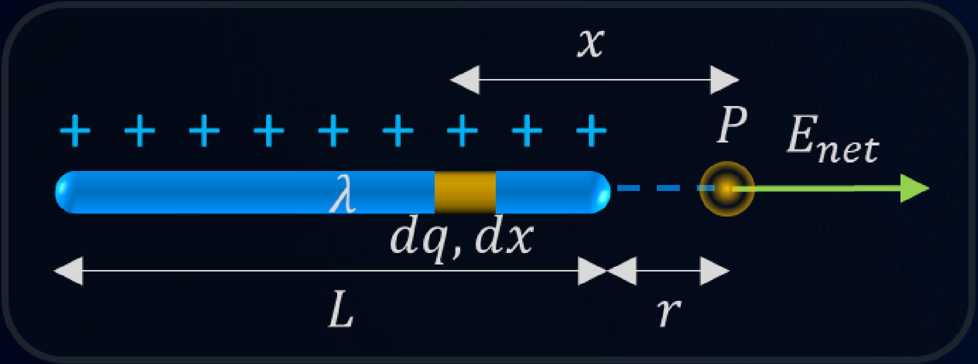
$$F = -\frac{1}{4\pi\epsilon_0}\frac{Qqx}{R^3}$$

$\Rightarrow F \propto -x$

$\Rightarrow$ Particle will execute SHM.



Electric Field at an Axial and Non - Axial Point due to a Rod

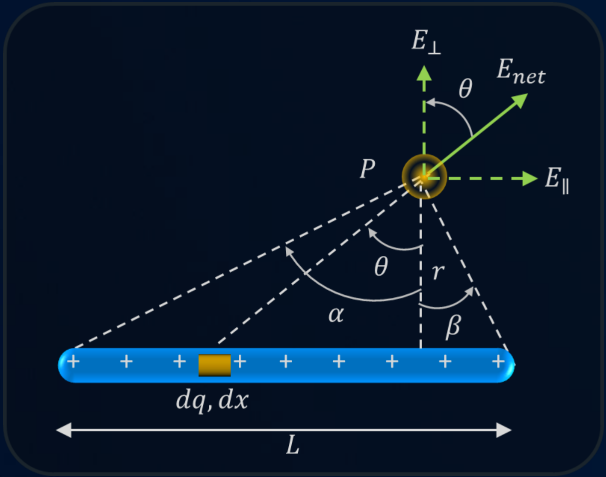


Find dq → $dq = \lambda\, dx$

Find dE → $dE = \dfrac{k\, dq}{x^2}$

Put dq in dE → $dE = \dfrac{k\lambda}{x^2} dx$

$\vec{E}_{net} = \int d\vec{E}$ → $E_{net} = k\lambda\left[\dfrac{1}{r} - \dfrac{1}{r+L}\right]$



$$\vec{E}_{net} = \int d\vec{E}$$

$$E_{\perp} = \frac{k\lambda}{r}[\sin\alpha + \sin\beta]$$

$$E_{\parallel} = \frac{k\lambda}{r}[\cos\beta - \cos\alpha]$$

$$E_{net} = \sqrt{E_{\perp}^2 + E_{\parallel}^2}$$

And

$$\tan\theta = \frac{E_{\parallel}}{E_{\perp}}$$

Special Cases: Summary

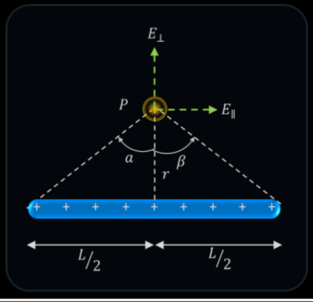
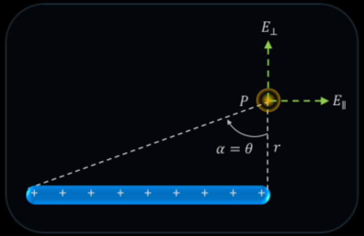
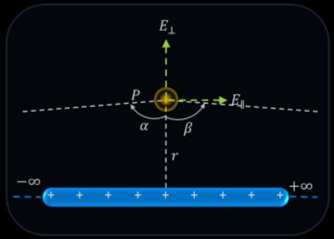
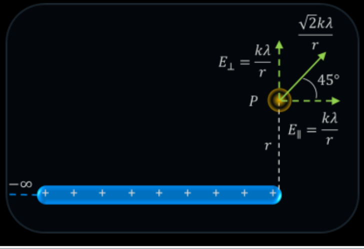
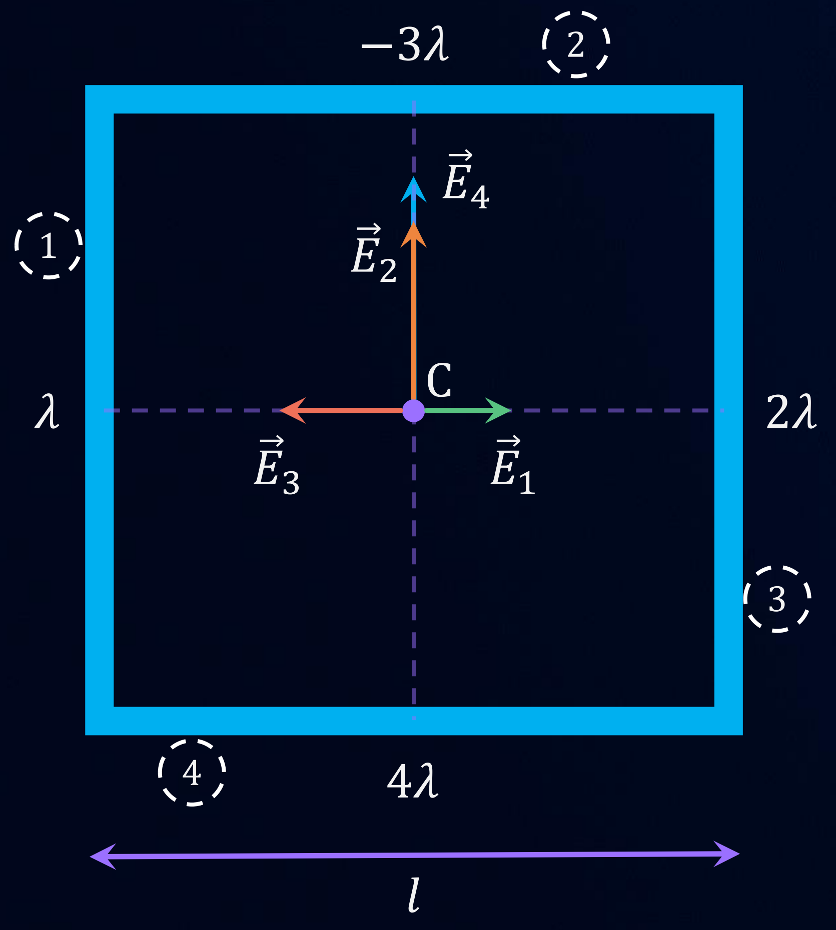
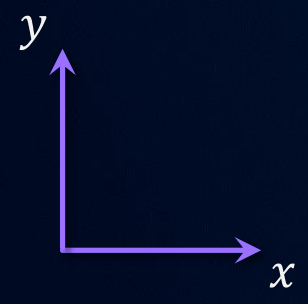
Case	$E_{\perp}$	$E_{\parallel}$	E_{net}	Diagram
On Perpendicular Bisector	$\frac{2k\lambda}{r}\sin\theta$	0	$\frac{2k\lambda}{r}\sin\theta$	
Finite Rod – Along ⊥ to an edge	$\frac{k\lambda}{r}\sin\theta$	$\frac{k\lambda}{r}(1-\cos\theta)$	$\sqrt{E_{\perp}^2+E_{\parallel}^2}$	
Infinite Rod	$\frac{2k\lambda}{r}$	0	$\frac{2k\lambda}{r}$	
Semi Infinite Rod - ⊥ to end point	$\frac{k\lambda}{r}$	$\frac{k\lambda}{r}$	$\frac{\sqrt{2}k\lambda}{r}$	

Figure shows a square of side l of which the four sides are charged with uniform linear charge density $\lambda, -3\lambda, 2\lambda$ and 4λ respectively. Find the electric field strength at the centre of square.

Solution :





$$\vec{E}_1 = \frac{2\sqrt{2}k\lambda}{l}\hat{i} \qquad \vec{E}_3 = \frac{4\sqrt{2}k\lambda}{l} - \hat{i}$$

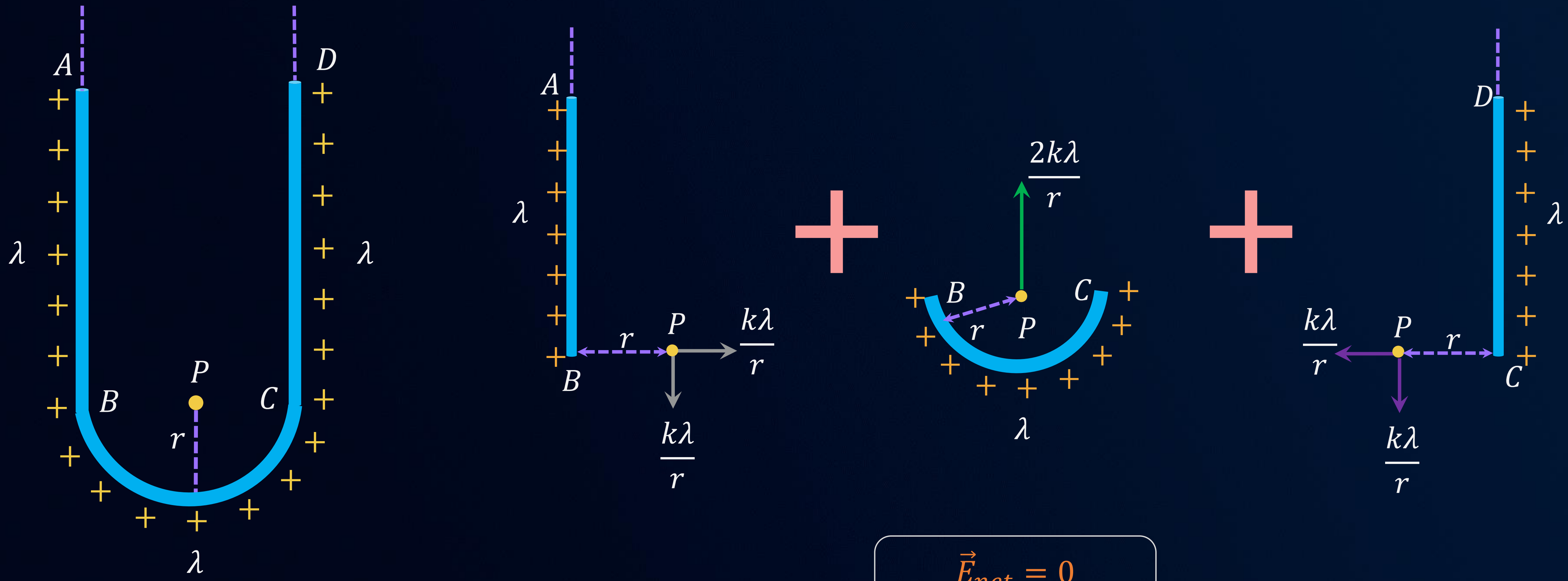
$$\vec{E}_2 = \frac{6\sqrt{2}k\lambda}{l}\hat{j} \qquad \vec{E}_4 = \frac{8\sqrt{2}k\lambda}{l}\hat{j}$$

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

$$\vec{E}_{net} = -\frac{2\sqrt{2}k\lambda}{l}\hat{i} + \frac{14\sqrt{2}k\lambda}{l}\hat{j}$$

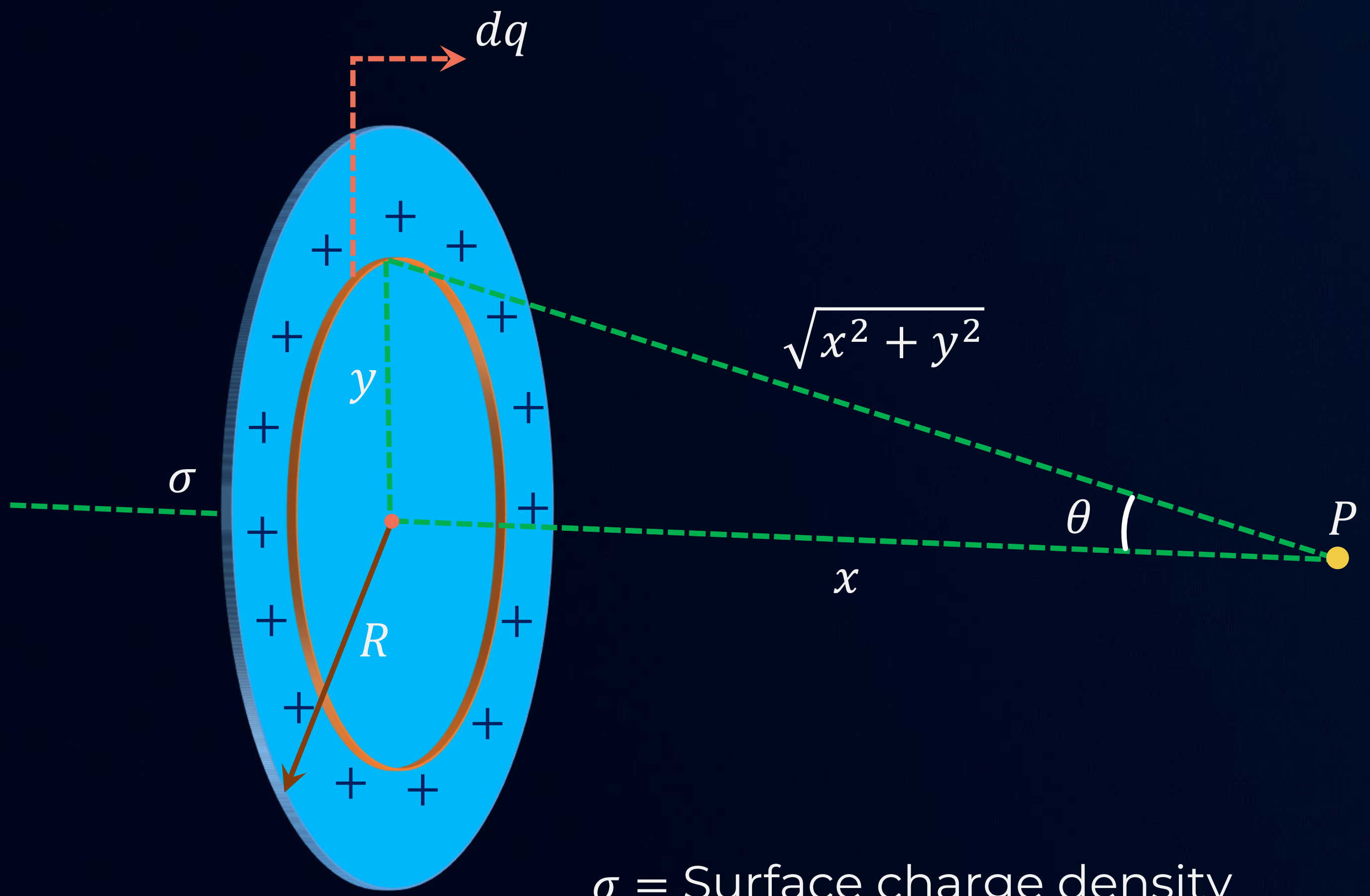
Find electric field at point P for the arrangement consisting of two uniform rods & one uniform semi-circular ring each of linear charge density λ.

Solution :



$$\vec{E}_{net} = 0$$

Electric field due to a thin disc of uniform charge distribution along its axis



σ = Surface charge density

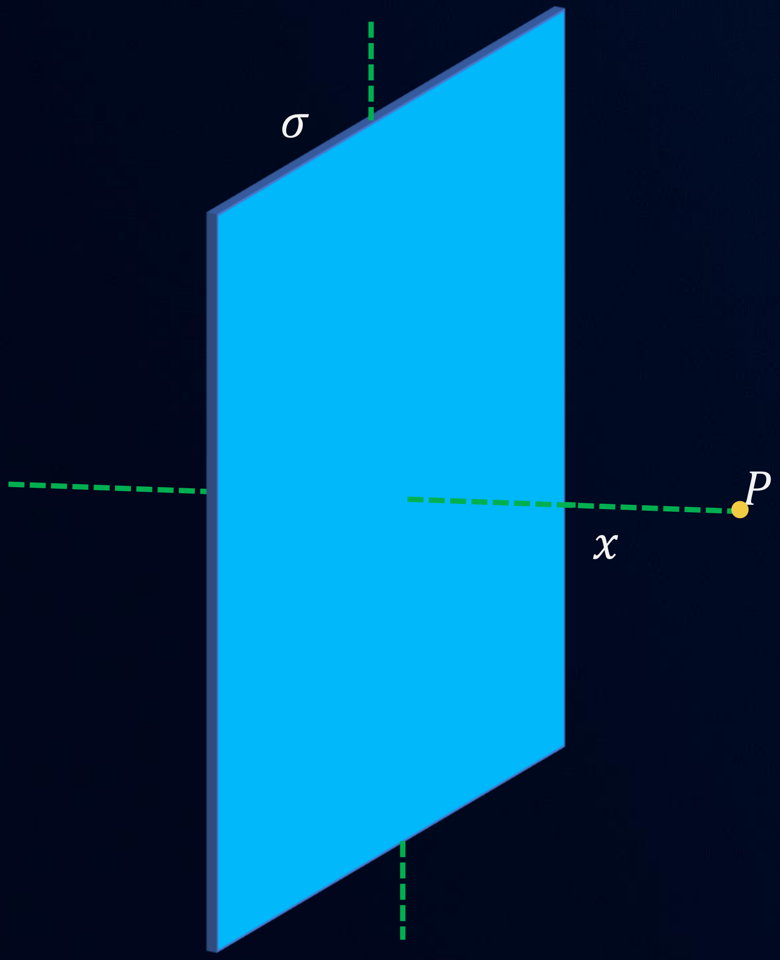
When $y = R, \theta = \phi$

$$E = \frac{\sigma}{2\epsilon_0}(1 - \cos\phi)$$

$$\cos\phi = \frac{x}{\sqrt{x^2 + y^2}}$$

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

Electric field due to an infinitely large thin sheet of uniform charge distribution



σ = Surface charge density

- An infinite sheet is nothing but a disc having an infinite radius.

