JEE Main 2020 Paper

Date : 5th September 2020

Time : 02 : 00 pm - 05 : 00 pm

Subject : Maths

Q.1 If x=1 is a critical point of the function $f(x)=(3x^2+ax-2-a)e^x$, then:

(1) x=1 is a local minima and $x=-\frac{2}{3}$ is a local maxima of f.

(2) x=1 is a local maxima and $x=-\frac{2}{3}$ is a local minima of f.

(3) x=1 and $x=-\frac{2}{3}$ are local minima of f.

(4) x=1 and $x=-\frac{2}{3}$ are local maxima of f.

Sol. **1**

$f(x) = (3x^2+ax-2-a)e^x$

$f'(x) = (3x^2+ax-2-a)e^x + (6x+a)e^x = 0$

$e^x\left[3x^2+(a+6)x-2\right]=0$

at x = 1, 3 +a+6 - 2 = 0

a=−7

$f(x) = (3x^2 - 7x + 5)e^x$

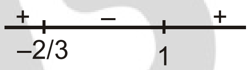
$f'(x) = (6x-7)e^x + (3x^2-7x+5)e^x$

$= e^x(3x^2-x-2) = 0$

$= 3x^2 - 3x + 2x - 2 = 0$

$= (3x+2)(x-1) = 0$

x = 1, -2/3



x = 1 is point of local minima.

x = −2/3 is point of local maxima.

Q.2 $\lim_{x\to 0}\frac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$

(1) is equal to $\sqrt{e}$ (2) is equal to 1 (3) is equal to 0 (4) does not exist

Sol. **2**

$\lim_{x\to 0}\frac{x\left[e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right]}{\left(\sqrt{1+x^2+x^4}-1\right)}$

$$\lim_{x\to 0} \frac{x\left[e^{\left[\frac{\left(\sqrt{1+x^2+x^4}\right)^2-1}{x\times 2}\right]}-1\right]\times\left(\sqrt{1+x^2+x^4}+1\right)}{\left(x^2+x^4\right)}$$

$$\lim_{x\to 0} \frac{e^{\left(\frac{x^3+x}{2}\right)}-1}{\left(\frac{x^3+x}{2}\right)\times 2}\times 2 = 1$$

Q.3 The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is:

(1) equivalent to $(p \vee q) \wedge (\sim p)$

(2) equivalent to $(p \wedge q) \vee (\sim p)$

(3) a contradiction

(4) a tautology

Sol. **4**

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to p \vee q$	$(p \to (q \to p)) \to$ $(p \to (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

Q.4 If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then:

(1) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(2) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$

(3) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

(4) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

Sol. **1**

$$L = \sin\left(\frac{3\pi}{16}\right)\sin\left(\frac{-\pi}{16}\right)$$

$$L = \frac{-1}{2}\left[\cos\frac{\pi}{8} - \cos\frac{\pi}{4}\right]$$

$$L = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\frac{\pi}{8}$$

$M = \cos\left(\frac{3\pi}{16}\right)\cos\left(\frac{\pi}{16}\right)$

$M = \frac{1}{2}\left[\cos\frac{\pi}{4}+\cos\frac{\pi}{8}\right]$

$M = \frac{1}{2\sqrt{2}}+\frac{1}{2}\cos\frac{\pi}{8}$

Q.5 If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460, then x is equal to:

(1) $7^{1/2}$ (2) 7^2 (3) e^2 (4) $7^{46/21}$

Sol. **2**

$(2 + 3 + 4 + \ldots + 21)\log_7 x = 460$

$\Rightarrow \frac{20\times(21+2)}{2}\log_7 x = 460$

$\Rightarrow 230 \log_7 x = 460 \Rightarrow \log_7 x = 2 \Rightarrow x = 7^2$

Q.6 There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is:

(1) 2250 (2) 2255 (3) 1500 (4) 3000

Sol. **1**

	S—1	S—2	S—3
	1,2,3,4,5	1,2,3,4,5	1,2,3,4,5
6 case	1	1	3
	1	2	2
	1	3	1
	2	2	1
	2	1	2
	3	1	1

$= 3({}^5C_1 \times {}^5C_2 \times {}^5C_2) + 3({}^5C_1 \times {}^5C_1 \times {}^5C_3)$
$= 3(5 \times 5 \times 2 \times 5 \times 2) + 3(5 \times 5 \times 10)$
$= 750 + 1500 = 2250$

Q.7 If the mean and the standard deviation of the data 3,5,7,a,b are 5 and 2 respectively, then a and b are the roots of the equation:

(1) $x^2-20x+18=0$ (2) $x^2-10x+19=0$ (3) $2x^2-20x+19=0$ (4) $x^2-10x+18=0$

Sol. **2**

$\text{S.D.} = \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

$$(2)^2 = \frac{83+a^2+b^2}{5} - (5)^2$$

$$4 = \frac{83+a^2+b^2}{5} - 25$$

$29\times5 - 83 = a^2+b^2 \Rightarrow a^2+b^2 = 62$

$$\frac{a+b+15}{5} = 5$$

$\Rightarrow \boxed{a+b=10}$ ...(1)

$2ab = 100 - 62 = 38$

$\boxed{ab=19}$ ...(2)

from eq.(1) & (2)

$x^2 - 10x + 19 = 0$

Q.8 The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \frac{1}{2}$ is:

(1) $\frac{2\sqrt{3}}{3}$ (2) $\frac{2\sqrt{3}}{5}$ (3) $\frac{\sqrt{3}}{12}$ (4) $\frac{\sqrt{3}}{10}$

Sol. **4**

$x = \tan\theta$

$$u = \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right) = \tan^{-1}(\tan\theta/_2) = \frac{\theta}{2} = \frac{\tan^{-1}x}{2}$$

$x = \sin\theta$

$$v = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{\cos 2\theta}\right) = 2\theta$$

$= 2\sin^{-1}x$

$$\frac{du}{dv} = \frac{1}{2(1+x^2)} \times \frac{\sqrt{1-x^2}}{2}$$

$$= \frac{\sqrt{3}}{2\times2} \times \frac{4}{5\times2} = \frac{\sqrt{3}}{10}$$

Q.9 If $\int \frac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}d\theta = A\log_e|B(\theta)| + C$ where C is a constant of integration, then $\frac{B(\theta)}{A}$ can be:

(1) $\frac{5(2\sin\theta+1)}{\sin\theta+3}$ (2) $\frac{5(\sin\theta+3)}{2\sin\theta+1}$ (3) $\frac{2\sin\theta+1}{\sin\theta+3}$ (4) $\frac{2\sin\theta+1}{5(\sin\theta+3)}$

Sol. **1**

$$\int \frac{\cos\theta}{5+7\sin\theta-2+2\sin^2\theta}d\theta$$

Put $\sin\theta = t$, $\cos\theta\, d\theta = dt$

$$\int \frac{dt}{2t^2 + 7t + 3}$$

$$= \frac{1}{2}\int \frac{dt}{t^2 + \frac{7t}{2} + \frac{3}{2}} = \frac{1}{2}\int \frac{dt}{t^2 + \frac{7}{2}t + \left(\frac{7}{4}\right)^2 - \frac{49}{16} + \frac{24}{16}}$$

$$= \frac{1}{2}\int \frac{dt}{(t + 7/4)^2 - (5/4)^2}$$

$$\frac{1}{2} \times \frac{1}{2 \cdot \frac{5}{4}} \ln\left|\left[\frac{t + 7/4 - 5/4}{t + 7/4 + 5/4}\right]\right|$$

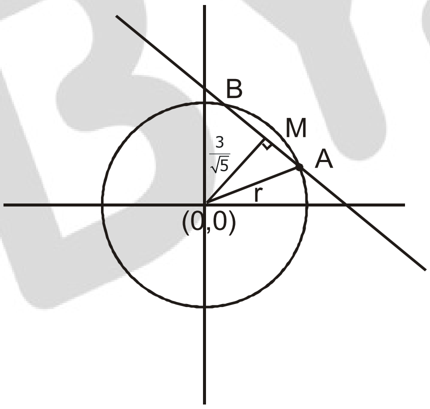
$$\frac{1}{5}\ln\left|\left(\frac{\sin\theta + 1/2}{\sin\theta + 3}\right)\right| + C$$

$$\frac{B(\theta)}{A} = 5\left(\frac{2\sin\theta + 1}{\sin\theta + 3}\right)$$

Q.10 If the length of the chord of the circle, $x^2+y^2 = r^2 (r>0)$ along the line, $y-2x=3$ is r, then r^2 is equal to:

(1) 12 (2) $\frac{24}{5}$ (3) $\frac{9}{5}$ (4) $\frac{12}{5}$

Sol. **4**



$$AB = 2\sqrt{r^2 - 9/5} = r$$

$$r^2 - 9/5 = \frac{r^2}{4}$$

$$3r^2/4 = 9/5$$

$$\boxed{r^2 = \frac{12}{5}}$$

Q.11 If α and β are the roots of the equation, $7x^2-3x-2=0$, then the value of $\frac{\alpha}{1-\alpha^2}+\frac{\beta}{1-\beta^2}$ is equal to:

