SCORING KEY

Qn. No.		Answer Key /Value Points	Score	Total
1	a.	Transitive only	1	1
	b.	$f(x) = y = \frac{7x-3}{4} \Rightarrow \frac{4y+3}{7} = x$		
		Let $g(y) = \frac{4y+3}{7}$, $g(x) = \frac{4y+3}{7}$		
		$fog(x) = f(g(x)) = f\left(\frac{4x+3}{7}\right) = \frac{\frac{7(4x+3)}{7} - 3}{4} = x$		(5)
		$gof(x) = g(f(x)) = g\left(\frac{7x-3}{4}\right) = \frac{\frac{4(7x-3)}{4} + 3}{7} = x$	1	
		$fog(x) = gof(x) = x$ $\therefore$ f is invertible		2
		$f^{-1}(x) = \frac{4x+3}{7}$	1	
	c.	$a * b = \frac{ab}{6} = b * a$. $\therefore$ * is commutative.	1	
		$(a * b) * c = \frac{abc}{36} = a*(b * c)$		2
		$\therefore$ * is associative.	1	
2.	a.	$\frac{\pi}{6}$	1	1
	b.	$2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$	1	(4)
		$= \tan^{-1}\left[\frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \times \frac{1}{7}}\right]$	1	3
		$= \tan^{-1}\left[\frac{31}{17}\right]$	1	
3.	a.	2	1	1
	b.	$A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ $A^2 = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$		
		$5A = \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix}$	1	
		$7I = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$		2

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		$A^2 - 5A + 7I = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} = 0$	1	
	c.	$A = IA$		(5)
		$\begin{pmatrix} 1 & 3 \\ 4 & 13 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A$		
		$R_2 \rightarrow R_2 - 4R_1$	1	
		$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -4 & 1 \end{pmatrix} A$		2
		$R_1 \rightarrow R_1 - 3R_2$	1	
		$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 13 & -3 \\ -4 & 1 \end{pmatrix} A$		
		$A^{-1} = \begin{pmatrix} 13 & -3 \\ -4 & 1 \end{pmatrix}$		
4.	a.	54	1	1
	b.	$R_1 \rightarrow R_1 + R_2 + R_3$		
		$\Delta = \begin{vmatrix} 3x-2 & 3x-2 & 3x-2 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$	1	(4)
		$= (3x-2)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{vmatrix}$		
		$C_2 \rightarrow C_2 - C_1,\ C_3 \rightarrow C_3 - C_1$	1	3
		$= (3x-2)\begin{vmatrix} 1 & 0 & 0 \\ 3 & 3x-11 & 3 \\ 0 & 0 & 3x-11 \end{vmatrix}$		
		$= (3x-2)\begin{vmatrix} 3x-11 & 0 \\ 0 & 3x-11 \end{vmatrix}$	1	
		$= (3x-2)(3x-11)^2$		
5.	a.	$\frac{1}{3}$	1	1
	b.	$AX = B$ $\quad \lvert A \rvert = 6$		1 (5)
		$X = A^{-1}B \quad A^{-1} = \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}$	2	4

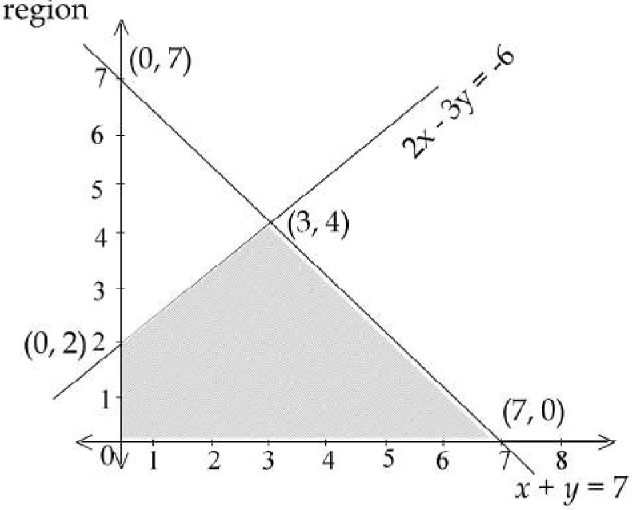
Qn. No.		Answer Key /Value Points	Score	Total
		$X = \frac{1}{6}\begin{bmatrix} 0 & 2 & 2 \\ 3 & -3 & 0 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$	1	
		$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = 1, y = 2, z = 3$		
6.	a.	3	1	1 (3)
	b.	$f(x)$ is continuous at $x = 0$		
		$\lim_{x\to 0} f(x) = f(0)$		
		$\lim_{x\to 0} k\frac{\sin x}{x} = 5$	1	
		$k.1 = 5$		2
		$k = 5$	1	
7.	a.	Put $x = \tan\theta$ $\theta = \tan^{-1}x$		
		$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$		
		$= \cos^{-1}(\cos 2\theta)$	1	
		$= 2\theta$		2
		$= 2\tan^{-1}x$		
		$\frac{dy}{dx} = \frac{2}{1+x^2}$	1	
	b.	$y = (\sin^{-1}x)^2$		
		$\frac{dy}{dx} = 2\sin^{-1}x\frac{1}{\sqrt{1-x^2}}$		1
		$(1-x^2)\left(\frac{dy}{dx}\right)^2 = 4(\sin^{-1}x)^2$	1	3
		$(1-x^2).2\frac{dy}{dx}\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 \times -2x = 4\frac{dy}{dx}$		
		$(1-x^2)\frac{d^2y}{dx^2} - x.\frac{dy}{dx} - 2 = 0$	1	
	c.	$f(x) = x^2 - 5x + 6$ on $[2, 3]$		
		$f(x)$ is a polynomial, $\therefore f(x)$ is continuous on $[2, 3]$		
		$f'(x) = 2x - 5$ exists on $(2, 3)$		1
		$f(2) = f(3) = 0$	1	3
		$f'(c) = 0 \Rightarrow 2c - 5 = 0$		
		$\Rightarrow c = \frac{5}{2} \in (2, 3)$	1	