  Electric field at a distance x from the centre along the axis of a disc is,

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

- For the case of infinite sheet,

  $x \ll R$

$$E = \frac{\sigma}{2\epsilon_0}$$

Two infinite plane parallel sheets, separated by a distance d have equal and uniform charge densities σ. Magnitude of Electric field at a point to the left, between, and right of the sheets are

Solution :

Electric Field at (1)

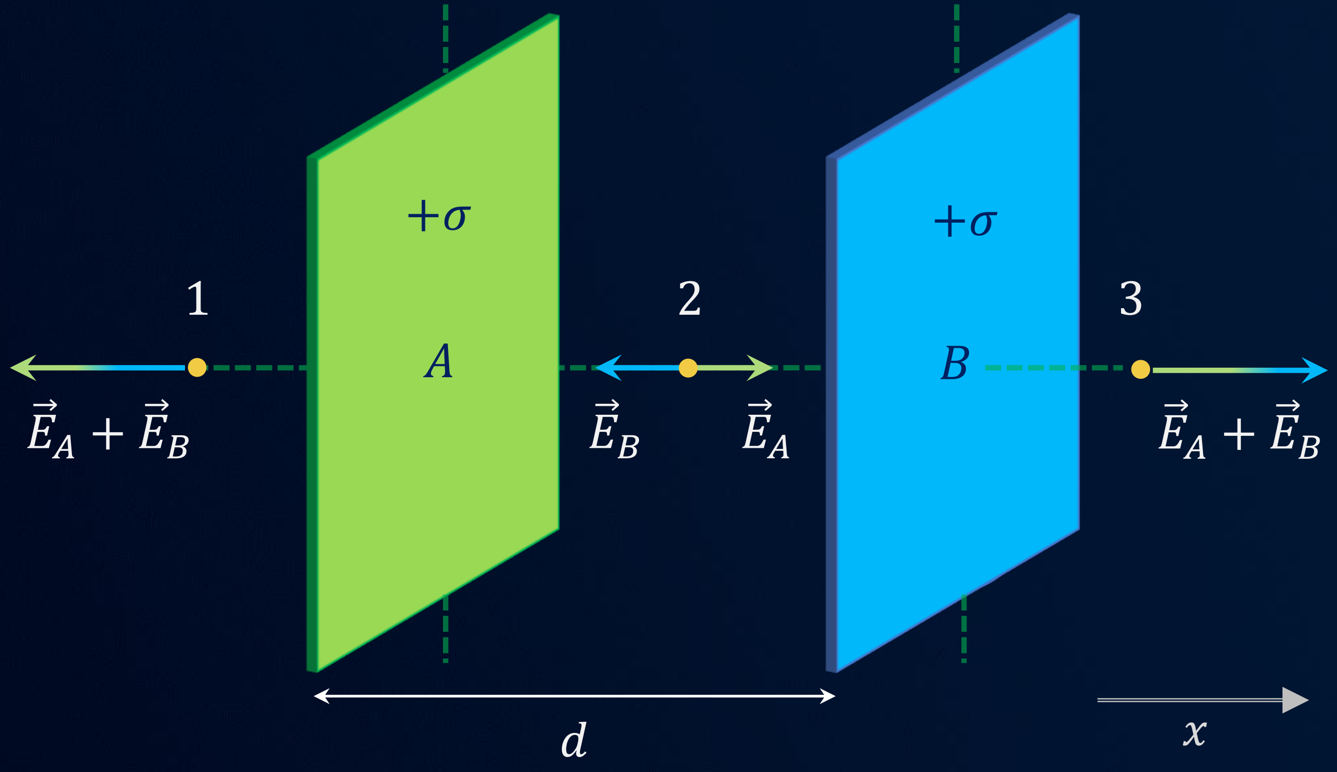
$$\vec{E}_1 = \vec{E}_A + \vec{E}_B = -\frac{\sigma}{2\epsilon_0}\hat{\imath} - \frac{\sigma}{2\epsilon_0}\hat{\imath} = -\frac{\sigma}{\epsilon_0}\hat{\imath}$$

Electric Field at (2)

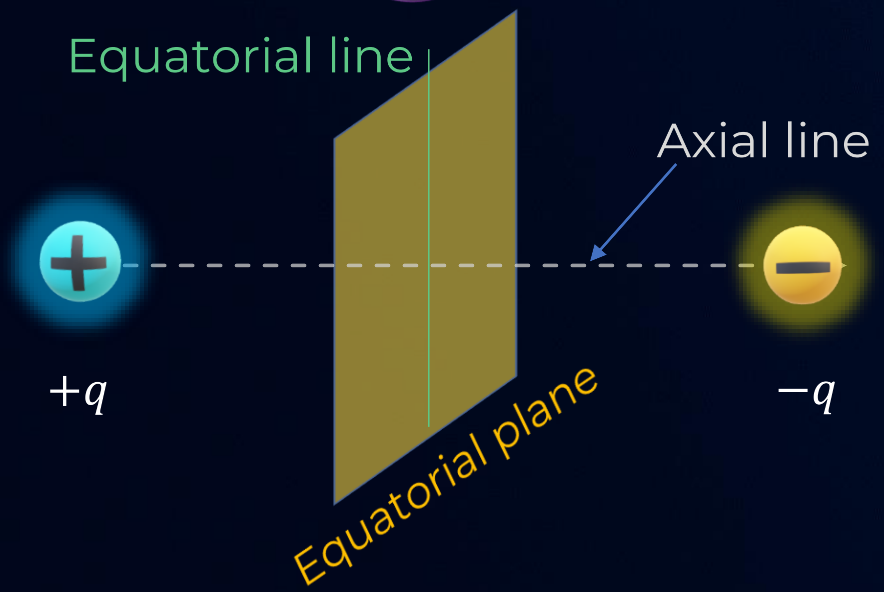
$$\vec{E}_2 = \vec{E}_A + \vec{E}_B = \frac{\sigma}{2\epsilon_0}\hat{\imath} - \frac{\sigma}{2\epsilon_0}\hat{\imath} = \vec{0}$$

Electric Field at (3)

$$\vec{E}_3 = \vec{E}_A + \vec{E}_B = \frac{\sigma}{2\epsilon_0}\hat{\imath} + \frac{\sigma}{2\epsilon_0}\hat{\imath} = \frac{\sigma}{\epsilon_0}\hat{\imath}$$



Electric Dipole



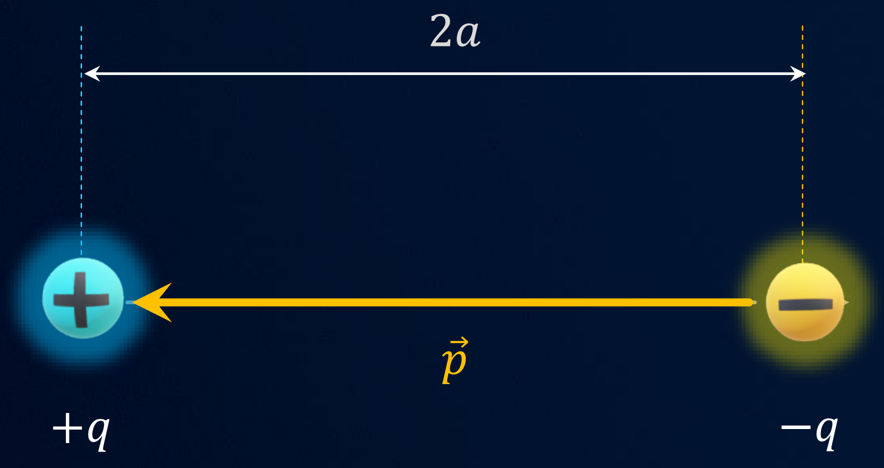
An electric dipole is a system consisting of two point charges, equal in magnitude but opposite in nature, and separated by a small distance.

Dipole Moment ($\vec{p}$)

The dipole moment of an electric dipole is a vector quantity.

$|\vec{p}| = q(2a)$

SI Unit: Coulomb-metre (Cm)



Magnitude: The product of the magnitude of either of the charges and the separation distance between them.

Direction: It is along the axis of the dipole (directed from the negative charge to positive charge)

Find the electric dipole moment of the equilateral triangle formed by three charges as shown in the figure.

Solution :

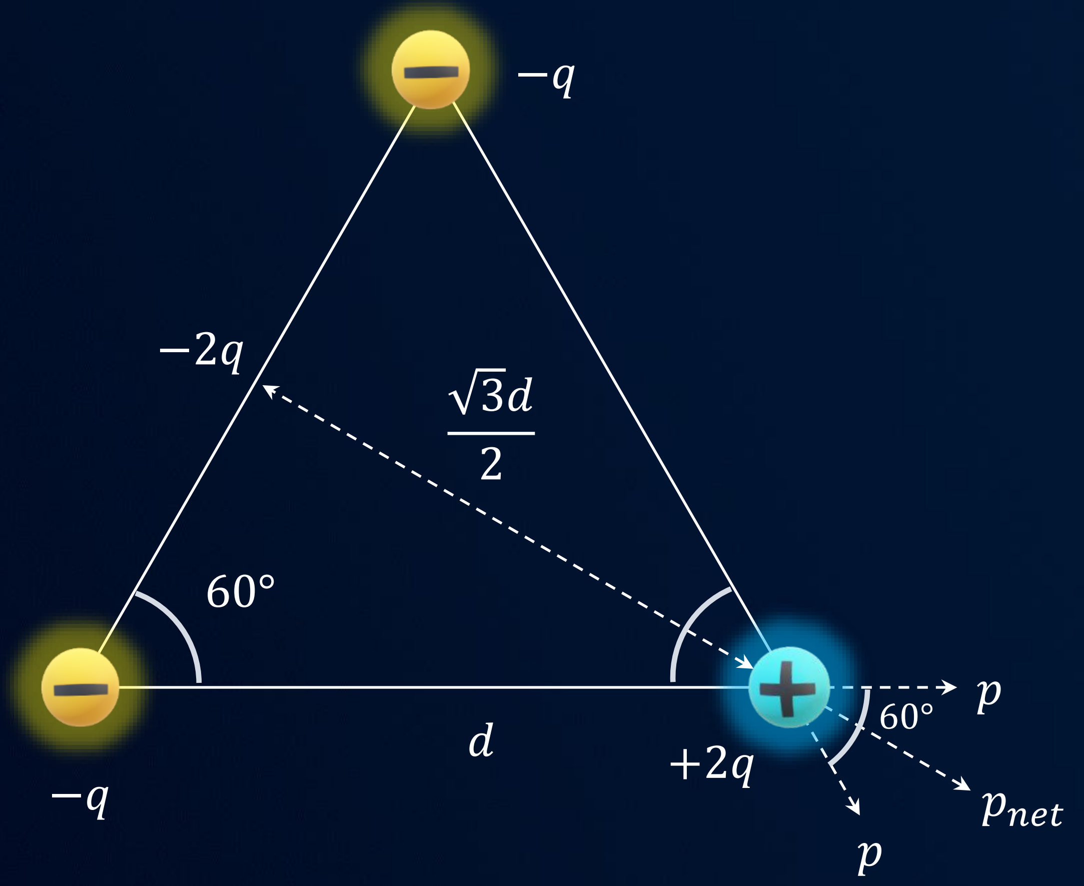
Since, $|\vec{p}| = qd$

The resultant dipole moment of the given configuration is,

$$p_{net} = \sqrt{p^2 + p^2 + 2(p)(p)\cos 60^\circ}$$

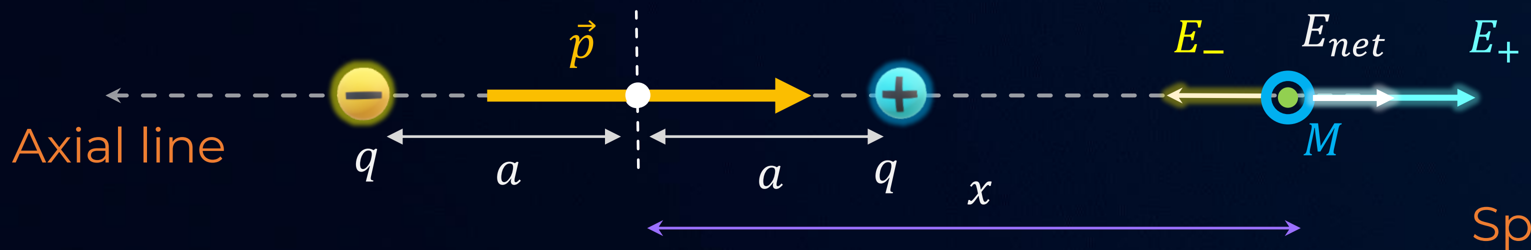
$$p_{net} = \sqrt{3p^2}$$

Thus, $p_{net} = \sqrt{3}qd$



Electric Dipole

Electric field due to a dipole at an axial point

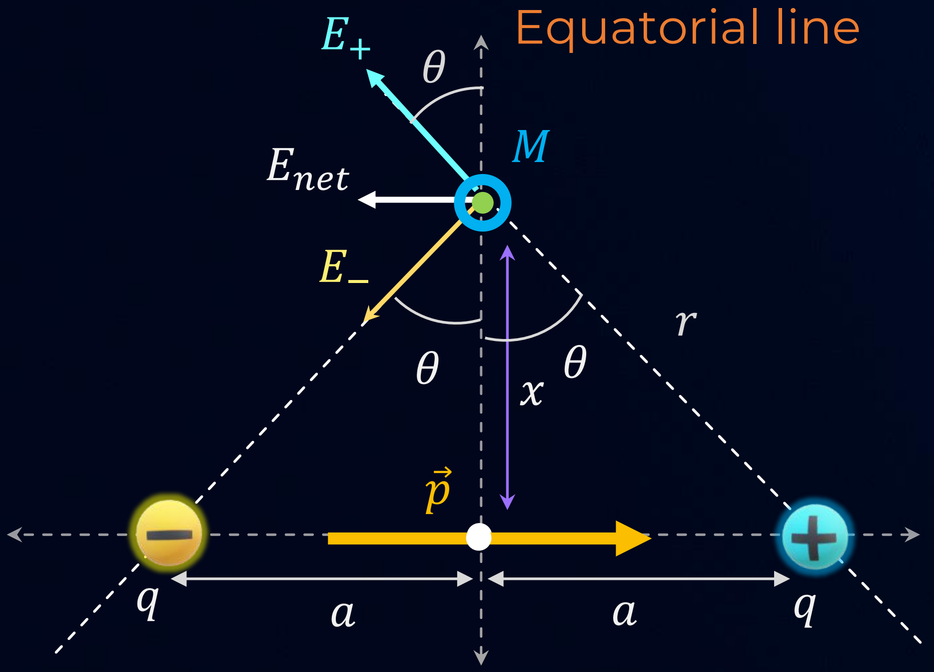


$$\vec{E}_{net} = \frac{2kx\vec{p}}{(x^2 - a^2)^2}$$

Special case : When $x \gg a$,

$$\vec{E}_{net} = \frac{2k\vec{p}}{x^3}$$

Electric field due to a dipole at an equatorial point



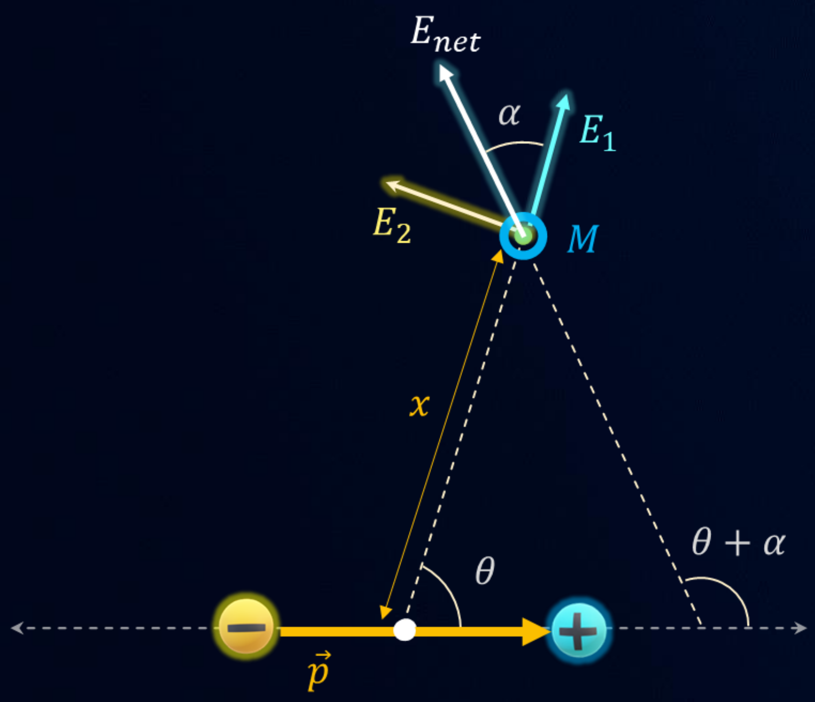
$$\vec{E}_{net} = \frac{-k\vec{p}}{(x^2 + a^2)^{\frac{3}{2}}}$$

Special case : When $x \gg a$,

$$\vec{E}_{net} = -\frac{k\vec{p}}{x^3}$$

Electric Dipole

Electric field due to a dipole at a general point

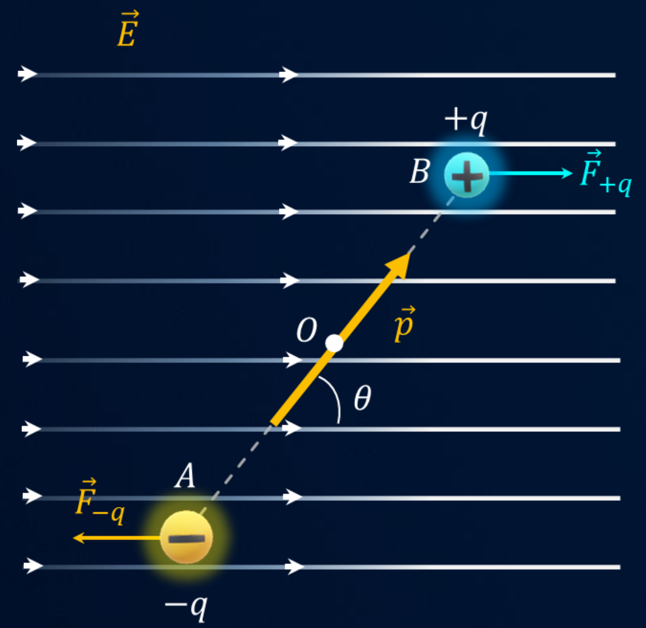


$$E_{net} = \frac{kp(1 + 3\cos^2\theta)^{\frac{1}{2}}}{x^3}$$

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right)$$

The angle between the dipole vector and the net electric field at M is $(\theta + \alpha)$

Torque on a dipole in a uniform electric field



- Net force on dipole is zero.
- Net torque on dipole is,

$$\vec{\tau}_{net} = \vec{p} \times \vec{E}$$

Stable Equilibrium $(\theta = 0°)$

Unstable Equilibrium $(\theta = 180°)$

Forces on a Dipole in a Non-Uniform Electric Field

Consider an electric dipole in a non-uniform electric field as shown.