(1) $\frac{27}{32}$ (2) $\frac{1}{24}$ (3) $\frac{27}{16}$ (4) $\frac{3}{8}$

Sol. 3

$\alpha + \beta = 3/7$, $\alpha\beta = -2/7$

$$\frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha^2+\beta^2)+(\alpha\beta)^2}$$

$$\frac{\frac{3}{7}+\frac{2}{7}\times\frac{3}{7}}{1-\left\{\frac{9}{49}+\frac{4}{7}\right\}+\frac{4}{49}}$$

$$\frac{\left(\frac{21+6}{49}\right)}{\frac{16}{49}} \Rightarrow \frac{27}{16}$$

Q.12 If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is:

(1) $\frac{2}{13}(3^{50}-1)$ (2) $\frac{1}{26}(3^{49}-1)$ (3) $\frac{1}{13}(3^{50}-1)$ (4) $\frac{1}{26}(3^{50}-1)$

Sol. **4**

$$\frac{ar+ar^2+ar^3}{ar^5+ar^6+ar^7}=\frac{3}{243}$$

$$\frac{1+r+r^2}{r^4(1+r+r^2)}=\frac{1}{81}$$

$\boxed{r=3}$

$a(3+9+27) = 3$

$$a=\frac{3}{39}=\boxed{\frac{1}{13}}$$

$$S_{50}=a\left(\frac{r^{50}-1}{r-1}\right)$$

$$=\frac{1}{13}\left\{\frac{3^{50}-1}{2}\right\}$$

Q.13 If the line y=mx+c is a common tangent to the hyperbola $\frac{x^2}{100}-\frac{y^2}{64}=1$ and the circle $x^2+y^2=36$, then which one of the following is true?

(1) $4c^2=369$ (2) $c^2=369$ (3) $8m+5=0$ (4) $5m=4$

Sol. **1**

$c = \pm\sqrt{a^2m^2-b^2}$

$c = \pm\sqrt{100m^2-64}$

$y = mx \pm\sqrt{100m^2-64}$

$d|_{(0,0)} = 6$

$\left|\frac{\sqrt{100m^2-64}}{\sqrt{m^2+1}}\right| = 6$

$100m^2 - 64 = 36m^2 + 36$

$64m^2 = 100$

$m = \pm\frac{10}{8}$

$c^2 = 100\times\frac{100}{64}-64 \Rightarrow \frac{(164)(36)}{64}$

$\boxed{4c^2=369}$

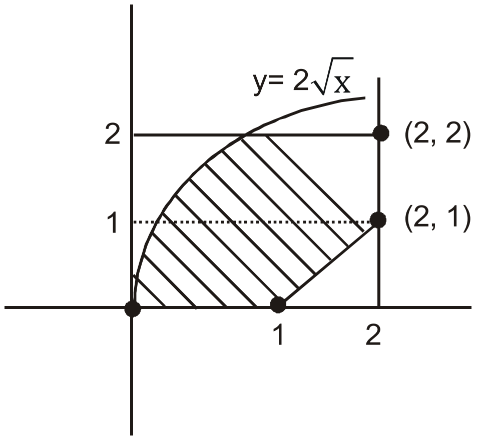
Q.14 The area (in sq. units) of the region $A=\{(x,y):(x-1)[x]\le y\le 2\sqrt{x}, 0\le x\le 2\}$ where [t] denotes the greatest integer function, is:

(1) $\frac{4}{3}\sqrt{2}-\frac{1}{2}$ (2) $\frac{8}{3}\sqrt{2}-\frac{1}{2}$ (3) $\frac{8}{3}\sqrt{2}-1$ (4) $\frac{4}{3}\sqrt{2}+1$

Sol. **2**

$$y = f(x) = (x-1)[x] = \begin{cases} 0 & 0\le x<1 \\ x-1 & 1\le x<2 \\ 2(x-1) & x=2 \end{cases}$$

$y^2 \le 4x$



$$\int_0^1 \left(2\sqrt{x}-0\right)+\int_1^2 \left(2\sqrt{x}-(x-1)\right)$$

$$\frac{2}{3}\times 2x^{3/2}\Big|_0^1+\left(\frac{4}{3}x^{3/2}-\frac{x^2}{2}+x\right)_1^2$$

$$\frac{4}{3}+\left\{\left(\frac{4}{3}\times 2\sqrt{2}-2+2\right)-\left(\frac{4}{3}+\frac{1}{2}\right)\right\}$$

$$\frac{4}{3}+\frac{8\sqrt{2}}{3}-\frac{4}{3}-\frac{1}{2} = \frac{8\sqrt{2}}{3}-\frac{1}{2}$$

