BYJU'S Home Learning Program

Topic covered:

- **Mathematical Tools (session - 2) - JEE**

Worksheet

1. Simplify: $\log_2 5 + \log_2 1.6$

2. Given that $x = 2^p$ and $y = 4^q$, show that $\log_2(x^3 y) = 3p + 2q$.

3. Find the derivative of x^5.

4. If $y = x^3 + 2x^2 + 7x + 8$, then $\frac{dy}{dx}$ will be
   a. $3x^2 + 2x + 15$
   b. $3x^2 + 4x + 7$
   c. $x^3 + 2x^2 + 15$
   d. $x^3 + 4x + 7$

5. If $y = 2\sin x$ then $\frac{dy}{dx}$ will be
   a. $2\cos x$
   b. $-2\cos x$
   c. $\cos x$
   d. None

6. If $y = \frac{1}{x^4}$ then, $\frac{dy}{dx}$ will be
   a. $\frac{4}{x^3}$
   b. $4x^3$
   c. $-\frac{4}{x^5}$
   d. $\frac{4}{x^5}$

7. If $y = e^x.\cot x$ then $\frac{dy}{dx}$ will be
   a. $e^x \cot x - \text{cosec}^2 x$
   b. $e^x \text{cosec}^2 x$
   c. $e^x[\cot x - \text{cosec}^2 x]$
   d. $e^x \cot x$

8. If $y = \frac{\ln x}{x}$ then $\frac{dy}{dx}$ will be
   a. $\frac{1-\ln x}{x}$
   b. $\frac{1+\ln x}{x^2}$
   c. $\frac{1-\ln x}{x^2}$
   d. $\frac{\ln x - 1}{x^2}$

9. Differentiation of $\sin(x^2)$ w.r.t. x is
   a. $\cos(x^2)$
   b. $2x\cos(x^2)$
   c. $x^2\cos(x^2)$
   d. $-\cos(2x)$

10. If $y = x^2 \sin x$, then $\frac{dy}{dx}$ will be
    a. $x^2\cos x + 2x\sin x$
    b. $2x\sin x$
    c. $x^2\cos x$
    d. $2x\cos x$

11. If $y = \tan x . \cos^2 x$ then $\frac{dy}{dx}$ will be
   a. $1 + 2\sin^2 x$
   b. $1 - 2\sin^2 x$
   c. 1
   d. $2\sin^2 x$

12. If $y = 2\sin^2\theta + \tan\theta$ then $\frac{dy}{d\theta}$ will be
   a. $4\sin\theta\cos\theta + \sec\theta\tan\theta$
   b. $2\sin 2\theta + \sec^2\theta$
   c. $4\sin\theta + \sec^2\theta$
   d. $2\cos^2\theta + \sec^2\theta$

13. Find the derivative of x^2 using first principle.

14. Find $f'(x)$ if $f(x)$ is
   (i) xe^x
   (ii) $x \sin x$

15. If $f(x) = \sqrt{x}\, g(x)$, where $g(4) = 2$ and $g'(4) = 3$, find $f'(4)$.

16. Find the derivative of $y = \frac{t^2-1}{t^2+1}$.

17. Find the derivative of $y(x) = \frac{x^3}{(x+1)^2}$ with respect to x.

18. Find $\frac{dy}{dx}$ of $y = \tan x^2$.

19. Find the coordinate of the point on curve $y = x^4 - 2x^2 + 2$ for which first derivative is 0.

20. Find the slope of the tangent to the curve $y = 2\sqrt{x}$ at the point (1,2).

BYJU'S Home Learning Program

Answer Key

Question Number	1	2	3	4	5
Answer Key	3	$3p + 2q$	$5x^4$	(b)	(a)

Question Number	6	7	8	9	10
Answer Key	(c)	(c)	(c)	(b)	(a)

Question Number	11	12	13	14	15
Answer Key	(b)	(b)	$2x$	(i) $(x+1)e^x$ (ii) $x.\cos x + \sin x$	6.5

Question Number	16	17	18	19	20
Answer Key	$\frac{4t}{(t^2+1)^2}$	$\frac{3x^2}{(x+1)^2} - \frac{2x^3}{(x+1)^3}$	$2x \sec^2 x^2$	(0, 2) (1, 1) (-1, 1)	1

Solutions

1. $\log_b m + \log_b n = \log_b(mn)$
   $\log_2 5 + \log_2 1.6 = \log_2 8 = \log_2 2^3 = 3\log_2 2 = 3$