The force on the charges are,

$$\vec{F}_{+q} = +q(\vec{E} + d\vec{E}) \qquad \vec{F}_{-q} = -q\vec{E}$$

Thus, net force on dipole is,

$$\vec{F}_{net} = \vec{F}_{-q} + \vec{F}_{+q} = qd\vec{E}$$

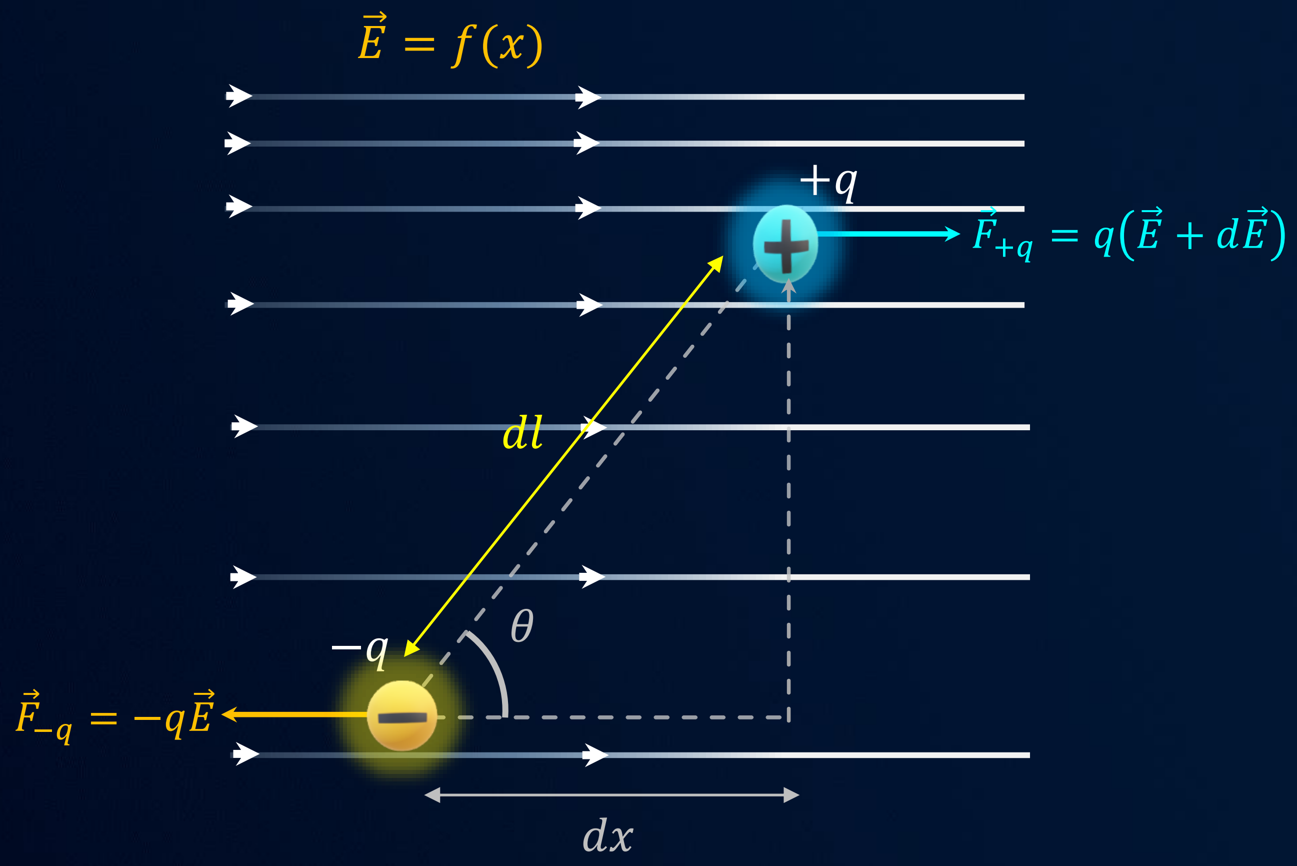
$$F_{net} = q\frac{dE}{dx}dx$$

$$= q\frac{dE}{dx}dl\cos\theta \qquad (\because p = qdl)$$

$$F_{net} = p\left(\frac{dE}{dx}\right)\cos\theta$$

If $\theta = 0°$,

$$F_{net} = p\left(\frac{dE}{dx}\right)$$



An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{\imath}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{\jmath}$, it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is:

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Solution :

$$\vec{p} = p\cos\theta\,\hat{\imath} + p\sin\theta\,\hat{\jmath}$$

$$\vec{E}_1 = E\hat{\imath} \;\;;\;\; \vec{E}_2 = \sqrt{3}E_1\hat{\jmath} = \sqrt{3}E\hat{\jmath}$$

$$\vec{T}_1 = \vec{p}\times\vec{E}_1 = (p\cos\theta\,\hat{\imath} + p\sin\theta\,\hat{\jmath})\times E\hat{\imath}$$

$$\vec{T}_1 = pE\sin\theta\,(-\hat{k})$$

$$\vec{T}_2 = \vec{p}\times\vec{E}_2 = (p\cos\theta\,\hat{\imath} + p\sin\theta\,\hat{\jmath})\times\sqrt{3}E\hat{\jmath}$$

$$\vec{T}_2 = \sqrt{3}pE\cos\theta\,(\hat{k})$$

$$\vec{T}_2 = -\vec{T}_1$$

$$\Rightarrow \sqrt{3}pE\cos\theta\,(\hat{k}) = -pE\sin\theta\,(-\hat{k})$$

$$\Rightarrow \tan\theta = \sqrt{3}$$

$$\Rightarrow \theta = 60^\circ$$

Consider two charges each of $10\ \mu C$ but opposite in sign separated by $5\ mm$. Find the electric field at a point $0.2\ m$ away from the midpoint on a line that is passing through the midpoint and at an angle of $60°$ to the axis of the dipole.

Solution :

We know that the net electric field is,

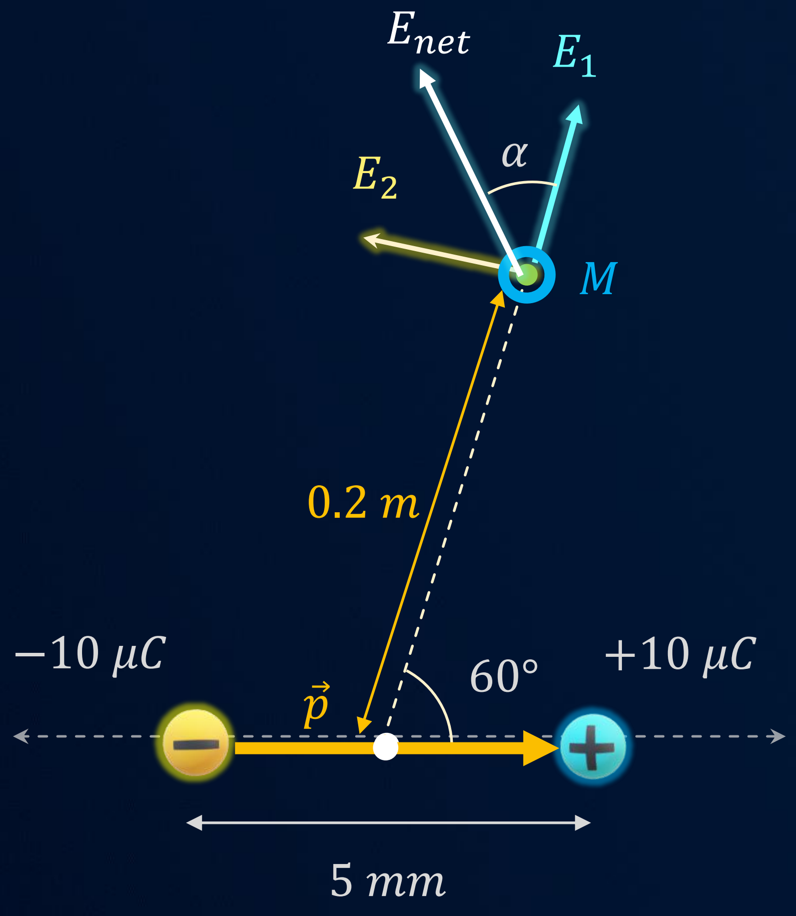
$$E_{net} = \frac{kp(1 + 3\cos^2\theta)^{\frac{1}{2}}}{x^3} = \frac{(9 \times 10^9)(10 \times 10^{-6} \times 5 \times 10^{-3})\left(1 + \frac{3}{4}\right)^{\frac{1}{2}}}{(0.2)^3}$$

$$\alpha = \tan^{-1}\left(\frac{\tan\theta}{2}\right) = \tan^{-1}\left(\frac{\tan 60°}{2}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

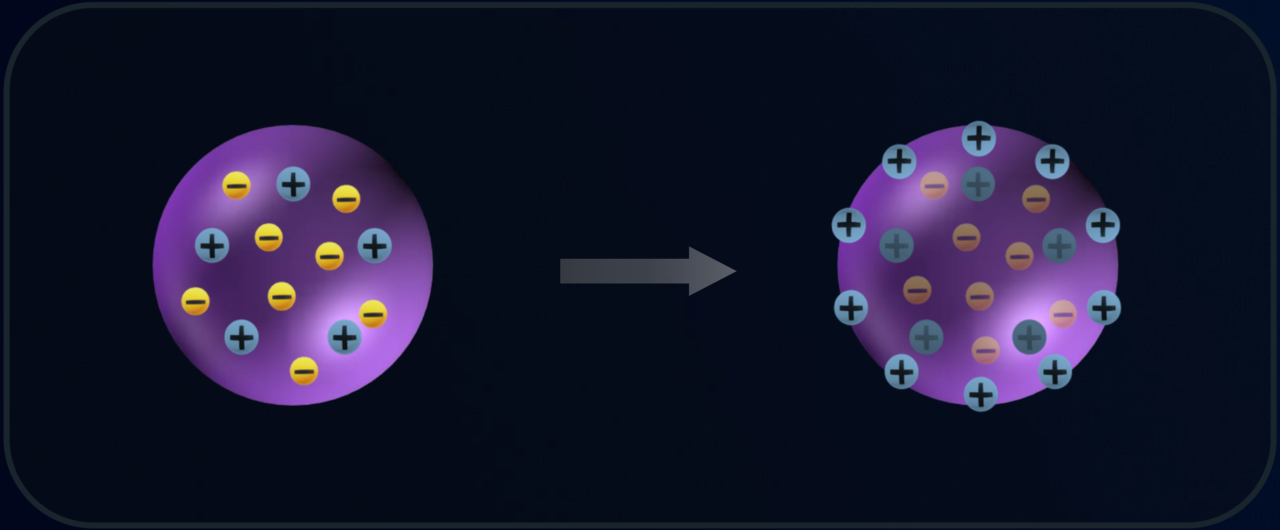
The electric field is, $E_{net} \approx 7.44 \times 10^4\ NC^{-1}$

The angle between the net electric field and dipole vector is,

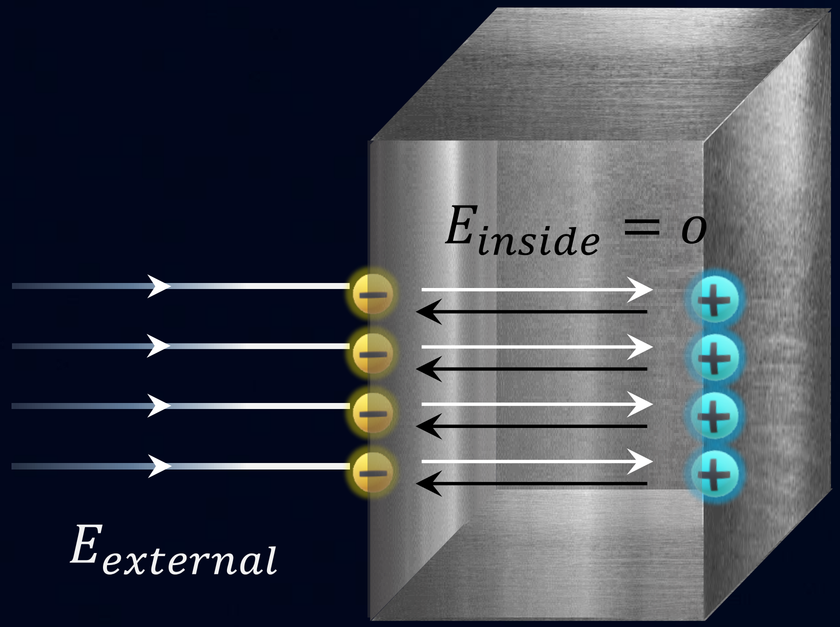
$$\alpha + \theta = 60° + \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$



Properties of a Conductor

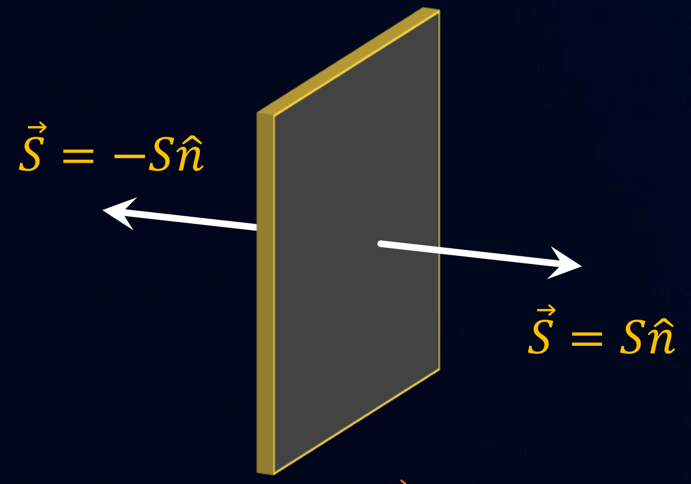


- Excess charge given to an isolated conductor redistributes itself on the surface in order to minimize the potential energy/maximize stability.



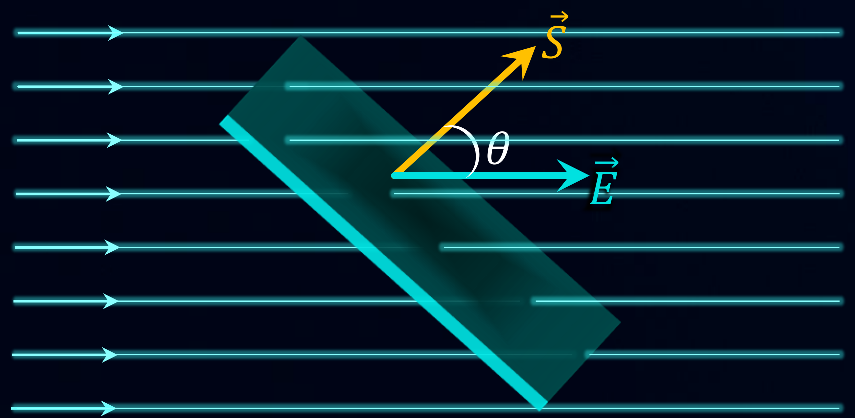
- Electric field lines never exist inside a conductor.
- At steady state, the net electric field inside a conductor is zero.

Electric Flux



$\vec{S} \rightarrow$ Area vector

- Area is a vector quantity.
- The direction of area vector is taken along normal to the surface.
- Direction of normal vector is along $\hat{n}$, magnitude is the area of surface.
- For an open surface, any one of the two normal directions can be considered as positive.



Uniform electric field

- Electric flux is the measure of the net electric lines of force normally crossing a surface.
- The electric flux of the uniform electric field $\vec{E}$ through an area $\vec{S}$ is given by:

$$\phi = \vec{E}.\vec{S}$$

If the electric field $\vec{E} = E\hat{\imath}$, then what will be the net electric flux through a cube of side a ?

