BYJU'S Home Learning Program

Topic covered:

- **Mathematical Tools (Session - 2) - JEE**

Daily Practice Problems

1. Given that $y = 4 \times 10^{2x}$, express x in terms of y, giving an exact simplified answer in terms of logarithmic base 10.

2. Find the derivative of following functions using first principle
   i. $\cos x$
   ii. $\operatorname{cosec} x$
   iii. $\sec x$
   iv. $\cot x$

3. If $y = \sin x$ and $x = 3t$, then $\frac{dy}{dt}$ will be
   a. $9\cos(x)$
   b. $\cos(x)$
   c. $3\cos(3t)$
   d. $-\cos x$

4. If $y = \sin(x) + \ln(x^2) + e^{2x}$, then $\frac{dy}{dx}$ will be
   a. $\cos x + \frac{2}{x} + e^{2x}$
   b. $\cos x + \frac{2}{x} + 2e^{2x}$
   c. $-\cos x + \frac{2}{x^2} + e^{2x}$
   d. $-\cos x - \frac{2}{x^2} + 2e^{2x}$

5. If $y = x^3$ then $\frac{d^2y}{dx^2}$ is
   a. $6x^2$
   b. $6x$
   c. $3x^2$
   d. $3x$

6. Show that $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$.

COMPREHENSION (Q.7 – Q.8)

If a function is given as $y = \ln(\sin x)$, then

7. The first derivative of y w.r.t. x will be
   a. $\sin x$
   b. $\tan x$
   c. $\ln(\tan x)$
   d. $\cot x$

8. Differentiation of above result w.r.t. x will be
   a. $\operatorname{cosec}^2 x$
   b. $-\operatorname{cosec}^2 x$
   c. $\sec^2 x$
   d. $-\operatorname{cosec}^2 x \cot x$

9. If a function is given by $v = 2t^4$, then its first derivative w.r.t. t will be given by
   a. $8t^3$
   b. $8t$
   c. $-8t^3$
   d. t^2

10. Find the derivative of $y = (x^2 + 1)(x^3 + 3)$.

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Answer Key

Question Number	1	2	3	4	5
Answer Key	$x = \frac{1}{2}\log_{10}\frac{y}{4}$	(i) $-\sin x$ (ii) $-\operatorname{cosec} x \cot x$ (iii) $\sec x \tan x$ (iv) $-\operatorname{cosec}^2 x$	(c)	(b)	(b)

Question Number	6	7	8	9	10
Answer Key	1	(d)	(b)	(a)	$5x^4 + 3x^2 + 6x$

Solutions

1. $y = 4 \times 10^{2x}$

$\frac{y}{4} = 10^{2x}$

We know,

If $y = a^x$, then $x = \log_a y$

So, $2x = \log_{10} \frac{y}{4}$

$x = \frac{1}{2} \log_{10} \frac{y}{4}$

2.

i. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\cos(x+h) - \cos x}{h}$

$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$

$= \lim_{h \to 0} \frac{\cos x (\cos h - 1) - \sin x \sin h}{h}$

$= \lim_{h \to 0} \frac{\cos x (\cos h - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$

$= \cos x \lim_{h \to 0} \frac{(\cos h - 1)}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$

$= \cos x \times 0 - \sin x \times 1 = -\sin x$

ii.

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{\operatorname{cosec}(x+h) - \operatorname{cosec} x}{h}$

$= \lim_{h \to 0} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h}$

$= \lim_{h \to 0} \frac{\frac{\sin x - \sin(x+h)}{\sin(x+h)\sin x}}{h}$

Using, $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$

$= \lim_{h \to 0} \frac{2\cos\left(\frac{x+x+h}{2}\right) \sin\left(\frac{x-x-h}{2}\right)}{h \sin(x+h)\sin x}$

$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x+h}{2}\right) \sin\left(\frac{-h}{2}\right)}{h \sin(x+h)\sin x}$

$= \lim_{h \to 0} \frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)\sin x} \quad \lim_{h \to 0} \frac{-\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Second limit has value unity, which is already derived in theory class.

