BYJU’S Home Learning Program

Topic covered:

- **Mathematical Tools (Session - 1) - JEE**

Daily Practice Problems

1. Find the equation of the line passing through $(2, 1)$ and parallel to the line $2x - y = 4$.
   a. $y = \frac{2}{5}x - 1$
   b. $y = 5x - 2$
   c. $y = 2x - 3$
   d. None of these

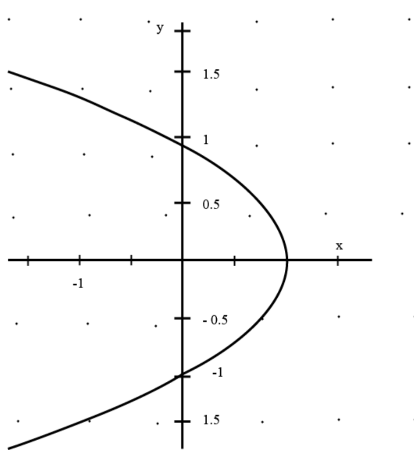
2. The equation $x^2 + y^2 - 12x - 8y + 27 = 0$ is equivalent to
   a. $(x - 6)^2 + (y - 4)^2 = 25$
   b. $(x + 6)^2 + (y + 4)^2 = 25$
   c. $(x - 6)^2 + (y - 4)^2 = 27$
   d. $(x + 6)^2 + (y + 4)^2 = 3\sqrt{3}$

3. The quadratic equation $2x^2 - 7x - 5 = 0$ has roots α and β. Find
   a. $\alpha + \beta$
   b. $\alpha\beta$
   c. $\propto^2 + \beta^2$
   d. $\frac{1}{\alpha} + \frac{1}{\beta}$

4. If $\cot(x) = 2$, then find $\frac{(2+2\sin x)(1-\sin x)}{(1+\cos x)(2-2\cos x)}$

5. Simplify: $\frac{\sin\alpha}{1+\cos\alpha} + \frac{1+\cos\alpha}{\sin\alpha}$

6. Is the graph shown below a function?



7. Compute maximum integer value of k that will make the discriminant of the quadratic equation $x^2 - 6x + k$ positive.

8. If the curve described by the equation $y = x^2 + bx + c$ cuts the x-axis at -4 and y- axis at 4, at which other point does it cut the x-axis?
   a. -1
   b. 4
   c. 1
   d. -4

9. Find the y-intercept of the line passing through the points $A(3, -2)$ and $B(-1,\ 3)$.

10. Find the minimum and maximum value of $y = 1 + 2\sin x$ respectively.

Answer Key

Question Number	1	2	3	4	5
Answer Key	(c)	(a)	3.5, -2.5, 17.25, 1.4	4	$\frac{2}{\sin\alpha}$

Question Number	6	7	8	9	10
Answer Key	No	8	(a)	$\frac{7}{4}$	-1, 3

Solutions

1. (d)
   Since line is parallel, slope of the unknown line is 2
   Now equation of line: $2 = \frac{y-1}{x-2}$
   $y = 2x - 3$

2. (a)
   $x^2 - 12x + 36 + y^2 - 8y + 16 + 27 - 36 - 16 = 0$
   $(x-6)^2 + (y-4)^2 = 25$

3.
   a. $\alpha + \beta = -\frac{(-7)}{2} = 3.5$
   b. $\alpha\beta = -\frac{-5}{2} = -2.5$
   c. For $\alpha^2 + \beta^2$, we need to recall that
   $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (3.5)^2 - 2(-2.5) = 17.25$
   d. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{3.5}{-2.5} = \frac{7}{5}$

4. $\frac{(2+2\sin x)(1-\sin x)}{(1+\cos x)(2-2\cos x)} = \frac{2[(1+\sin x)(1-\sin x)]}{2[(1+\cos x)(1-\cos x)]}$
   $\frac{1-\sin^2 x}{1-\cos^2 x} = \frac{\cos^2 x}{\sin^2 x} = \left(\frac{\cos x}{\sin x}\right)^2 = \cot^2 x = 2^2 = 4$

5. $\frac{\sin^2\alpha+(1+\cos\alpha)^2}{\sin\alpha(1+\cos\alpha)} = \frac{\sin^2\alpha+1+2\cos\alpha+\cos^2\alpha}{\sin\alpha(1+\cos\alpha)} = \frac{2(1+\cos\alpha)}{\sin\alpha(1+\cos\alpha)} = \frac{2}{\sin\alpha}$

6. No. Because for one value of x, there are two value of y.

7. The given quadratic equation is $x^2 - 6x + k = 0$
   In this equation, $a = 1, b = -6$ and $c = k$
   The value of the discriminant, $D = 6^2 - 4 \times 1 \times k$
   i.e., $36 - 4k > 0$
   or $36 > 4k$ or $k < 9$

8. (a)
$y = x^2 + bx + c$ is a quadratic equation and the equation represents a parabola.
The x coordinate of the point where it cuts the y axis = 0
Therefore, (0, 4) is a point on the curve and will satisfy the equation.
Substitute y = 4 and x = 0 in the quadratic equation, $4 = 0^2 + b(0) + c$ or $c = 4$.
The y coordinate of the point where it cuts the x axis = 0. Therefore (-4, 0) lies on the curve,
Substituting the values in equation, $0 = (-4)^2 + b(-4) + 4$. Therefore $b = 5$
Quadratic equation: $y = x^2 + 5x + 4$
If we substitute $x = -1$, quadratic equation becomes 0.

9. Using the two-point form, the equation of the line is :

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow \frac{y-(-2)}{x - 3} = \frac{3 - (-2)}{-1 - 3}$$

$$\Rightarrow \frac{y + 2}{x - 3} = \frac{5}{-4}$$

$$\Rightarrow -4y - 8 = 5x - 15$$

$$\Rightarrow 5x + 4y - 7 = 0$$

Now, we rearrange this equation into the slope-intercept form:

$$4y = -5x + 7$$

$$\Rightarrow y = \left(-\frac{5}{4}\right) x + \frac{7}{4}$$

It is evident that the y-intercept is

$$c = \frac{7}{4}$$

10. Maximum value of $\sin x = 1$
Therefore maximum value of y is $y = 1 + 2(1) = 3$
Minimum value of $\sin x = -1$
Therefore minimum value of y is $y = 1 + 2(-1) = -1$