MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Shortest distance between lines

$$\frac{x-5}{4}=\frac{y-3}{6}=\frac{z-2}{4} \text{ and } \frac{x-3}{7}=\frac{y-2}{5}=\frac{z-9}{6} \text{ is}$$

(1) $\frac{190}{37}$ (2) $\frac{190}{\sqrt{756}}$

(3) $\frac{37}{190}$ (4) $\frac{756}{\sqrt{190}}$

Answer (2)

Sol. $\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 6 & 4 \\ 7 & 5 & 6 \end{vmatrix}$

$= 16\hat{i} + 4\hat{j} - 22\hat{k}$

$$d = \left|\frac{(\vec{a}-\vec{b})\cdot(\vec{n}_1\times\vec{n}_2)}{|\vec{n}\times\vec{n}_2|}\right|$$

$$= \left|\frac{\left(2\hat{i}+\hat{j}-7\hat{k}\right)\cdot\left(16\hat{i}+4\hat{j}-22\hat{k}\right)}{\sqrt{16^2+4^2+(22)^2}}\right|$$

$$= \left|\frac{32+4+154}{\sqrt{256+16+484}}\right|$$

$$= \frac{190}{\sqrt{756}}$$

2. Consider the word "INDEPENDENCE". The number of words such that all the vowels are together, is

(1) 16800 (2) 15800

(3) 17900 (4) 14800

Answer (1)

Sol. Vowels: I E E E E

Consonants: N N N D D P C

| I E E E E | 3N, 2D, P, C

Number of required words $= \frac{8!}{3!2!} \times \frac{5!}{4!}$

$= 16800$

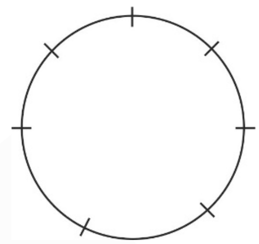
3. 7 boys and 5 girls are to be seated around a circular table such that no two girls sits together is

(1) $126(5!)^2$ (2) 720(5!)

(3) 720(6!) (4) 720

Answer (1)

Sol. $B_1, B_2, B_3, B_4, B_5, B_6, B_7$



Boys can be seated in (7 – 1)! ways = 6!

Now ways in which no two girls can be seated together is

$6! \times {}^7C_5 \times 5!$

$6! \times \frac{7!}{5!2!} \times 5!$

$= 126(5!)^2$

4. Consider the data : x, y, 10, 12, 4, 6, 8, 12. If mean is 9 and variance is 9.25, then the value of $3x - 2y$ is $(x > y)$

(1) 25 (2) 1

(3) 24 (4) 13

Answer (1)

Sol. $9 = \frac{52+x+y}{8}$

$\Rightarrow x + y = 20$

$$9.25 = \frac{x^2+y^2+100+144+16+36+64+144}{8} - 81$$

$\Rightarrow 722 = x^2 + y^2 + 504$

$\Rightarrow x^2 + y^2 = 218$

$(x + y)^2 - 2xy = 218$

$\Rightarrow xy = 91$

$\therefore x = 13, y = 7$

$3x - 2y = 39 - 14$

= 25

5. Coefficient independent of x in the expansion of $\left(3x^2 - \frac{1}{2x^5}\right)^7$ is

(1) $\frac{5103}{4}$

(2) $\frac{5293}{6}$

(3) $\frac{6715}{3}$

(4) $\frac{7193}{4}$

Answer (1)

Sol. $T_{r+1} = {}^7C_r\left(3x^2\right)^{7-r}\left(\frac{-1}{2x^5}\right)^r$

$= {}^7C_r 3^{7-r}\left(\frac{-1}{2}\right)^r x^{14-7r}$

$\Rightarrow$ $14 - 7r = 0$

$\Rightarrow$ $r = 2$

$\therefore$ Coefficient of x^0 is

${}^7C_2 3^5 \times \frac{1}{4}$

$\frac{7\times6\times3^5}{2\times1\times4}$

$= \frac{5103}{4}$

6. Dot product of two vectors is 12 and cross product is $4\hat{i} + 6\hat{y} + 8\hat{k}$ find product of modulus of vectors

(1) $4\sqrt{35}$

(2) $2\sqrt{65}$

(3) $5\sqrt{37}$

(4) $6\sqrt{37}$

Answer (2)

Sol. Let the vectors be $\vec{a}$ and $\vec{b}$

$$\left|\left(\vec{a}\times\vec{b}\right)\right|^2 = |\vec{a}|^2|\vec{b}|^2 - \left(\vec{a}\cdot\vec{b}\right)^2$$

$$116 + 144 = \left(|\vec{a}||\vec{b}|\right)^2$$

$\Rightarrow$ $|\vec{a}||\vec{b}| = \sqrt{260}$

7. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term of the expansion is

(1) 3654

(2) 3658

(3) 3600

(4) 1000

Answer (1)

Sol. Given ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 1 : 5 : 20$

$$\therefore \quad \frac{n!}{(r-1)!(n-r+1)!}\times\frac{r!(n-r)!}{n!} = \frac{1}{5}$$

$$\frac{r}{n-r+1} = \frac{1}{5}$$

$\Rightarrow$ $n - r + 1 = 5r$

$n = 6r - 1$ ...(i)