Solution :

ϕ due to $\vec{E}$:

$$\vec{A}_1 = -a^2\hat{\imath} \longrightarrow \phi_1 = -Ea^2$$

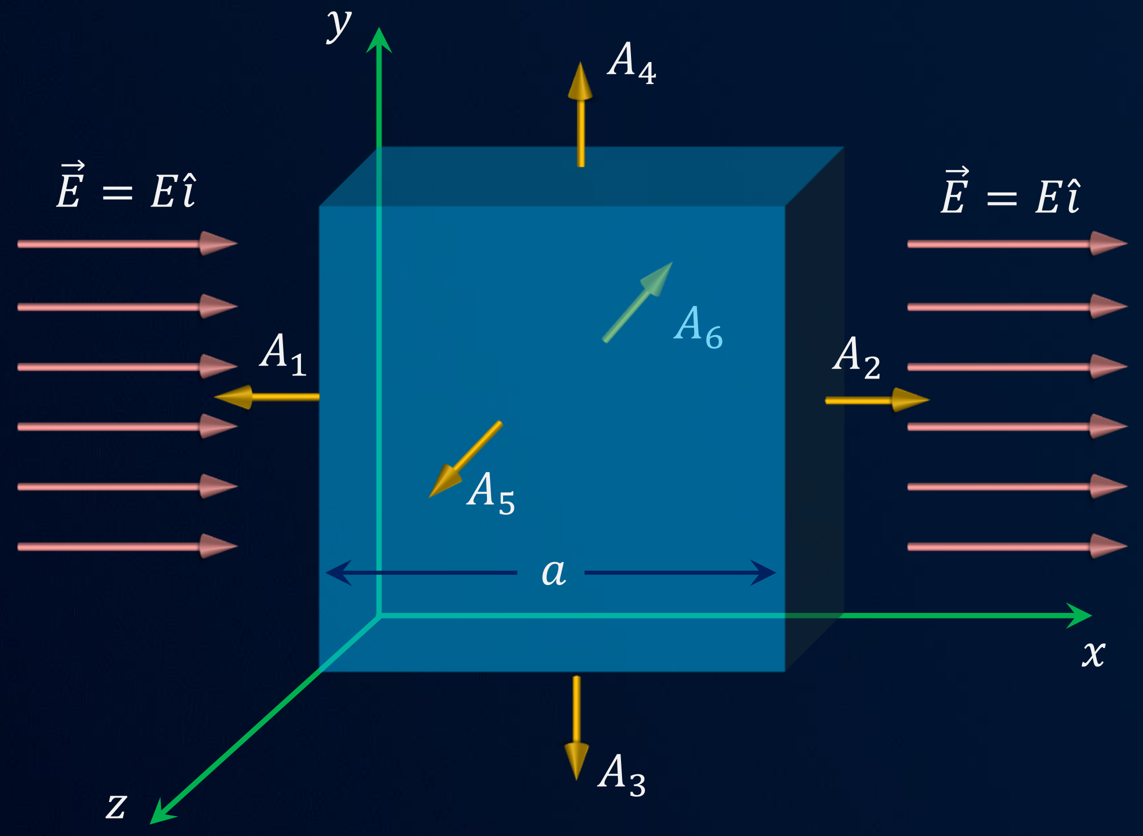
$$\vec{A}_2 = a^2\hat{\imath} \longrightarrow \phi_2 = Ea^2$$

$$\vec{A}_3 = -a^2\hat{\jmath} \longrightarrow \phi_3 = 0$$

$$\vec{A}_4 = a^2\hat{\jmath} \longrightarrow \phi_4 = 0$$

$$\vec{A}_5 = a^2\hat{k} \longrightarrow \phi_5 = 0$$

$$\vec{A}_6 = -a^2\hat{k} \longrightarrow \phi_6 = 0$$

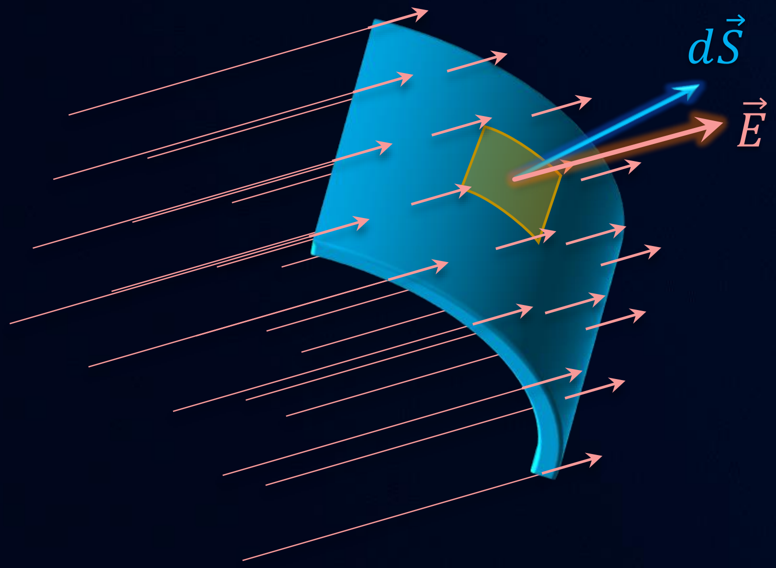


Therefore, the net electric flux through the cube is,

$$\phi_{net} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 0$$

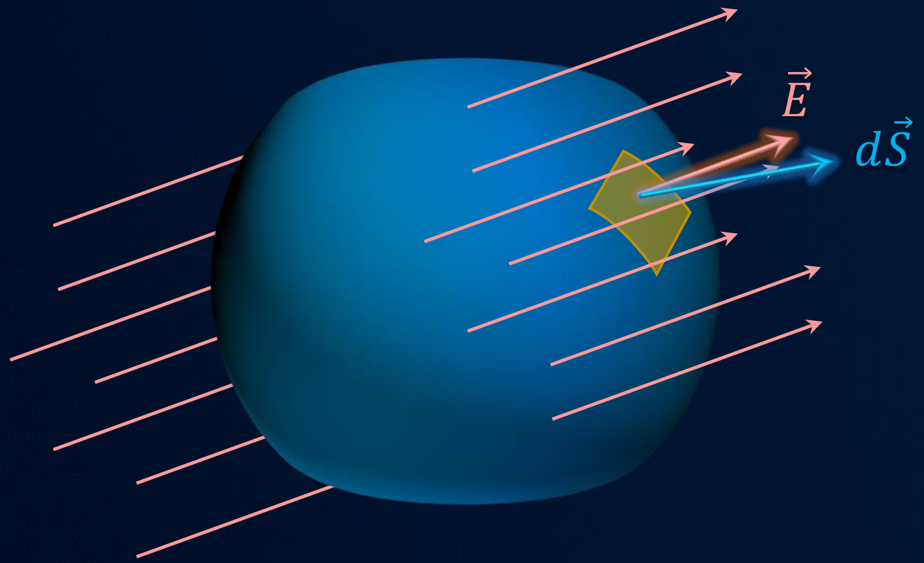
Electric Flux

Electric flux through any curved surface



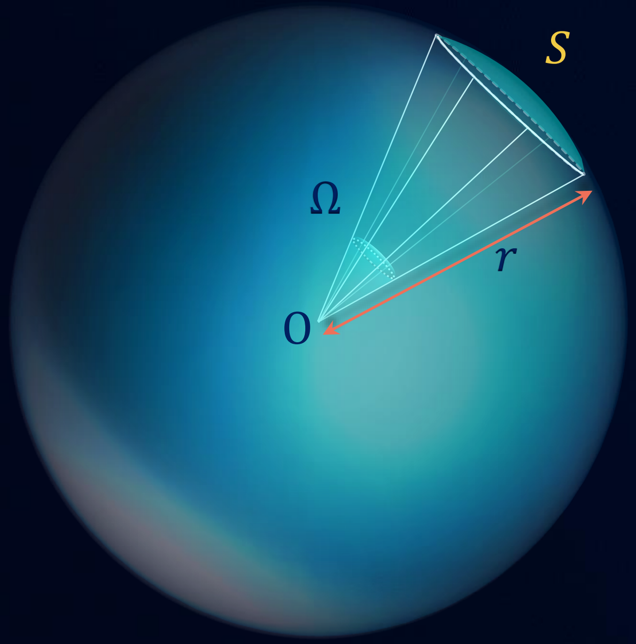
$$\phi = \int \vec{E}.d\vec{S}$$

Electric flux through a closed surface



$$\phi = \oint \vec{E}.d\vec{S}$$

Solid Angle



S → Area of spherical surface intercepted by the cone

r → Radius of spherical surface

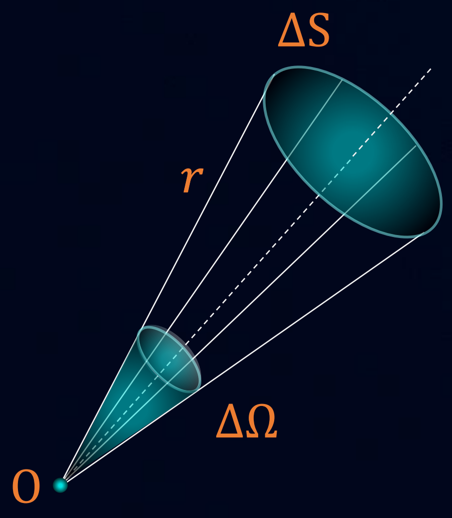
- The solid angle is defined as an angle subtended by an area at a point.

$$\Omega = \frac{area}{radius^2} = \frac{S}{r^2}$$

- SI unit: Steradian (sr)

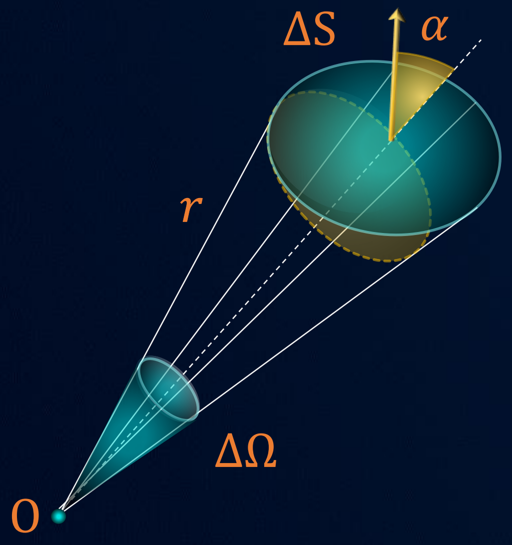
Solid Angle

- If the normal of the small planar surface passes through the point at which the surface subtends the solid angle, then the solid angle will be,



$$\Delta\Omega = \frac{\Delta S}{r^2}$$

- If the normal of the small planar surface does not pass through the point at which the surface subtends the solid angle, and the normal rather makes an angle of α as shown in the figure, then the solid angle will be,

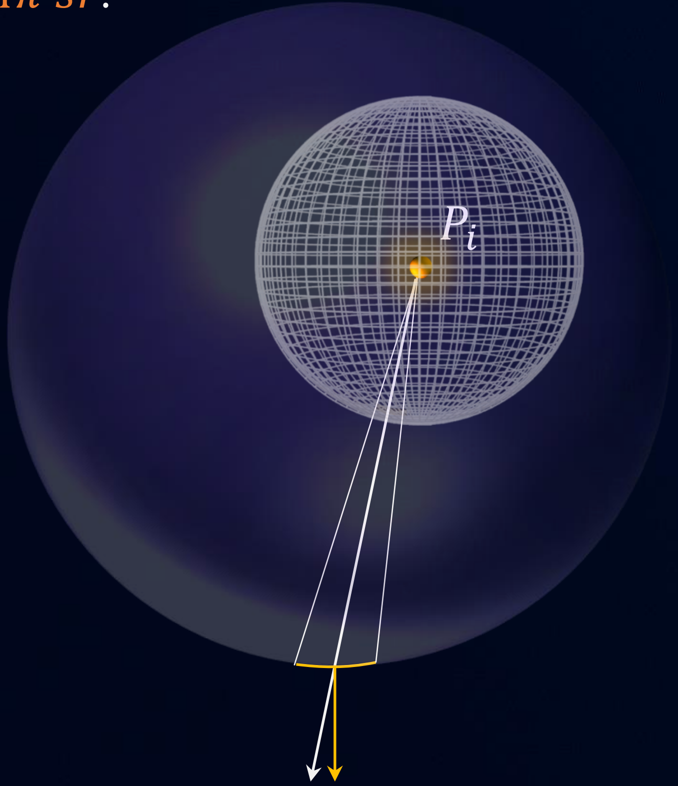


$$\Delta\Omega = \frac{\Delta S \cos\alpha}{r^2}$$

Solid Angle

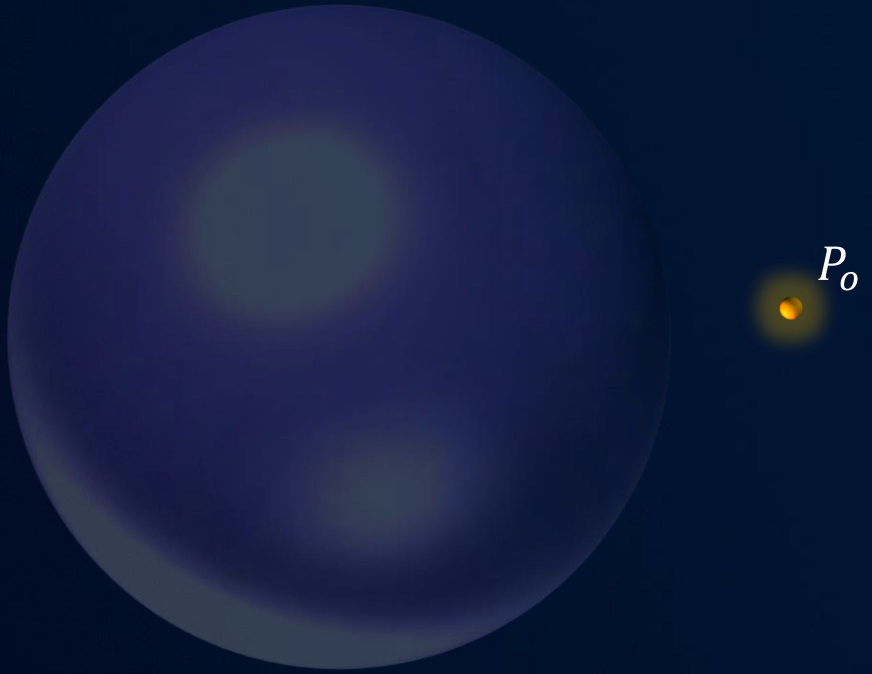
Solid Angle at any Interior Point

- As long as a point is inside any closed volume of any arbitrary shape, the solid angle subtended at that point will be $4\pi\ sr$.



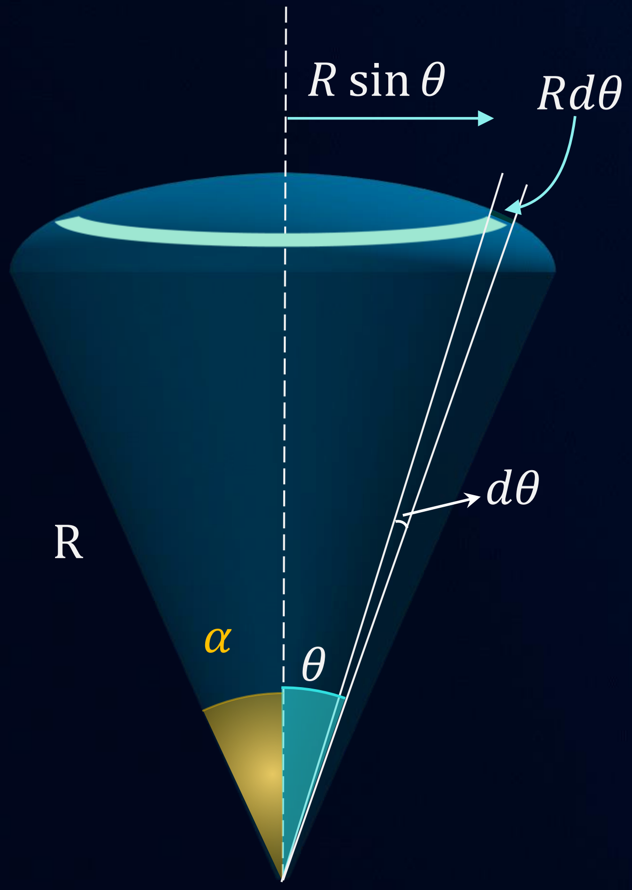
Solid Angle at any Exterior Point

- The solid angle subtended by any random closed surface at any exterior point is zero.



Find the relation between the solid angle at the vertex and half angle of the cone. α = half angle of the cone & l = slant height of the cone.