Qn. No.		Answer Key /Value Points	Score	Total
8.	a.	$60\ \pi cm^2/sec$	1	1
	b.	i. $p(x) = R(x) - C(x)$		
		$= -80 + 30x - x^2$	1	
		ii. $p'(x) = 30 - 2x$		(4)
		for max. or minima, $p'(x) = 0$		
		$p'(x) = 0 \Rightarrow 2x - 30 = 0$	1	
		$\Rightarrow x = 15$		
		at $x = 15$ $p''(x) = -2 < 0$		
		p(x) is maximum at $x = 15$	1	3
		maximum profit $= p(15)$		
		$= 145$		
		OR		
	a.	$(-\pi/2, \pi/2)$	1	1
	b.	$x = 25 \quad \Delta x = 0.2 \quad f(x) = x^{1/2}$	1	
		$f(x + \Delta x) \approx f(x) + \frac{dy}{dx}.\Delta x$	1	(4)
		$\sqrt{25.2} \approx \sqrt{25} + \frac{1}{2\sqrt{25}} \times 0.2$		3
		≈ 5.02	1	
9.	a.	$\tan^{-1}x + c$	1	1
	b.	$t = \tan^{-1}x \qquad dt = \frac{1}{1+x^2}dx$		
		$\int \frac{(\tan^{-1}x)^2}{1+x^2}dx = \int t^2 dt$	1	
		$= \frac{t^3}{3} + c$		2
		$= \frac{(\tan^{-1}x)^3}{3} + c$	1	
	c.	$\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$		(5)
		$A = -1 \qquad B = 2$		
		$\int \frac{x}{(x+1)(x+2)}dx = \int \frac{-1}{(x+1)}dx + \int \frac{2}{(x+2)}dx$	1	2

Qn. No.		Answer Key /Value Points	Score	Total
		$= -\log\lvert x+1\rvert + 2\log\lvert x+2\rvert + C$	1	
		$= \log\left\lvert\frac{(x+2)^2}{x+1}\right\rvert + C$		
		OR		
	a.	0	1	1
	b.	$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....(1)		(5)
		$I = \int_0^{\pi/2} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$	1	
		$= \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....(2)	1	
		(1)+(2) $\Rightarrow 2I = \int_0^{\pi/2} 1 \,.\, dx$	1	
		$= [x]_0^{\pi/2} = \frac{\pi}{2}$	1	4
		$I = \frac{\pi}{4}$		
10	a.	$\int_a^b y\,dx$	1	1
	b.	$\frac{x^2}{16}+\frac{y^2}{9} = 1 \Rightarrow y = \frac{3}{4}\sqrt{16-x^2}$		(4)
		Required Integral $= 4\int_0^4 y\,dx$	1	
		$= 4\int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$	2	3
		$= 12\pi$ sq. units		
11	a.	2, 1	1	1
	b.	I.F $= e^{2x}$	1	
		solution is IF y = $\int (Q.IF)dx$		(4)
		$e^{2x}.y = \int 6e^{x}.e^{2x}\,dx$		