Now,

$$\frac{n!}{r!(n-r)!}\times\frac{(r+1)!(n-r-1)!}{n!} = \frac{5}{20}$$

$\Rightarrow$ $\frac{r+1}{n-r} = \frac{1}{4}$

$\Rightarrow$ $4r + 4 = n - r$

$n = 5r + 4$ ...(ii)

By (i) and (ii)

$5r + 4 = 6r - 1$

$\Rightarrow$ $r = 5$

and $n = 29$

Now coefficient of fourth term

$= {}^nC_3 = {}^{29}C_3 = 3654$

8. The area under the curve of equations: $x^2 \leq y$, $y \leq 8 - x^2$ and $y \leq 7$, is

(1) $\frac{16}{3}$

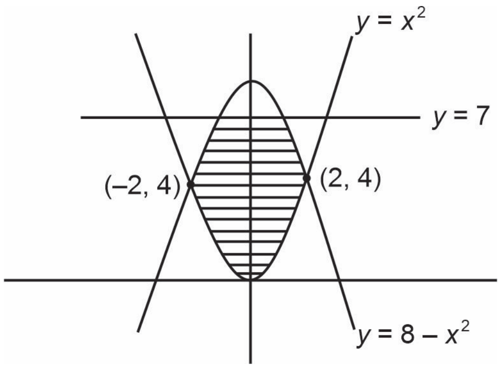
(2) 18

(3) 20

(4) $\frac{22}{3}$

Answer (3)

Sol :



Required area = $2\left[\int_0^4 \sqrt{y}\,dy + \int_4^7 \left(\sqrt{8-y}\right)dy\right]$

$= 2\left[\left.\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^4 - \left.\frac{(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right|_4^7\right]$

$= \frac{4}{3}\left(8-(1-8)\right)$

$= \frac{4}{3}(15)$ = 20 sq. units

9. $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, Q = PAP^T$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $P^TQ^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find 2$a$ + b + 3c – 4d.

(1) 2005

(2) 2007

(3) 2006

(4) 2008

Answer (1)

Sol. $P \times P^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Similarly $P^TP = I$

Now, $Q^{2007} = (PAP^T)(PAP^T)$ ..... 2007 times

$= PA^{2007}P^T$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$P^TQ^{2007}P = P^TPA^{2007}P^TP = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow a = 1, b = 2007, c = 0, d = 1$

$2a + b + 3c - 4d = 2 \times 1 + 2007 + 3 \times 0 - 4 \times 1$

$= 2005$

10. A bolt manufacturing factory has three products A, B and C. 50% and 30% of the products are A and B type respectively and remaining are C type. Then probability that the product A is defective is 4%, that of B is 3% and that of C is 2%. A product is picked randomly picked and found to be defective, then the probability that it is type C.

(1) $\frac{4}{33}$

(2) $\frac{1}{33}$

(3) $\frac{2}{33}$

(4) $\frac{9}{33}$

Answer (1)

Sol. Product A is 50%, B is 30% and C is 20%

Let A_1 is the event that product A is selected

B_1 is the event that product B is selected

C_1 is the event that product C is selected

and D is the event that product is defective

then,

$$P\left(\frac{C_1}{D}\right) = \frac{P(C_1)P\left(\frac{D}{C_1}\right)}{P(A_1)P\left(\frac{D}{A_1}\right)+P(B_1)P\left(\frac{D}{B_1}\right)+P(C_1)P\left(\frac{D}{C_1}\right)}$$

$$= \frac{\frac{20}{100}\times\frac{2}{100}}{\frac{50}{100}\times\frac{4}{100}+\frac{30}{100}\times\frac{3}{100}+\frac{20}{100}\times\frac{2}{100}}$$

$$= \frac{40}{200+90+40} = \frac{4}{33}$$

11. A has 5 elements and B has 2 elements. The number of subsets of $A \times B$ such that the number

of elements in subset is more than or equal to 3 and less than 6, is

(1) 602 (2) 484

(3) 582 (4) 704

Answer (3)

Sol. $n(A) = 5$, $n(B) = 2$

$\Rightarrow$ $n(A \times B) = 10$

Number of subsets having 3 elements = ${}^{10}C_3$

Number of subsets having 4 elements = ${}^{10}C_4$

Number of subsects having 5 elements = ${}^{10}C_5$

$\therefore$ ${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5$

= 120 + 210 + 252

= 582

12. Check whether the function $f(x) = \frac{(1+2^x)^7}{2^x}$ is

(1) Even

(2) Odd

(3) Neither even nor odd

(4) None of these

Answer (3)

Sol. $f(x) = \frac{(1+2^x)^7}{2^x}$

$$f(-x) = \frac{(1+2^{-x})^7}{2^{-x}} = \frac{(2^x+1)^7}{2^{6x}}$$

$\therefore$ $f(x)$ is neither even nor odd.