Solution :



$$S = \int_0^{\alpha} (2\pi R \sin\theta)\, R d\theta$$

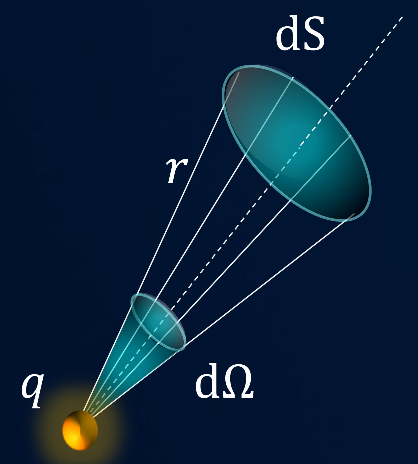
$$S = 2\pi R^2 (1 - \cos\alpha)$$

$$\Omega = \frac{S}{r^2} = 2\pi(1 - \cos\alpha)\ sr$$

Flux Due to a Point Charge

Flux due to a charge through a small surface dS,

$$d\phi = \frac{q}{\epsilon_0}\frac{d\Omega}{4\pi}$$

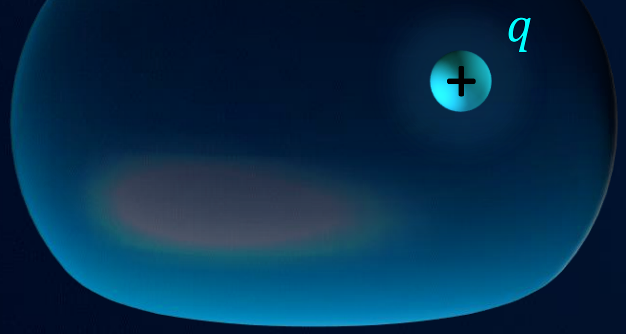


Flux through a closed surface due to an outside charge,

$$\phi = zero$$

Flux through surface area of a closed surface produced by enclosed charge,

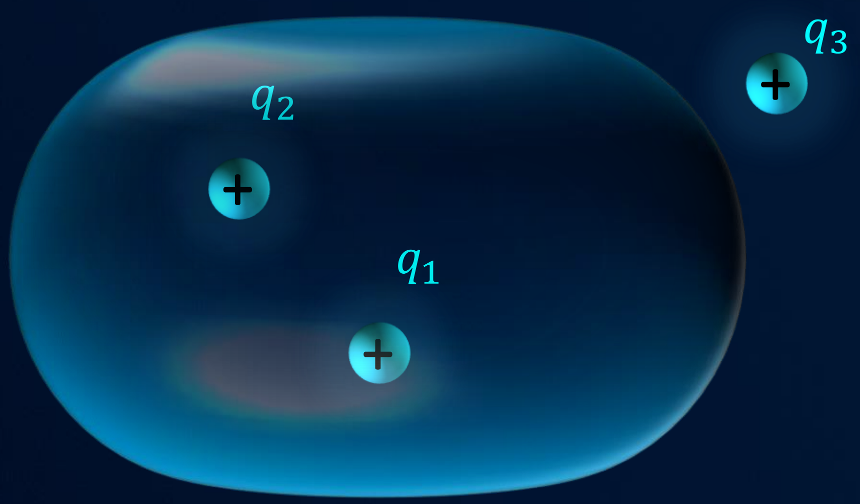
$$\phi = \frac{q}{\varepsilon_o}$$



Gauss's law

Flux through Gaussian surface due to Charges q_1, q_2 & q_3,

$\phi_1 = \frac{q_1}{\varepsilon_0}$, $\phi_2 = \frac{q_2}{\varepsilon_0}$ & $\phi_3 = 0$



Total Flux through the gaussian surface,

$$\phi_{total} = \frac{q_1 + q_2 + 0}{\varepsilon_0} = \frac{q_1 + q_2}{\varepsilon_0}$$

$$\phi = \frac{\sum q_{in}}{\varepsilon_0}$$

Gauss's law

Statement :

The flux of the net electric field through a closed surface is equal to the net charge enclosed by the surface divided by ε_o.

Mathematical form of Gauss's law :

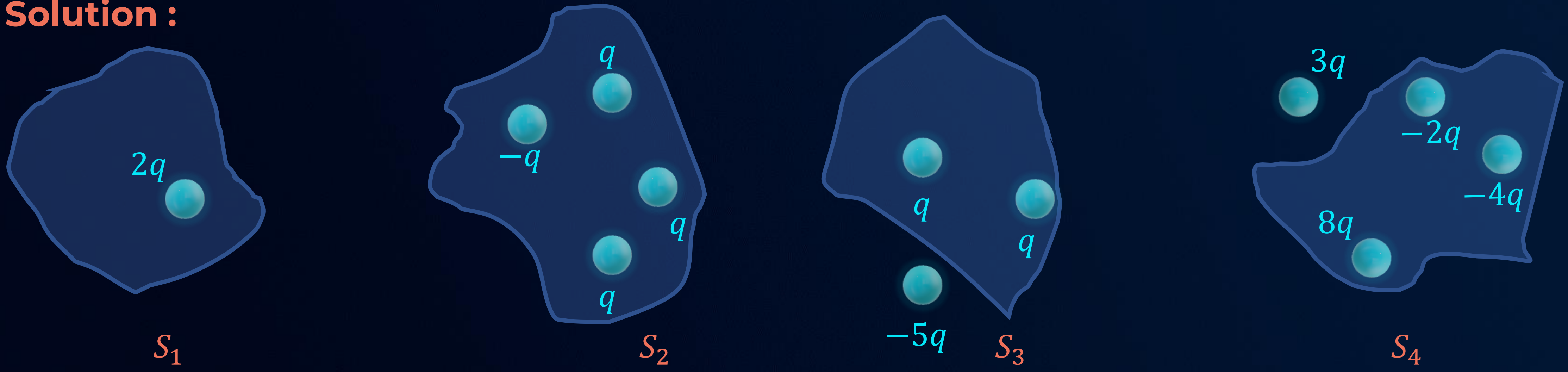
$$\phi = \oint \vec{E} \, . \, \vec{dS} = \frac{q_{in}}{\varepsilon_0}$$

$\vec{E}$ is due to all charges.

- The closed surface or the periphery of a volume on which Gauss's law is applied is known as the Gaussian surface.
  - It can be real or hypothetical.
  - ϕ is independent of its shape.

Four closed surfaces and corresponding charge distributions are shown below. Let the respective electric fluxes through the surfaces be ϕ_1, ϕ_2, ϕ_3 & ϕ_4 . Find relation among the fluxes.

Solution :



$$\Phi_1 = \frac{2q}{\varepsilon_o}$$

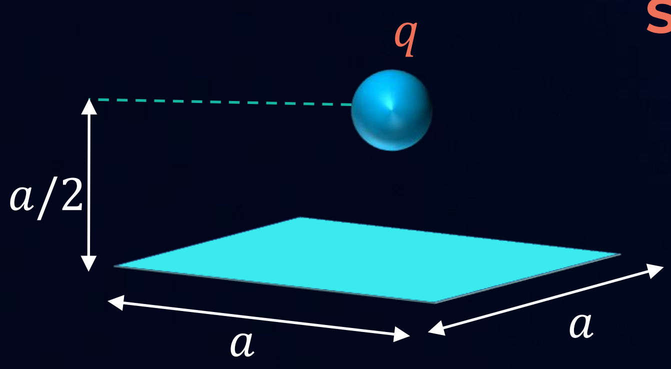
$$\Phi_2 = \frac{(q+q-q+q)}{\varepsilon_o} = \frac{2q}{\varepsilon_o}$$

$$\Phi_3 = \frac{(q+q)}{\varepsilon_o} = \frac{2q}{\varepsilon_o}$$

$$\Phi_4 = \frac{(8q-2q-4q)}{\varepsilon_o} = \frac{2q}{\varepsilon_o}$$

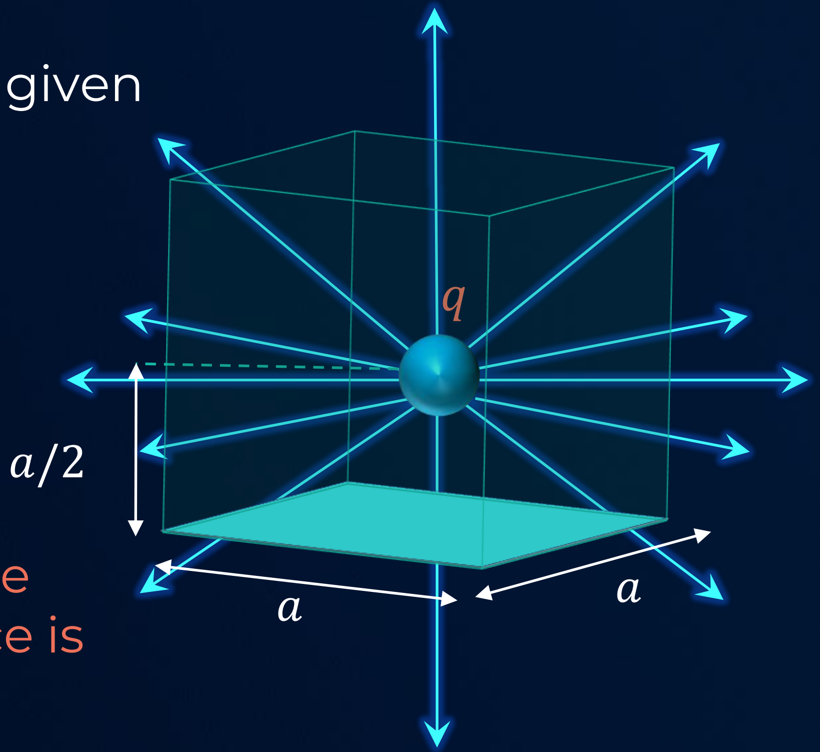
$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ Flux through these surfaces are equal.

A charge q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown. Find the flux of electric field through the square surface.



Solution :

Gaussian Surface – a cube where the given charge is at its centre.



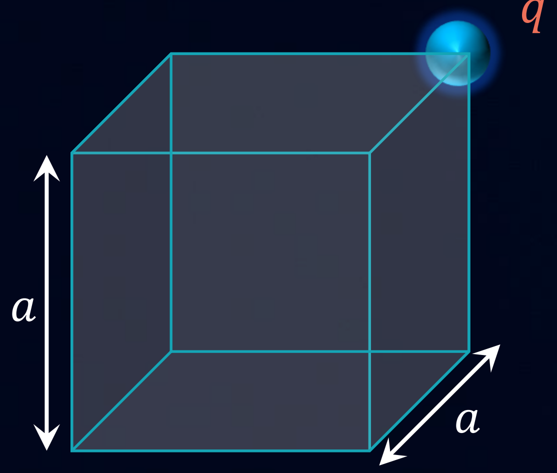
Net flux passing through the cube,

$$\Rightarrow \phi_{cube} = \frac{q}{\varepsilon_0}$$

Cube contains six similar equal square faces. So, flux through one square face is

$$\phi_{face} = \frac{q}{6\varepsilon_0}$$

What is the electric flux through a cube of side a ,if a charge q is placed at one of its corners?



Solution :

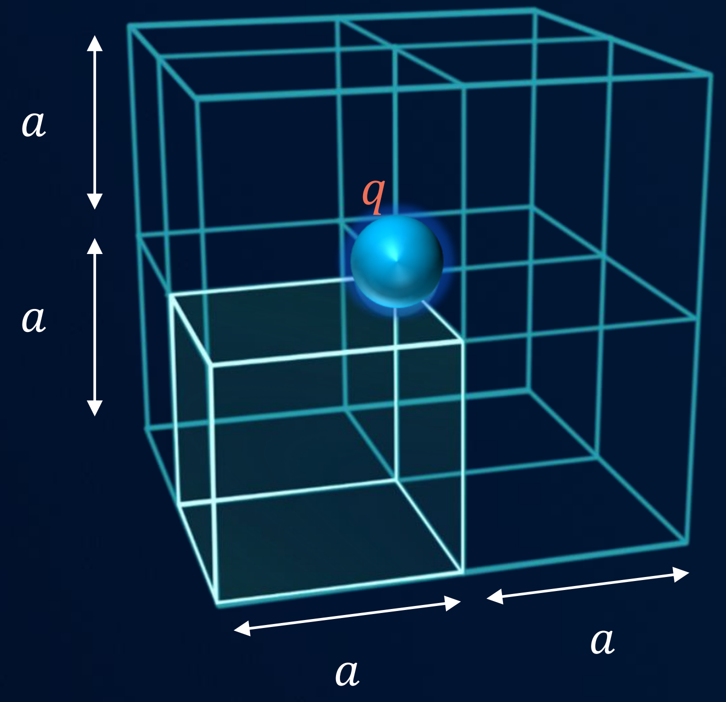
Gaussian Surface – a cube of side $2a$ where the charge is at the centre of cube.

Electric Flux passing through the big Cube.

$$\Rightarrow \phi_{8\ cube} = \frac{q}{\varepsilon_0}$$

Big cube of side $2a$ is made of 8 similar cubes of side a. Hence, flux through a single cube of side a is,

$$\phi_{1\ cube} = \frac{q}{8\varepsilon_0}$$

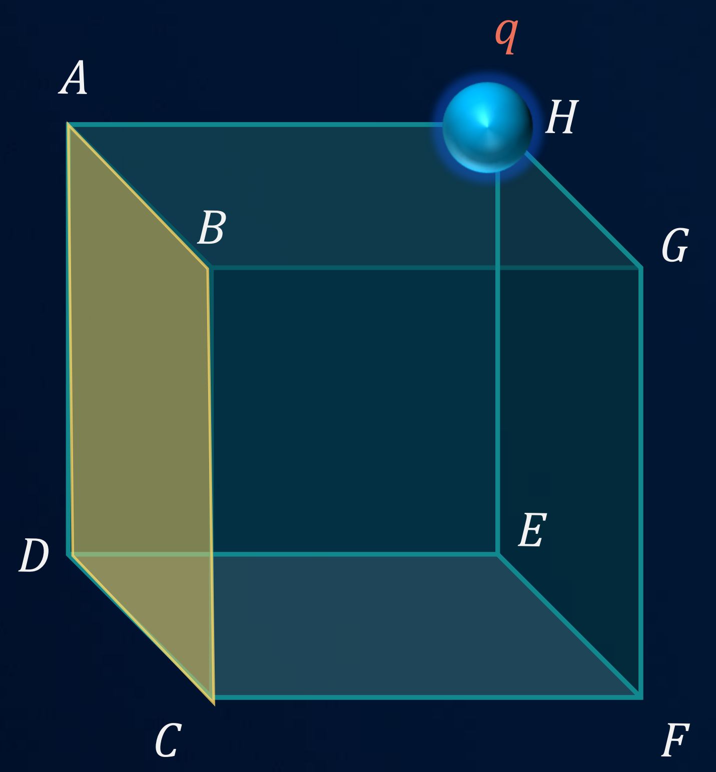


Find the electric flux through the left face ($ABCD$) of the cube, due to charge q.

Solution :

Flux through a cube if charge q is placed at its corner as shown is,

$$\Rightarrow \phi = \frac{q}{8\varepsilon_0}$$



The point charge $+q$ is placed at the corner H. Therefore, flux associated with the surfaces that consists of the corner H is zero.

$$\Rightarrow \phi_{ADEH} = \phi_{ABGH} = \phi_{EFGH} = Zero$$

$$\Rightarrow \phi_{ABCD} = \phi_{CDEF} = \phi_{CBGF} = \frac{1}{3} \times \frac{q}{8\varepsilon_0}$$

$$\Rightarrow \boxed{\phi_{ABCD} = \frac{q}{24\varepsilon_0}}$$

Determine the flux of electric field across a disc of radius R due to a point charge q placed at a distance l from its centre.

Solution :

Flux passing through the disc,

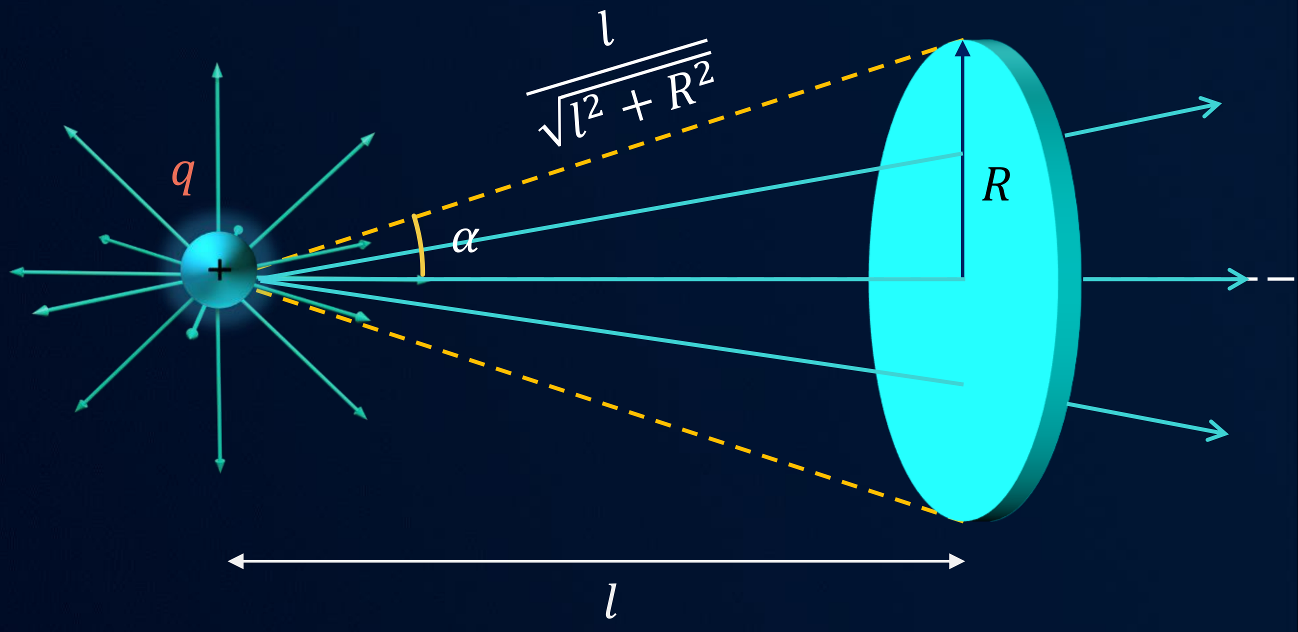
$$\Rightarrow \ \phi_{Disc} = \frac{q}{4\pi\varepsilon_o}\Omega_{Disc}$$

Ω_{Disc} : Solid Angle subtended by disc at the charge(Similar to cone)

$\Omega_{Disc} = 2\pi(1 - \cos\alpha)$, Therefore

$$\Rightarrow \ \phi_{Disc} = \frac{q}{4\pi\varepsilon_o} \times 2\pi(1 - \cos\alpha)$$

$$\phi_{Disc} = \frac{q}{2\varepsilon_o} \times \left(1 - \frac{l}{\sqrt{l^2 + R^2}}\right)$$



Determine the flux of electric field through the curved surface of the cylinder (length = l and radius = R) due to a point charge q placed at its center.