Qn. No.		Answer Key /Value Points	Score	Total
		$= 6.\int e^{3x}\,dx$	1	3
		$e^{2x}.y = 6.\frac{e^{3x}}{3} + c$		
		$e^{2x}.y = 2e^{3x} + c$	1	
12	a.	$\frac{\pi}{4}$	1	1
	b.	$\overrightarrow{AB} = \hat{i} + 3\hat{j} + 3\hat{k}$	1	
		$\overrightarrow{AC} = \hat{i} + 0\hat{j} + 3\hat{k}$		
		Area of ΔABC $= \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\|$		6
		$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 3 \\ 1 & 0 & 3 \end{vmatrix} = 9i + 0j - 3k$	1	
		Area of ΔABC $= \frac{1}{2}\sqrt{90}$ sq. units	1	3
	c.	$\vec{a}, \vec{b}, \vec{c}$ are coplannar, then $\left[\vec{a}, \vec{b}, \vec{c}\right] = 0$ or $\vec{a}.\left(\vec{b} \times \vec{c}\right) = 0$		
		$\begin{vmatrix} 3 & 2 & -\lambda \\ 7 & -1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = 0$	1	
		$\Rightarrow \lambda = \frac{15}{8}$	1	2
13.	a.	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{i} - \hat{j} + 6\hat{k}$	1	
		$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} i & j & k \\ 2 & -3 & 3 \\ 3 & -2 & 2 \end{vmatrix}$	1	
		$= 0i + 5j + 5k$	1	
		SD $= \left\|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\|}\right\|$		3
		$= \left\|\frac{\left(2\hat{i} - \hat{j} + 6\hat{k}\right).(0i + 5j + 5k)}{\sqrt{0 + 25 + 25}}\right\|$	1	
		$= \frac{5\sqrt{2}}{2}$		

Qn. No.		Answer Key /Value Points	Score	Total
	b.	distance $= \left\lvert \dfrac{\vec{a}.\vec{n} - d}{\lvert\vec{n}\rvert} \right\rvert$		(5)
		$= \left\lvert \dfrac{6 \times 2 - 3 \times 5 + 2 \times -3 - 4}{\sqrt{36 + 9 + 4}} \right\rvert$	1	
		$= \dfrac{13}{7}$ units	1	2
		OR		
	a.	$\cos\theta = \left(\dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} . \sqrt{a_2^2 + b_2^2 + c_2^2}} \right)$	1	
		$= \dfrac{3 + 5 + 8}{\sqrt{50} . \sqrt{6}}$	1	3
		$= \dfrac{8\sqrt{3}}{15}$	1	
		$\theta = \cos^{-1}\left(\dfrac{8\sqrt{3}}{15}\right)$		
	b.	$x + 2y + 3z + 7 = 0$		(5)
		$x + 2y + 3z + \dfrac{7}{2} = 0$		
		distance between planes		
		$= \left\lvert \dfrac{d - d'}{\lvert\vec{m}\rvert} \right\rvert$	1	
		$= \left\lvert \dfrac{7 - 7/2}{\sqrt{1 + 4 + 9}} \right\rvert$		2
		$= \dfrac{\sqrt{14}}{4}$ units	1	
14	a.	i. Objective function		(4)
		$z = 150x + 210y$	1	1
		ii. $x + y \leq 500$	1	
		$21x + 32y \leq 1200$	1	3
		$x \geq 0$	1	
		$y \geq 0$		

Qn. No.		Answer Key /Value Points	Score	Total
15		$x + y = 7$		
		$\begin{array}{c\|c\|c} x & 0 & 7 \\ \hline y & 7 & 0 \end{array}$ (0, 7), (7, 0) lie on the line		
		$2x - 3y = -6$		
		$\begin{array}{c\|c\|c} x & 0 & 3 \\ \hline y & 2 & 4 \end{array}$ (0, 2), (3, 4) lie on the line	1	
		feasible region 	1	2
		Corner points (0, 0) (0, 2) (3, 4) (7, 0)		
		Corner points — Value of objective function	1	
		(0, 0) — $z = 0$		
		(0, 2) — $z = -30$		
		(3, 4) — $z = -21$		
		(7, 0) — $z = 91$	1	2
		z is minimum at $x = 0$, $y = 2$ and minimum value is -30.		
16.	a.	$\frac{4}{9}$	1	1
	b.	$P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$ $P(A') = \frac{1}{2}$ $P(B') = \frac{2}{3}$		(5)
		$P(A \cap B) = P(A).P(B) = \frac{1}{6}$		
		i. P(the problem is solved) = $P(A \cup B)$	1	
		$= P(A) + P(B) - P(A \cap B)$		
		$= \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$	1	2
		$= \frac{2}{3}$		

Qn. No.		Answer Key /Value Points	Score	Total
		ii. P(exactly one of them solves) $= P(A'\cap B) + P(A\cap B')$ $= P(A').P(B) + P(A).P(B')$ $= \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \times \frac{2}{3}$ $= \frac{1}{2}$	1 1	2
17		$P(B) = \frac{10}{18} = \frac{5}{9}$ $P(R) = \frac{8}{18} = \frac{4}{9}$ i. P(Both balls are red) = P(1st Red & 2nd Red) = P(1st Red) × P(2nd Red) $= \frac{4}{9} \times \frac{4}{9}$ $= \frac{16}{81}$ ii. P(One Black and One Red Balls) = P(One Black).P(One Red) + P(One Red).P(One Black) $= 2 \times \frac{5}{9} \times \frac{4}{9}$ $= \frac{40}{81}$ iii. P(at least one ball is red) = 1 - P(no red ball) = 1 - P(B).P(B) $= 1 - \frac{10}{18} \times \frac{10}{18}$ $= \frac{56}{81}$	1 1 1 1	2 2