13. Let $I(x) = \int \frac{(x+1)\,dx}{x(1+xe^x)^2}$, then $\lim_{x\to\infty} I(x) = 1$. The value of $I(1)$ is

(1) $\frac{1}{e+1} - \ln(e+1) + 1$

(2) $\frac{1}{e+1} - \ln(e+1)$

(3) $\frac{1}{e+1} - \ln(e+1) + 2$

(4) $\frac{1}{e+1} + 2$

Answer (3)

Sol. $I(x) = \int \frac{(x+1)\,dx}{x(1+xe^x)^2}$

$$\int \frac{e^x(x+1)}{xe^x(1+xe^x)^2} = dx$$

Let $1 + xe^x = t$

$\Rightarrow$ $e^x(1+x)\,dx = dt$

$$= \int \frac{dt}{(t-1)\,t^2} = -\ln t + \frac{1}{t} + \ln(t-1) + c$$

$$= -\ln(1+xe^x) + \frac{1}{x.e^x+1} + \ln(x.e^x) + c$$

$$= \ln\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{xe^x+1} + c$$

$\lim_{x\to\infty}(I(x)) = c = 1$

$\therefore$ $I(x) = \ln\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{xe^x+1} + 1$

$$I(1) = \ln\left(\frac{e}{1+e}\right) + \frac{1}{e+1} + 1$$

$$= 2 + \frac{1}{e+1} - \ln(1+e)$$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g., 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If a_α is the maximum value of $a_n = \frac{n^3}{n^4+147}$.

Then find α

Answer (5)

Sol. $f(n) = \frac{n^3}{n^4+147}$

$$f'(n) = \frac{(3n^2)(n^4+147)-(n^3)(4n^3)}{(n^4+147)^2} = 0$$

$f'(n) = 0$

$\Rightarrow \quad n = \sqrt{21}$

$4 < \sqrt{21} < 5$

$a_5 > a_4$

$\therefore$ for $n = 5$ the value is maximum

$\boxed{\alpha = 5}$

22. Maximum value n such that (66)! is divisible by 3^n

Answer (31)

Sol. $\because$ 3 is a prime number

$$\left[\frac{66}{3}\right]+\left[\frac{66}{3^2}\right]+\left[\frac{66}{3^3}\right]+\left[\frac{66}{3^4}\right]+....$$

22 + 7 + 2 + 0….

= 31

$(66)! = (3)^{31}....$

Maximum value of n = 31

23. If $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ and $|\text{adj}(\text{adj}(\text{adj}(A))| = 16^n$ then the value of n is

Answer (06)

Sol. $|A| = 2(5) - 1(2) = 8$

$\therefore$ Now $|\text{adj}(\text{adj}(\text{adj}(A))| = |A|^{(n-1)^3}$

$= 8^8 = 16^6$

$\therefore \quad \boxed{n = 6}$

24. The value of $\frac{8}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}}(8[\text{cosec}x]-5[\cot x])dx$ is ([·] represents greatest integer function) ________.

Answer (56)

Sol. $\frac{8}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 8[\text{cosec}x]dx - \frac{8}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 5[\cot x]dx$

$$= \frac{8}{\pi}\times 8\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 1\cdot dx - \frac{8}{\pi}\times 5\left(\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 1dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 0\cdot dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}}(-1)dx + \int_{\frac{3\pi}{4}}^{\frac{5\pi}{6}}(-2)dx\right)$$

$$= \frac{64}{\pi}\left(\frac{2\pi}{3}\right) - \frac{40}{\pi}\left(\left(\frac{\pi}{4}-\frac{\pi}{6}\right)+0-\left(\frac{3\pi}{4}-\frac{\pi}{2}\right)-2\left(\frac{5\pi}{6}-\frac{3\pi}{4}\right)\right)$$

$$= \frac{128}{3} - \frac{40}{\pi}\left(\frac{\pi}{12}-\frac{\pi}{4}-\frac{2\pi}{12}\right)$$

$$= \frac{128}{3} - 40\left(\frac{1}{12}-\frac{1}{4}-\frac{1}{6}\right)$$

$$= \frac{128}{3} - 40\left(\frac{1-3-2}{12}\right) = \frac{128}{3} - 40\left(-\frac{1}{3}\right)$$

$$= \frac{168}{3}$$

= 56

25. If $\lim_{x\to 0}\frac{1-\cos^2 3x}{\cos^3 4x}\times\frac{\sin^3 4x}{(\log(1+2x))^5} = t$ then $[t]$ is (where [.] represents greatest integer fraction)

Answer (18)

Sol. $\lim_{x\to 0}\frac{1-\cos^2 3x}{\cos^3 4x}\times\frac{\sin^3 4x}{(\log(1+2x))^5}$

$$\lim_{x\to 0}\frac{\sin^2 3x\sin^3 4x}{\cos^3(4x)(\log(1+2x))^5}$$

$$\lim_{x\to 0}\frac{\frac{\sin^2 3x}{(3x)^2}\cdot\frac{\sin^3 4x}{(4x)^3}.(3x)^2.(4x)^3}{\cos^3 4x.\left(\frac{\log(1+2x)}{2x}\right)^5(2x)^5}$$

$$= \frac{9\times 64}{32} = 18$$

26.

27.

28.

29.

30.

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