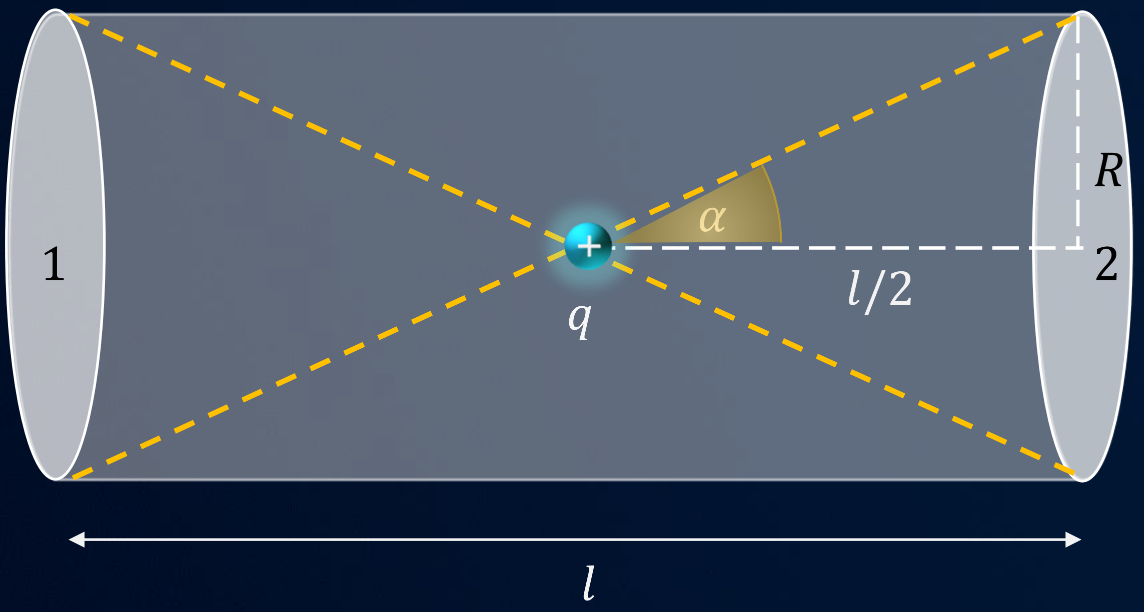
Solution :

If we assume a conical surface at face 2 then,

Flux through face 2 is,

$$\phi = \frac{q}{4\pi\epsilon_0} \times 2\pi(1 - \cos\alpha)$$

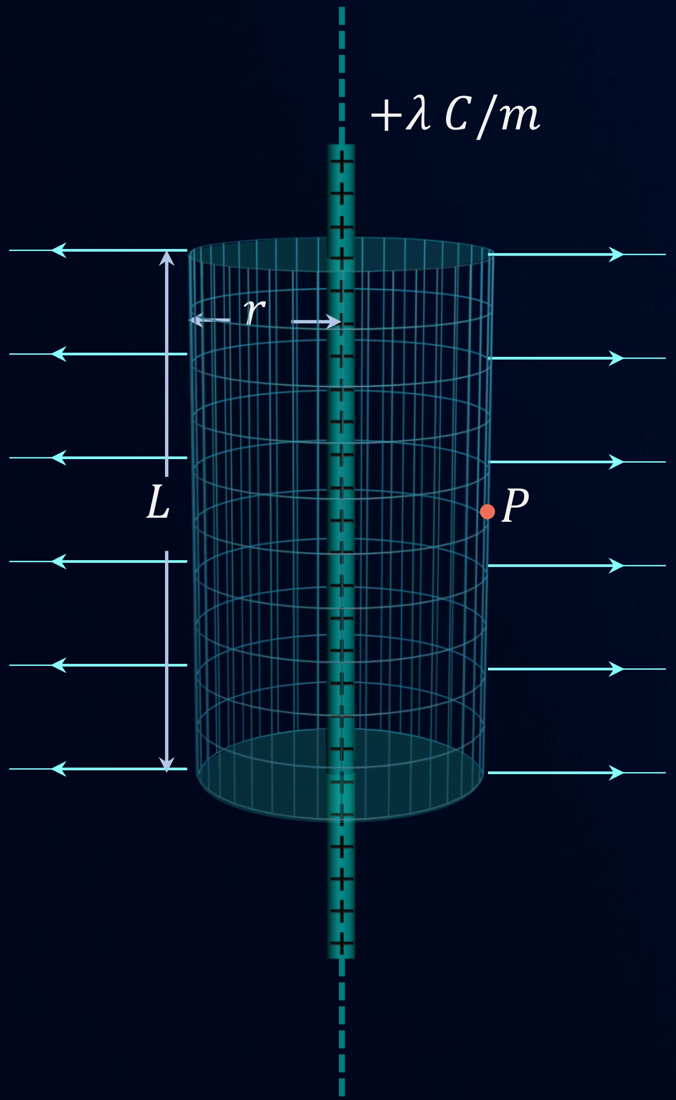
$$\phi = \frac{q}{2\epsilon_0} \times \left(1 - \frac{\frac{l}{2}}{\sqrt{\left(\frac{l}{2}\right)^2 + R^2}}\right) = \frac{q}{2\epsilon_0}\left(1 - \frac{l}{\sqrt{l^2 + 4R^2}}\right)$$



$$\phi_1 = \phi_2 = \phi$$

$$\phi_{total} = \phi_1 + \phi_2 + \phi_{curved} = \frac{q}{\epsilon_0} \Rightarrow \phi_{curved} = \frac{q}{\epsilon_0} - 2\phi = \frac{q}{\epsilon_0}\left(\frac{l}{\sqrt{l^2 + 4R^2}}\right)$$

Electric Field - Uniformly Charged Infinitely Long, Thin wire



$$\oint \vec{E}.d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint \vec{E}.d\vec{S} = \int_{curved} \vec{E}.d\vec{S} = \frac{q_{encl}}{\epsilon_0} = \frac{\lambda L}{\epsilon_0}$$

$$\left(\because \int_{top} \vec{E}.d\vec{S} = 0 = \int_{bottom} \vec{E}.d\vec{S} \right)$$

$$E \int dS = \frac{\lambda L}{\epsilon_0} \Rightarrow E(2\pi r L) = \frac{\lambda L}{\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi r \epsilon_0}}$$

A very long cylindrical volume contains a uniformly distributed charge of density ρ . Find the electric field at a point P inside the cylindrical volume at a distance x from its axis.

Solution :

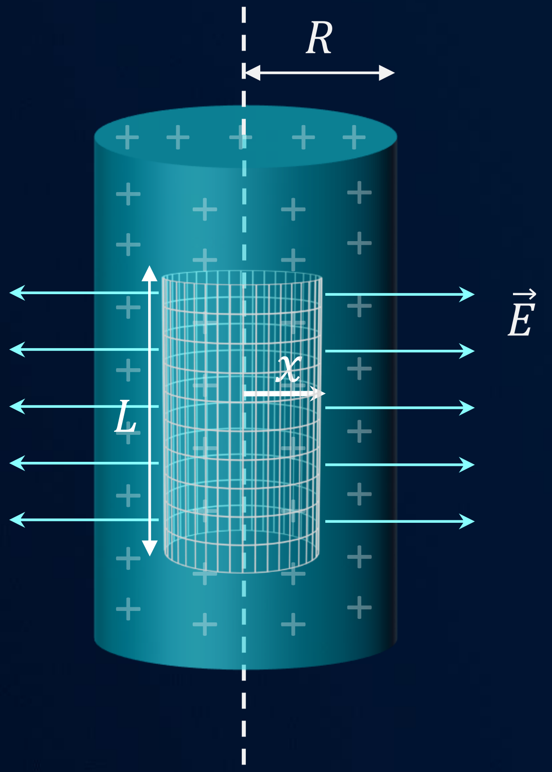
For $x < R$

$$\oint \vec{E}.d\vec{S} = \int_{curved} \vec{E}.d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow \int_{curved} E\,(dS) = \frac{q_{enc}}{\epsilon_0} \qquad \left(\because \int_{top} \vec{E}.d\vec{S} = 0 = \int_{bottom} \vec{E}.d\vec{S}\right)$$

$$\Rightarrow E(2\pi x L) = \frac{\rho\,(\pi x^2)L}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho\, x}{2\epsilon_0} \Rightarrow \vec{E} = \frac{\rho \vec{x}}{2\epsilon_0}$$



A very long cylindrical volume contains a uniformly distributed charge of density ρ . Find the electric field at a point P outside the cylindrical volume at a distance x from its axis.

Solution :

For $x > R$

$$\oint \vec{E}.d\vec{S} = \int_{curved} \vec{E}.d\vec{S} = \frac{q_{enc}}{\epsilon_0} \qquad \left(\because \int_{top} \vec{E}.d\vec{S} = 0 = \int_{bottom} \vec{E}.d\vec{S}\right)$$

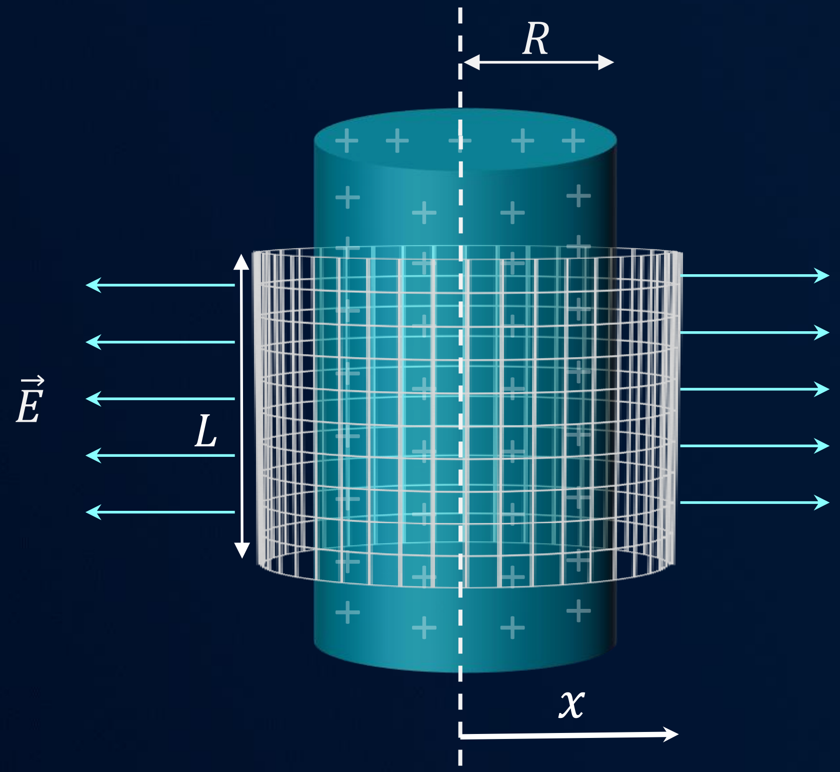
$$\Rightarrow \int_{curved} E\,(dS) = \frac{q_{enc}}{\epsilon_0}$$

$$\Rightarrow E(2\pi x L) = \frac{\rho\,(\pi R^2)L}{\epsilon_0}$$

$$\Rightarrow E = \frac{\rho R^2}{2x\epsilon_0}$$

For $x = R$

$$\Rightarrow E = \frac{\rho R}{2\epsilon_0}$$



The electric field in a region is given by , $\vec{E} = \frac{E_0 x}{l}\hat{\imath}$. Find the charge contained inside a cubical volume bounded by surfaces $x = 0, x = l, y = 0, y = l, z = 0$ and $z = l$.

Solution :

Applying Gauss's law to the cube, $\oint \vec{E} \cdot \overrightarrow{ds} = \frac{Q_{enc}}{\varepsilon_0}$

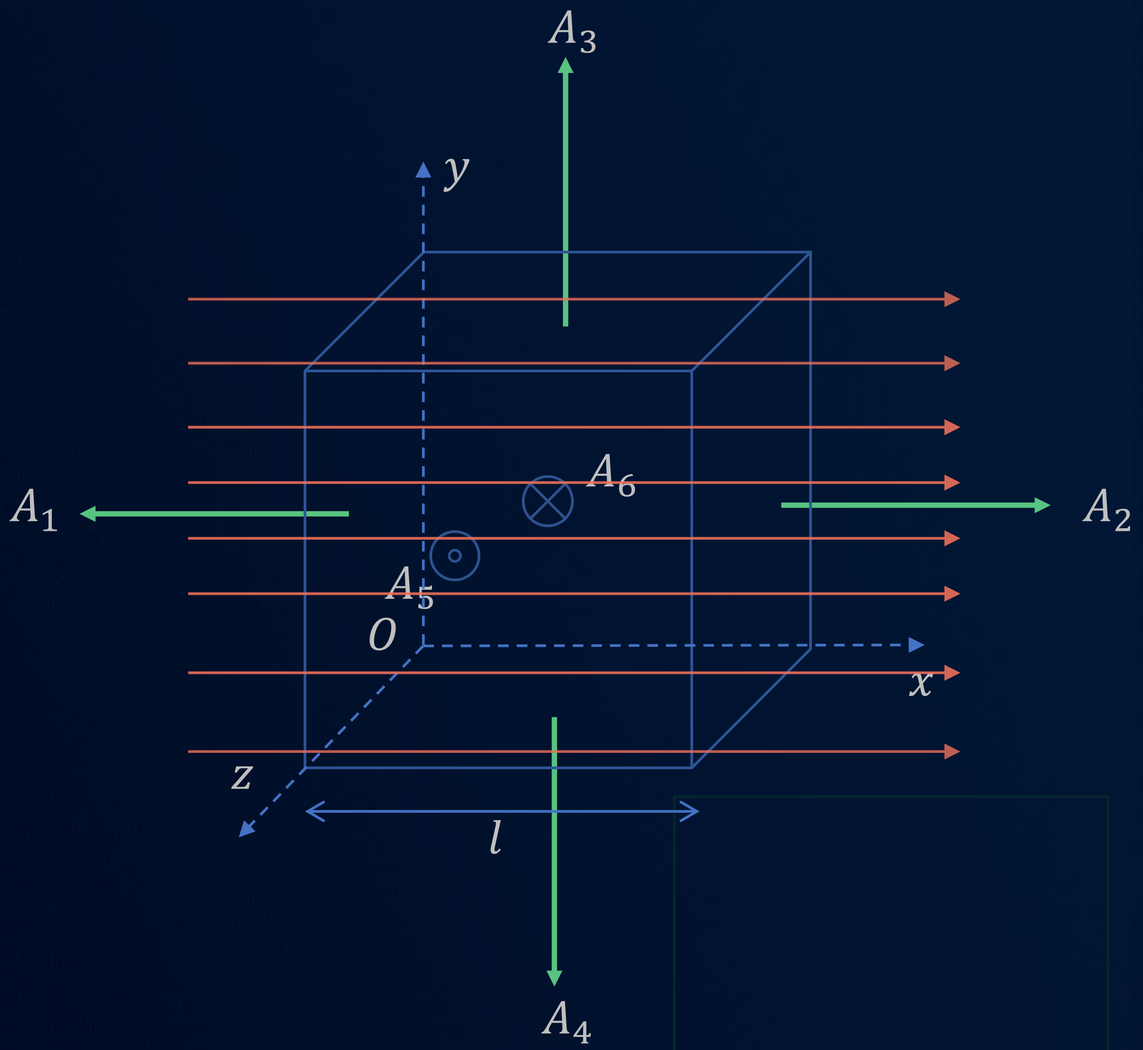
Flux along surfaces 3,4,5 and 6 will be zero as the electric field vector is perpendicular to area vector.

$$\oint \overrightarrow{E_1} \cdot \overrightarrow{ds} + \oint \overrightarrow{E_2} \cdot \overrightarrow{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