Q.15 If a+x=b+y=c+z+1, where a,b,c,x,y,z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to:

(1) y(a–b) (2) 0 (3) y(b–a) (4) y(a–c)

Sol. **1**

Given a+x=b+y=c+z+1

Now, $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$

$$= \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix} (C_3 \to C_3 - C_1)$$

$$= \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} (C_2 \to C_2 - C_3)$$

$$= y\begin{vmatrix} x & 1 & a \\ y & 1 & b \\ z & 1 & c \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= y\begin{vmatrix} x & 1 & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

$$= y[x\times 0-1\{(y-x)(c-a)-(b-a)(z-x)\}+a\times 0]$$

$$= y\left[bz-bx-az+ax-(cy-ay-cx+ax)\right]$$

$$= y\left[bz-bx-az-cy+ay+cx\right]$$

$$= y\left[b(z-x)+a(y-z)+c(x-y)\right]$$

$= y\left[b\{a-c-1\}+a(c-b+1)+c(b-a)\right]$

$= y\left[ab-bc-b+ac-ab+a+bc-ac\right]$

$= \boxed{y(a-b)}$

Q.16 If for some $\alpha \in R$, the lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar, then the line L_2 passes through the point:

(1) (2, −10, −2) (2) (10, −2, −2) (3) (10, 2, 2) (4) (−2, 10, 2)

Sol. **1**

A (-1,2,1), B(-2,-1, -1)

$\left[\overrightarrow{AB}\ \overrightarrow{b_1}\ \overrightarrow{b_2}\right] = 0$

$$\begin{vmatrix} -1 & -3 & -2 \\ 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \end{vmatrix} = 0$$

$-1(-1+\alpha-5) + 3(2-\alpha)-2(10-2\alpha+\alpha)=0$

$6-\alpha +6-3\alpha + 2\alpha - 20 = 0$

$-8 -2\alpha = 0$

$\boxed{\alpha = -4}$

$L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$

any point on L_2 is

$(-4\lambda-2, 9\lambda-1, \lambda-1)$

Q.17 The value of $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$ is:

(1) $2^{15}i$ (2) -2^{15} (3) $-2^{15}i$ (4) 6^5

Sol. **3**

$$\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30} \Rightarrow \left[\left(\frac{-1+i\sqrt{3}}{2}\right)(1+i)\right]^{30}$$

$\omega^{30}(1+i)^{30} = 2^{15}(-i)$

Q.18 Let $y=y(x)$ be the solution of the differential equation $\cos x\frac{dy}{dx}+2y\sin x=\sin 2x, x\in\left(0,\frac{\pi}{2}\right)$. If $y(\pi/3)=0$, then $y(\pi/4)$ is equal to:

(1) $2+\sqrt{2}$ (2) $\sqrt{2}-2$ (3) $\frac{1}{\sqrt{2}}-1$ (4) $2-\sqrt{2}$

Sol. **2**

$\frac{dy}{dx}+(2\tan x)y=2\sin x$

I.F. = $e^{2\ln(\sec x)}=\sec^2 x$

$y(\sec^2 x)=2\int\frac{\sin x}{\cos^2 x}dx$

$=2\int\sec x\tan x\,dx=2\sec x+c$

$y\left(\frac{\pi}{3}\right)=0$

$0 = 2 \times 2 + c$

$\Rightarrow c = -4$

$y(\sec^2 x) = 2\sec x - 4$

$x = \pi/4$

$2y = 2\sqrt{2}-4$

$y = \boxed{\sqrt{2}-2}$

Q.19 If the system of linear equations

$x+y+3z=0$

$x+3y+k^2z=0$

$3x+y+3z=0$

has a non-zero solution (x,y,z) for some $k\in R$, then $x+\left(\frac{y}{z}\right)$ is equal to:

(1) −9 (2) 9 (3) −3 (4) 3

Sol. **3**

$$\begin{vmatrix}1&1&3\\1&3&k^2\\3&1&3\end{vmatrix}=0$$

$(9-k^2)-(3-3k^2)+3(-8)=0$

$9-k^2-3+3k^2-24=0$

$2k^2-18=0$

$k^2=9$

$\boxed{k=3,-3}$

$$\begin{array}{r}x+y+3z=0\\x+3y+9z=0\\\hline 2y+6z=0\end{array}$$

$\boxed{y = -3z}$

$\boxed{y / z = -3}$

2x=0

$\boxed{x = 0}$

$x + \left(\frac{y}{z}\right) = -3$

Q.20 Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1,0)?