$= \frac{\cos(x)}{\sin(x)\sin x}(-1) = -\operatorname{cosec} x \cot x$

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iii. $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\sec(x+h)-\sec x}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)}-\frac{1}{\cos x}}{h}$

$= \lim_{h\to 0} \frac{\frac{\cos x-\cos(x+h)}{\cos(x+h)\cos x}}{h}$

Using, $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{h\cos(x+h)\cos x}$

$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{h\cos(x+h)\cos x}$

$= \lim_{h\to 0} \frac{-\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)\cos x} \quad \lim_{h\to 0} \frac{-\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$

Second limit has value unity, which is already derived in theory class.

$= \lim_{h\to 0} \frac{-\sin\left(\frac{2x+h}{2}\right)}{\cos(x+h)\cos x} \quad \lim_{h\to 0} \frac{-\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = \frac{-\sin(x)}{\cos(x)\cos x}(-1) = \sec x \tan x$

iv.

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\cot(x+h)-\cot x}{h}$

$= \lim_{h\to 0} \frac{\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}}{h}$

$= \lim_{h\to 0} \frac{\frac{\cos(x+h)\sin x-\sin(x+h)\cos x}{\sin(x+h)\sin x}}{h}$

Using, $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$= \lim_{h\to 0} \frac{\sin(x-x-h)}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0} \frac{\sin(-h)}{h\sin(x+h)\sin x}$

$= \lim_{h\to 0} \frac{1}{\sin(x+h)\sin x} \quad \lim_{h\to 0} \frac{-\sin(h)}{h}$

Second limit has value unity, which is already derived in theory class.

$= \frac{1}{\sin x \sin x}(-1) = -\text{cosec}^2 x$

3. (c)

$y = \sin x, \qquad x = 3t$

$\frac{dy}{dx} = \cos x, \qquad \frac{dx}{dt} = 3$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 3\cos x = 3\cos(3t)$

Alternative:

Replace the value of x in $\sin x$ by $3t$

$y = \sin(3t)$

$\frac{dy}{dt} = 3\cos(3t)$

4. (b)

$y = \sin x + \ln x^2 + e^{2x}$

$\frac{dy}{dx} = \cos x + \frac{2x}{x^2} + 2e^{2x}$

$= \cos x + \frac{2}{x} + 2e^{2x}$

5. (b)

$y = x^3$

$\frac{dy}{dx} = 3x^2$

$\frac{d^2y}{dx^2} = 6x$

6. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1.$

$= \lim_{\theta \to 0} \frac{1}{\cos\theta} \frac{\sin\theta}{\theta} = \frac{1}{\cos\theta} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \frac{1}{1} \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1.$

7. (d)

$y = \ln(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x}(\cos x) = \cot x$

8. (b)

$\frac{\mathrm{dy}}{\mathrm{d}x} = \cot x$

$\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2} = -\operatorname{cosec}^2 x$

9. (a)

$v = 2t^4$

$\frac{dv}{dt} = 2\,.4t^3 = 8t^3$

10. From the product Rule with $u = x^2 + 1$ and $v = x^3 + 3$, we find

We find, $\frac{d}{dx}[(x^2+1)(x^3+1)] = (x^2+1)(3x^2) + (x^3+3)(2x)$

$$= 3x^4 + 3x^2 + 2x^4 + 6x$$

$$= 5x^4 + 3x^2 + 6x$$

Alternatively, it can be done as well (perhaps better way) by multiplying the original expression for y and differentiating the resulting polynomial.

$y = (x^2+1)(x^3+3) = x^5 + x^3 + 3x^2 + 3$

$\frac{dy}{dx} = 5x^4 + 3x^2 + 6x$

This is in agreement with our first calculation.