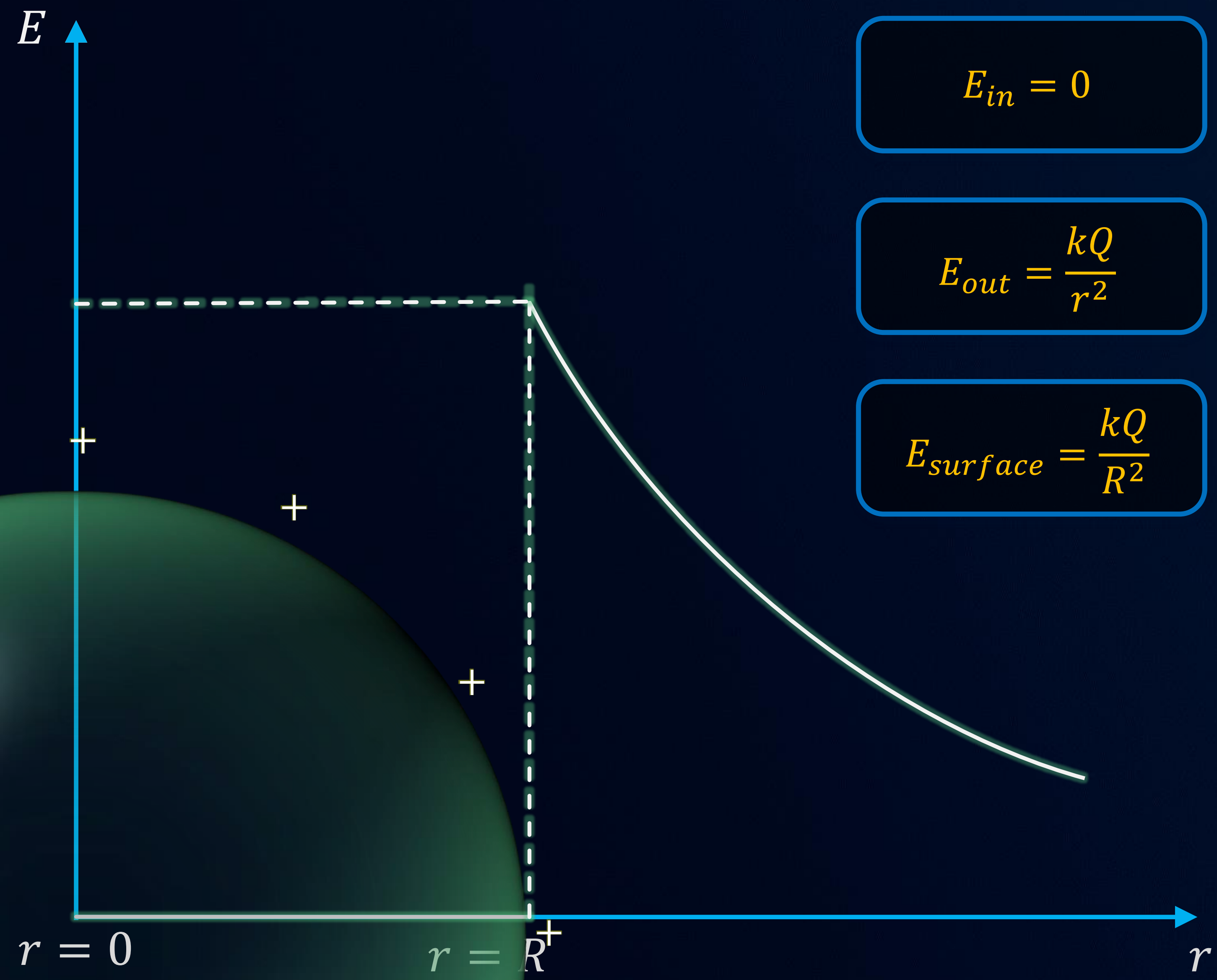
At $x = 0 \Rightarrow E_1 = 0$ and $x = l \Rightarrow E_2 = E_0$

$$E_0 l^2 = \frac{Q_{enc}}{\varepsilon_0}$$

$$Q_{enc} = \varepsilon_0 E_0 l^2$$



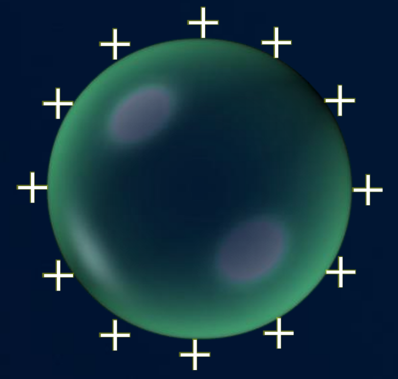
Uniformly charged Conducting/Nonconducting spherical shell



$$E_{in} = 0$$

$$E_{out} = \frac{kQ}{r^2}$$

$$E_{surface} = \frac{kQ}{R^2}$$



Choose a Gaussian surface

↓

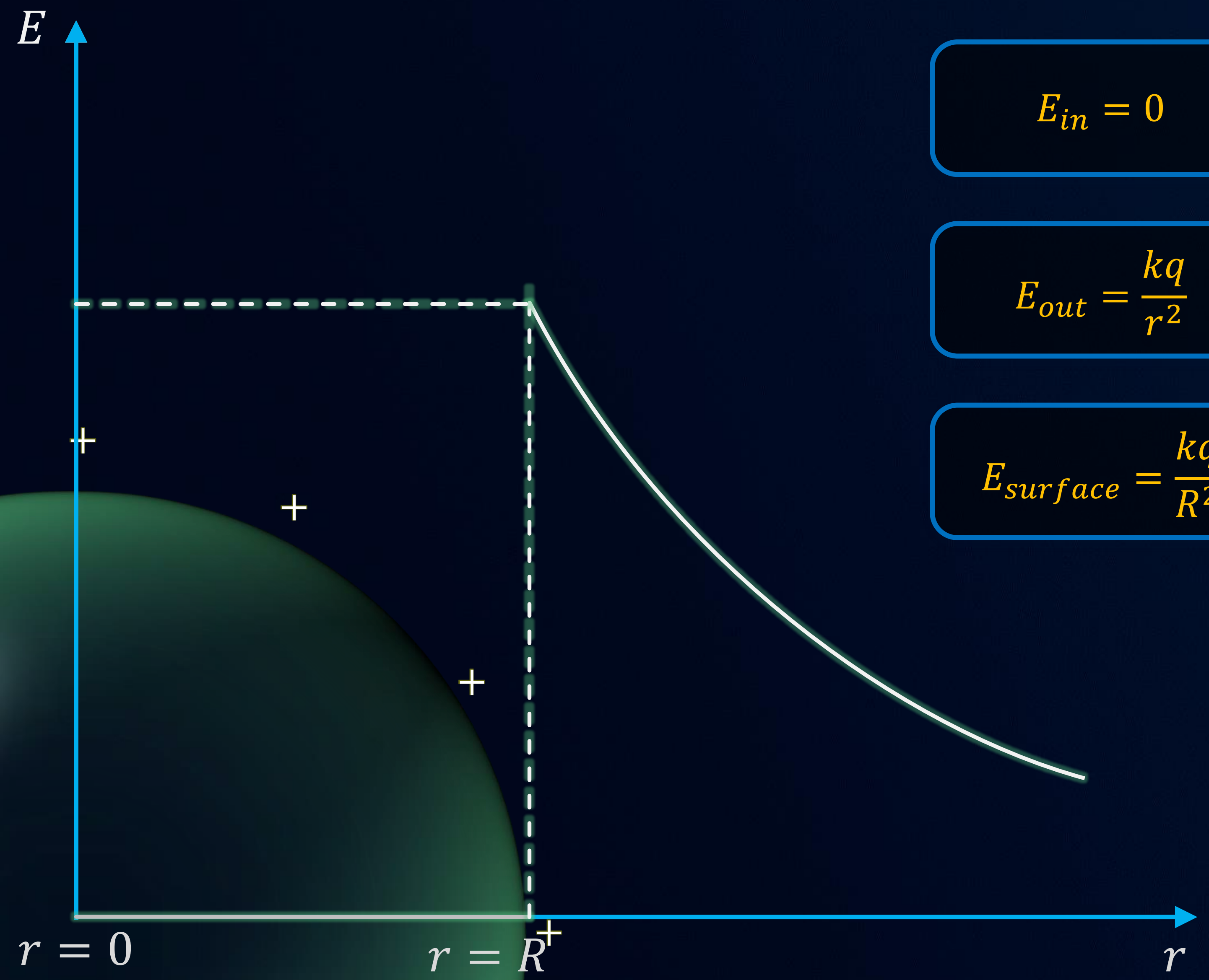
Identify the charges inside gaussian surface

↓

Apply Gauss law,

$$\oint \vec{E} \cdot \overrightarrow{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

Uniformly charged Conducting sphere



$$E_{in} = 0$$

$$E_{out} = \frac{kq}{r^2}$$

$$E_{surface} = \frac{kq}{R^2}$$

Choose a Gaussian surface

↓

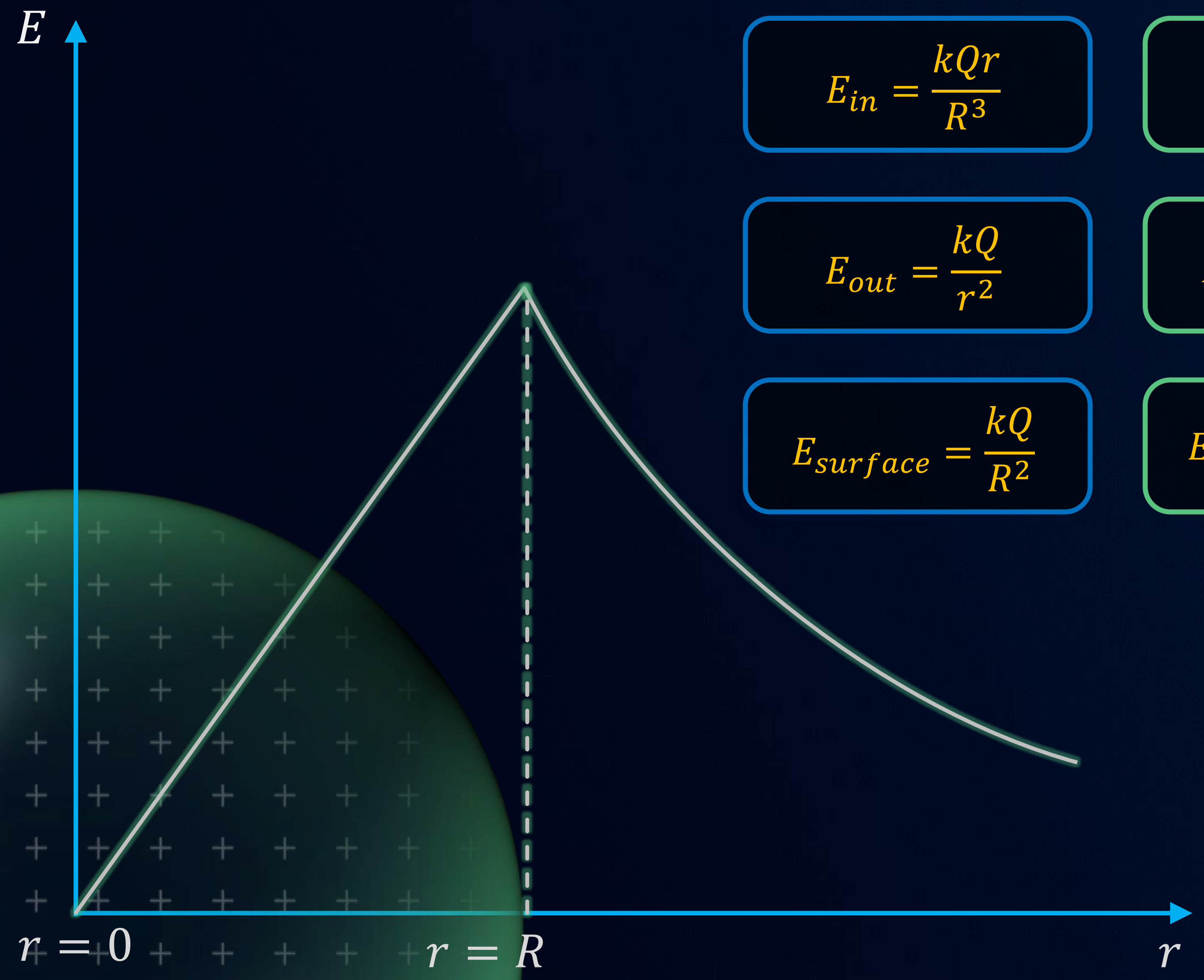
Identify the charges inside gaussian surface

↓

Apply Gauss law,

$$\oint \vec{E} \cdot \overrightarrow{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

Uniformly charged Non-conducting sphere



$$E_{in} = \frac{kQr}{R^3}$$

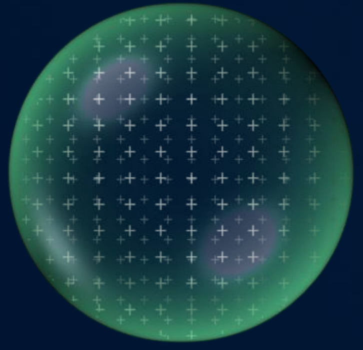
$$E_{in} = \frac{\rho r}{3\varepsilon_0}$$

$$E_{out} = \frac{kQ}{r^2}$$

$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2}$$

$$E_{surface} = \frac{kQ}{R^2}$$

$$E_{surface} = \frac{\rho R}{3\varepsilon_0}$$



Choose a Gaussian surface

↓

Identify the charges inside gaussian surface

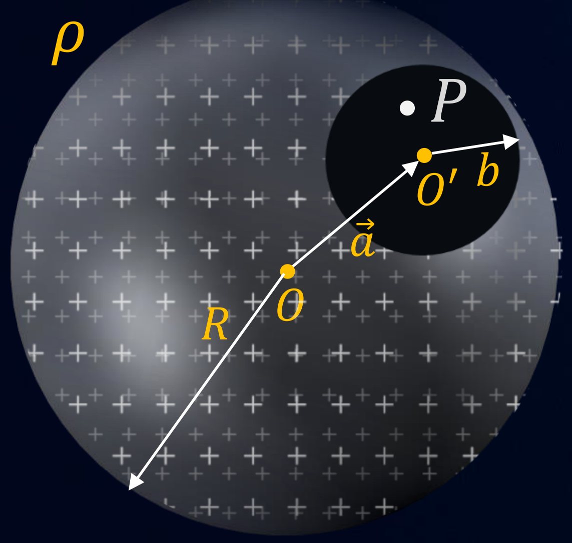
↓

Apply Gauss law,

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

A non-conducting sphere of radius R has a uniform volume charge density ρ. A spherical cavity of radius b, whose center lies at $\vec{a}$ from sphere, is removed from the sphere. Find the electric field at any point inside the cavity?

Solution :



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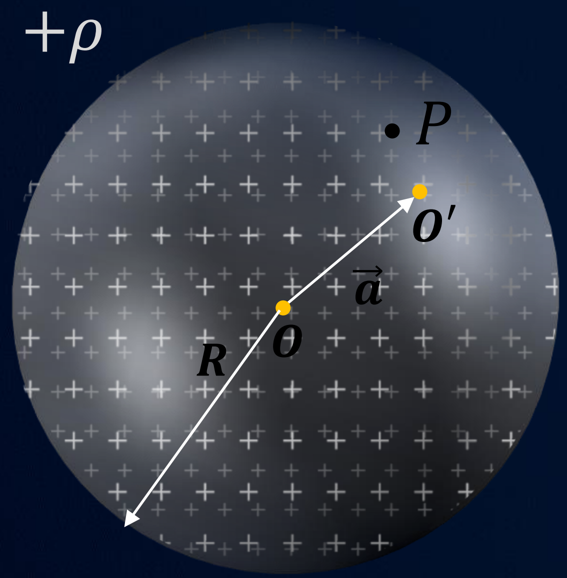

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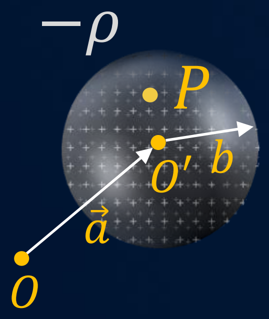
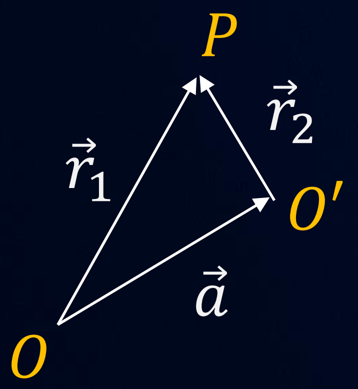




$$E_P = \frac{\rho \vec{r}_1}{3\varepsilon_0} + \frac{-\rho \vec{r}_2}{3\varepsilon_0} = \frac{\rho(\vec{r}_1 - \vec{r}_2)}{3\varepsilon_0}$$

At any point inside the given cavity, $E_{net} = \dfrac{\rho \vec{a}}{3\varepsilon_0}$

Very large thin sheet of uniform charge distribution

Non- Conducting Sheet

- uniform electric field
- In terms of charge density

$$E = \frac{\sigma}{2\varepsilon_0}$$

- In terms of charge

$$E = \frac{Q}{2A\varepsilon_0}$$

Conducting Sheet

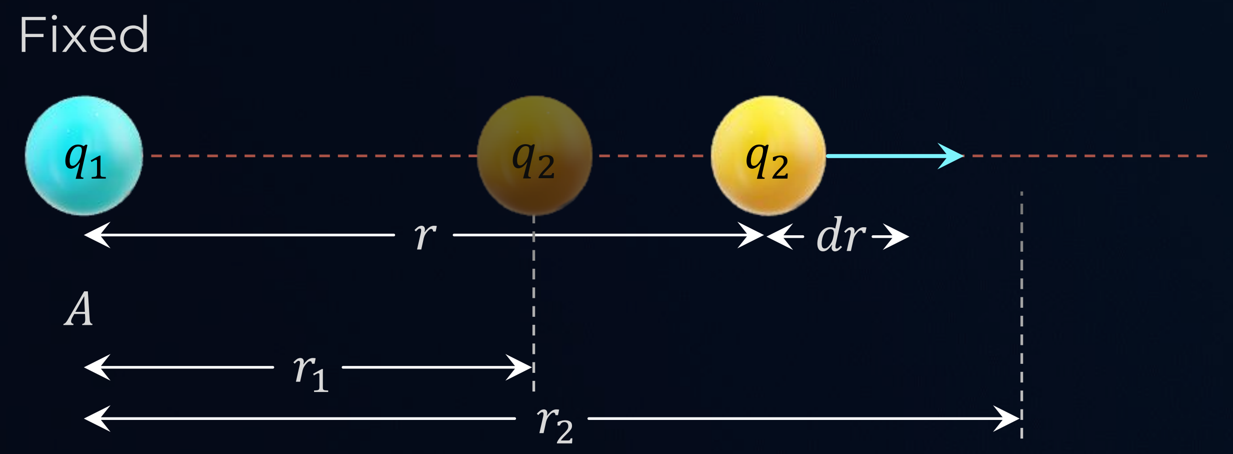
- uniform electric field
- In terms of charge density

$$E = \frac{\sigma^*}{\varepsilon_0}$$

- In terms of charge

$$E = \frac{Q}{2A\varepsilon_0}$$

Electric Potential Energy



Work done by electric force in moving the charge q_2 from $r = r_1$ to $r = r_2$:

$$W = kq_1q_2 \int_{r_1}^{r_2} \frac{dr}{r^2} = -kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