(1) (2,6) (2) (2,2) (3) (−2,6) (4) (−2,4)

Sol. **3**

$4x^3e^y + x^4e^yy' + \frac{2y'}{2\sqrt{y+1}} = 0$

at (1,0)

$4 + y' + \frac{2y'}{2} = 0$

$2y' = -4 \Rightarrow y' = -2$

E.O.T. :

y = −2(x-1)

$\boxed{2x + y = 2}$

Q.21 Let A={a,b,c} and B={1,2,3,4}. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one–one} is________

Sol. **19**

$C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one}

Case-I : If f(x) = 2 $\forall$ x $\in$ A then numberof function = 1

Case-II : If f(x) = 2 for exactly two elements then total number of many-one function $= {}^3C_2\,{}^3C_1 = 9$

Case-III : If f(x) = 2 for exactly one elementthen total number of many-one functions $= {}^3C_1\,{}^3C_1 = 9$

Total = 19

Q.22 The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$ in powers of x, is ______

Sol. **120**

$(1+x)^6(1+x^2)^6$

$6_{c_r}\, x^r \quad 6_{c_s}\, x^{2S}$

$6_{c_r} 6_{c_s} \quad x^{r+2S}$

r	s
0	2
4	0
2	1

$\Rightarrow 6_{c_0} 6_{c_2} + 6_{c_4} 6_{c_0} + 6_{c_2} 6_{c_1}$

$\Rightarrow$ 15+15+15×6

$\Rightarrow$ 120

Q.23 Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is_______

Sol. **6**

$\Rightarrow \dfrac{\vec{b}.\vec{a}}{2} = \dfrac{\vec{c}.\vec{a}}{2}$ $\boxed{\vec{b}.\vec{a} = \vec{c}.\vec{a}}$

$\Rightarrow \boxed{\vec{b}.\vec{c} = 0}$

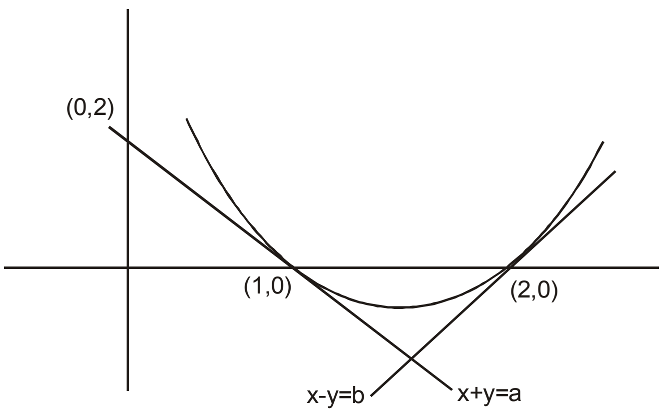
$\Rightarrow \left|\vec{a} + \vec{b} - \vec{c}\right| = \sqrt{a^2 + b^2 + c^2 + 2\vec{a}.\vec{b} - 2\vec{b}.\vec{c} - 2\vec{a}.\vec{c}}$

$= \sqrt{4 + 16 + 16}$

$= 6$

Q.24 If the lines x+y=a and x-y=b touch the curve $y=x^2-3x+2$ at the points where the curve intersects the x–axis, then $\frac{a}{b}$ is equal to_______

Sol. **0.5**



$y - 0 = -1(x-1)$
$x + y = 1 \Rightarrow a = 1$
$y - 0 = x - 2$
$x - y = 2 = b = 2$

$$\boxed{\frac{a}{b} = \frac{1}{2}}$$

Q.25 In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is________

Sol. **11**

Let 'n is total no. of bombs being dropped
at least 2 bombs should hit

$\Rightarrow$ prob ≥ 0.99

$p(x \geq 2) \geq 0.99$

$1 - p(x<2) \geq 0.99$

$1 - (p(x=0) + p(x=1)) \geq 0.99$

$$1 - \left[\ {}^{n}C_{o}(p)^{0} q^{n} + {}^{n}C_{1}(p)^{1}(q)^{n-1}\right] \geq 0.99$$

$$1 - \left[q^{n} + pnq^{n-1}\right] \geq 0.99$$

$$1 - \left[\frac{1}{2^{n}} + \frac{n}{2} \times \frac{1}{2^{n-1}}\right] \geq 0.99$$

$$1 - \frac{1}{2^{n}}(n+1) \geq 0.99$$

$$0.01 \geq \frac{1}{2^{n}}(n+1)$$

$2^{n} \geq 100 + 100n$

$n \geq 11$