2. $\log_2(x^3y) = \log_2(2^{3p}4^q) = \log_2\left(2^{(3p+2q)}\right) = 3p + 2q$

3. $y = x^5$
   $\frac{dy}{dx} = 5x^4$

4. (b)
   $y = x^3 + 2x^2 + 7x + 8$
   $\frac{dy}{dx} = 3x^2 + 4x + 7$

5. (a)
   $y = 2\sin x$
   $\frac{dy}{dx} = 2\cos x$

6. (c)
   $y = x^{-4}$
   $\frac{dy}{dx} = (-4)x^{-4-1} = -\frac{4}{x^5}$

7. (c)
   $y = e^x.\cot x$
   $\frac{dy}{dx} = e^x\frac{d}{dx}(\cot x) + \cot x\frac{d}{dx}(e^x)$
   $= e^x(-\mathrm{cosec}^2 x) + \cot x(e^x)$
   $= e^x[\cot x - \mathrm{cosec}^2 x]$

8. (c)
   $y = \frac{\ln x}{x}$
   $\frac{dy}{dx} = \frac{x\frac{d}{dx}(\ln x) - \ln x\frac{d}{dx}(x)}{x^2}$
   $= \frac{x\left(\frac{1}{x}\right) - \ln x}{x^2}$
   $\frac{dy}{dx} = \frac{1-\ln x}{x^2}$

9. (b)
   $y = \sin x^2$
   $\frac{dy}{dx} = \cos(x^2)\frac{d}{dx}\ (x^2)$
   $= 2x\cos(x^2)$

10. (a)

$y = x^2 \sin x$

$\frac{dy}{dx} = x^2 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^2)$

$= x^2 \cos x + 2x \sin x$

11. (b)

$y = \tan x \cos^2 x$

$\frac{dy}{dx} = \tan x \frac{d}{dx}(\cos^2 x) + \cos^2 x \frac{d}{dx}(\tan x)$

$= \tan x (2\cos x)(-\sin x) + \cos^2 x \sec^2 x$

$= 1 - 2\sin^2 x$

12. (b)

$y = 2\sin^2 \theta + \tan \theta$

$\frac{dy}{d\theta} = 2 \times 2 \sin\theta \cos\theta + \sec^2 \theta$

$= 2\sin 2\theta + \sec^2 \theta$

13. $\underset{h\to 0}{\text{Limit}} \frac{(x+h)^2 - x^2}{h}$

$= \underset{h\to 0}{\text{Limit}} \frac{(x+h-x)(x+h+x)}{h}$

$= \underset{h\to 0}{\text{Limit}} (2x+h) = 2x + 0 = 2x$

14. (i) $f'(x) = xe^x + e^x.1 = (x+1)e^x$

(ii) $f'(x) = x.\frac{d}{dx}(\sin x) + \sin x.\frac{dx}{dx}$

$f'(x) = x.\cos x + \sin x$

15. $f'(4) = \sqrt{x} g'(x) + \frac{g(x)}{2\sqrt{x}}$

$f'(4) = \sqrt{4} g'(4) + \frac{g(4)}{2\sqrt{4}} = 2 \times 3 + \frac{2}{2\times 2} = 6.5$

16. We apply the Quotient Rule with $u = t^2 - 1$ and $v = t^2 + 1$

$$\frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dt}\right) - u\left(\frac{dv}{dt}\right)}{v^2}$$

$\frac{dy}{dt} = \frac{(t^2+1).2t - (t^2-1).2t}{(t^2+1)^2}$

$= \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}$

17. We can rewrite this function as $y(x) = x^3(x+1)^{-2}$ and apply product rule,

$$\frac{dy}{dx} = (x+1)^{-2}\frac{d}{dx}(x^3) + x^3\frac{d}{dx}((x+1)^{-2})$$
$$= (x+1)^{-2}3x^2 + x^3(-2)(x+1)^{-3}$$
$$\frac{dy}{dx} = \frac{3x^2}{(x+1)^2} - \frac{2x^3}{(x+1)^3}$$

18. $\frac{d}{dx}(\tan x^2) = (\sec^2 x^2).2x = 2x\sec^2 x^2$

19. $\frac{dy}{dx} = \frac{d}{dx}(x^4 - 2x^2 + 2) = 4x^3 - 4x$

$$\frac{dy}{dx} = 0$$
$$4x^3 - 4x = 0$$
$$4x(x^2 - 1) = 0$$
$$x = 0, 1, -1$$

The corresponding points on the curve are $(0, 2), (1, 1)$ and $(-1, 1)$.

20. $y = 2\sqrt{x}$

$$\frac{dy}{dx} = 2\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{\sqrt{x}}$$

$\frac{dy}{dx}$ is called slope of the tangent to the curve $y = 2\sqrt{x}$ at the point (x, y). For the point $x = 1$ and $y = 2$, putting $x = 1$ and $y = 2$ in the equation $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$, we get $\frac{dy}{dx} = \frac{1}{\sqrt{1}} = 1$. Hence slope is 1.