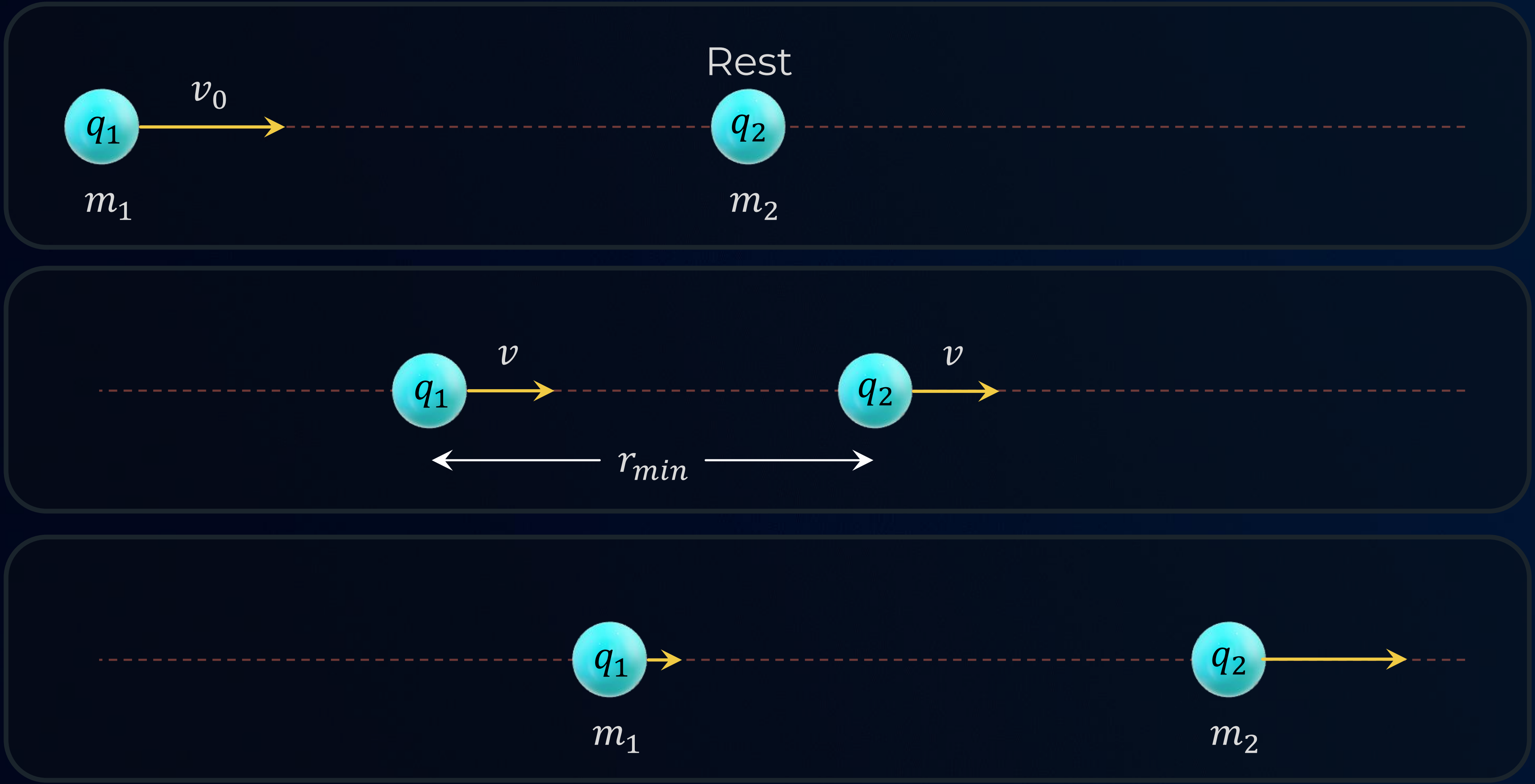
$$U(r_2) - U(r_1) = -W = kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

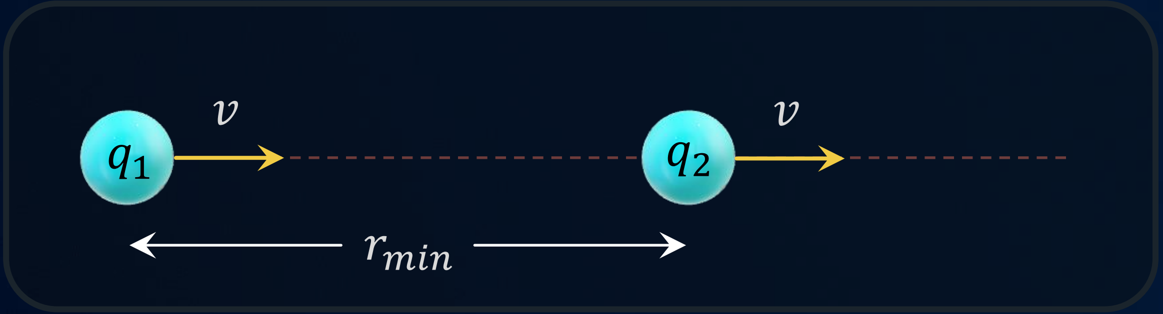
Potential energy of the system when the separation between the charges is r :

$$U(r) - U(\infty) = \frac{kq_1q_2}{r} - 0 \qquad U(r) = \left.\frac{kq_1q_2}{r}\right|_{U(\infty)=0}$$

- It is defined as the amount of work needed to move a charge from a reference point ($U = 0$) to a specific point.
- While calculating the potential energy of two charge systems using the formula $U(r) = \frac{kq_1q_2}{r}$, q_1 and q_2 are to be taken with signs.
- $\Delta U = W_{ext}$ if and only if $\Delta KE = 0$
- $\Delta U = -W_{electric}$

A charge $+q_1$ comes from infinity with an initial speed v_0 towards the charge $+q_2$ which is initially at rest (not fixed). Find the closest distance of approach between these two charges.





Applying conservation of mechanical energy for the system -

$$(KE + PE)_i = (KE + PE)_f$$

$$\frac{1}{2}m_1 v_0^2 + 0 = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{kq_1 q_2}{r_{min}}$$

$$\frac{1}{2}m_1 v_0^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{kq_1 q_2}{r_{min}} \qquad -(1)$$

Applying conservation of linear momentum for the system -

$$p_i = p_f$$

$$m_1 v_0 + 0 = m_1 v + m_2 v$$

$$v = \frac{m_1 v_0}{m_1 + m_2} \qquad -(2)$$

Solving 1 and 2 we get -

$$r_{min} = \frac{2kq_1 q_2 (m_1 + m_2)}{m_1 m_2 v_0^2}$$

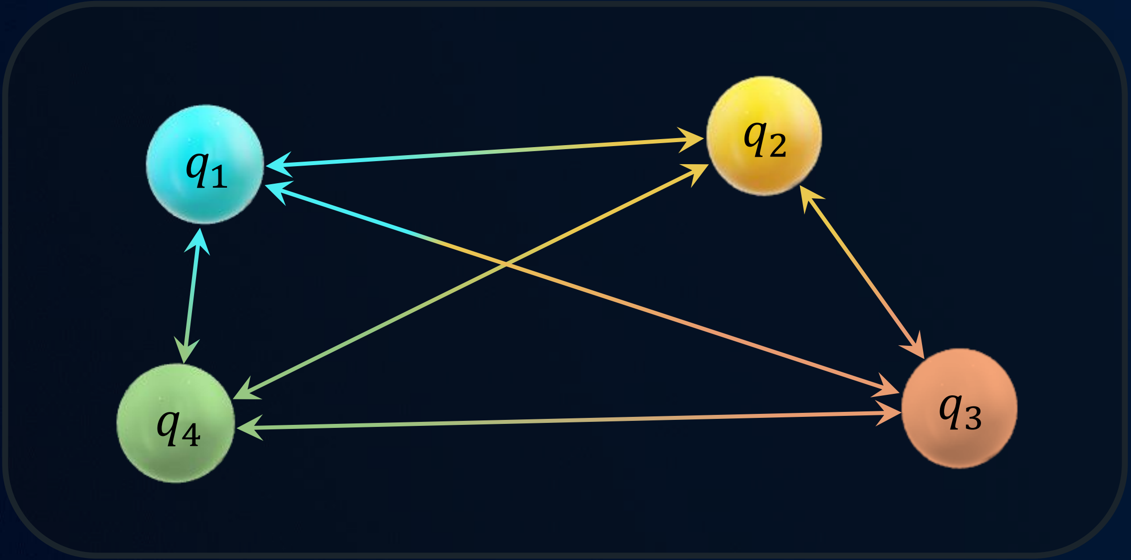
Multiple-Charge Systems

Electric potential energy of a multiple charge system

$$U_{sys} = U_{21} +$$

$$U_{31} + U_{32} +$$

$$U_{41} + U_{42} + U_{43} +$$



For n-charge system

$$U_{sys} = \sum_{i=1}^{n-1} U_{i+1,\,i}$$ (A total of ${}^nC_2$ terms)

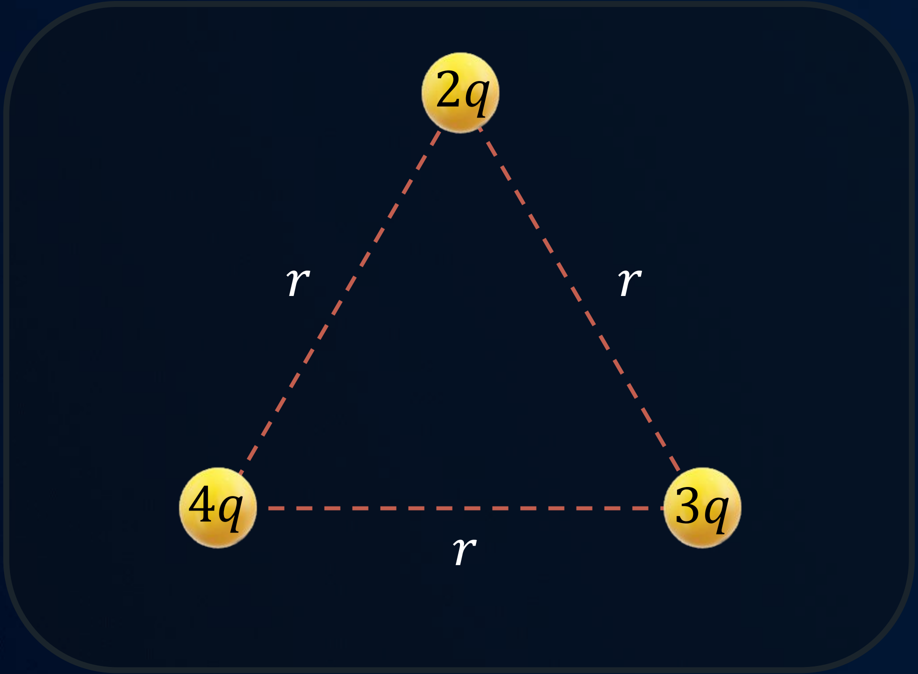
How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in the figure?

Solution :

$$W_{ext} = U_f$$

$$= \frac{k(2q)(4q)}{r} + \frac{k(2q)(3q)}{r} + \frac{k(3q)(4q)}{r}$$

$$W_{ext} = \frac{26kq^2}{r}$$



Change in Potential Energy

- Concept of potential energy is defined only in the case of conservative forces.

$$\Delta U = U_f - U_i = (-W_{cons})_{i\to f}$$

$$= -\int_i^f \vec{F}.d\vec{r}$$

?

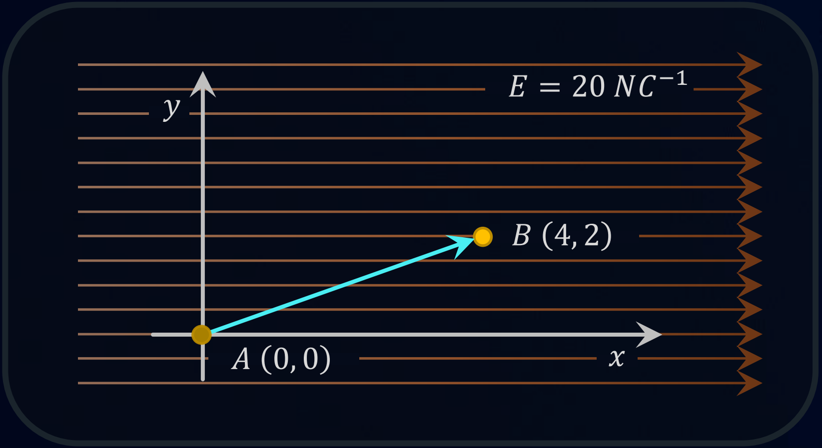
An electric field $E = 20\ NC^{-1}$ exists along the x-axis in space. A charge of $-2 \times 10^{-4}\ C$ is moved from point A to B. Find the change in electrical potential energy $U_B - U_A$ when the points A and B are given by

a. $A = (0, 0); B = (4\ m, 2\ m)$
b. $A = (4\ m, 2\ m); B = (6\ m, 5\ m)$
c. $A = (0, 0); B = (6\ m, 5\ m)$

Summary

$q = -2 \times 10^{-4}\ C \qquad \vec{E} = 20\ NC^{-1}\ \hat{\imath} \qquad \vec{F} = q\vec{E} = -4 \times 10^{-3}\ N\ \hat{\imath}$

a. $A = (0, 0); B = (4\ m, 2\ m)$

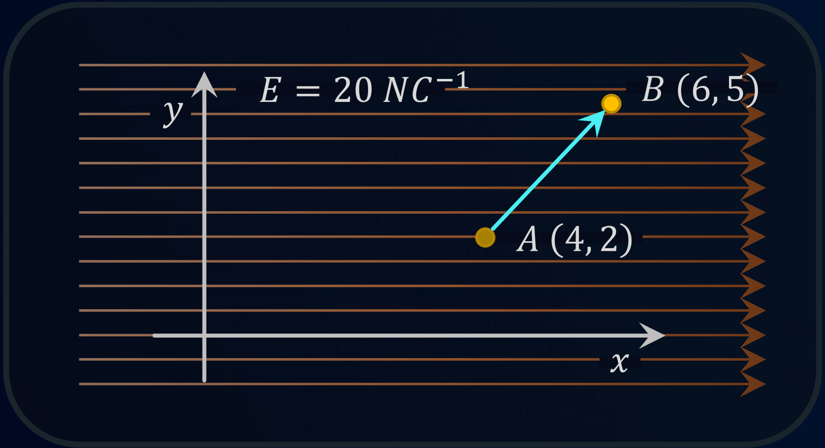


$$W_{el} = \vec{F} \cdot \overrightarrow{AB}$$
$$= (-4 \times 10^{-3}\ \hat{\imath}) \cdot (4\hat{\imath} + 2\hat{\jmath})$$
$$= -16 \times 10^{-3}\ J$$

$$U_B - U_A = -W_{el}$$

$$U_B - U_A = 0.016\ J$$

b. $A = (4\ m, 2\ m); B = (6\ m, 5\ m)$

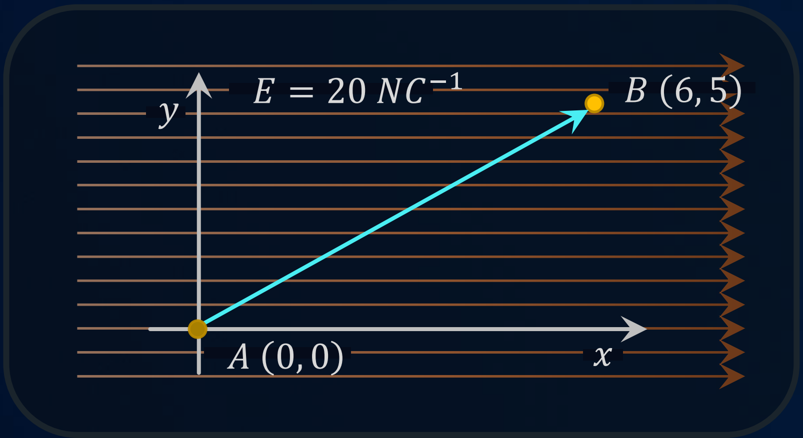


$$W_{el} = \vec{F} \cdot \overrightarrow{AB}$$
$$= (-4 \times 10^{-3}\ \hat{\imath}) \cdot (2\hat{\imath} + 3\hat{\jmath})$$
$$= -8 \times 10^{-3}\ J$$

$$U_B - U_A = -W_{el}$$

$$U_B - U_A = 0.008\ J$$

c. $A = (0, 0); B = (6\ m, 5\ m)$



$$W_{el} = \vec{F} \cdot \overrightarrow{AB}$$
$$= (-4 \times 10^{-3}\ \hat{\imath}) \cdot (6\hat{\imath} + 5\hat{\jmath})$$
$$= -24 \times 10^{-3}\ J$$

$$U_B - U_A = -W_{el}$$

$$U_B - U_A = 0.024\ J$$

Electric Dipole in a Uniform Electric Field

Potential energy of a dipole in a uniform electric field

$$U_{\theta_2} - U_{\theta_1} = -W_{el}$$

$$= -pE(\cos\theta_2 - \cos\theta_1)$$

$$U(\theta) - U\left(\frac{\pi}{2}\right) = -pE\left(\cos\theta - \cos\frac{\pi}{2}\right)$$

$$U(\theta) = -pE\cos\theta \qquad \left[\text{Considering } U = 0 \text{ for } \theta = \frac{\pi}{2}\ rad\right]$$

$$U(\theta) = -\vec{p}\cdot\vec{E}$$