

Verifying Cryptographic Protocols

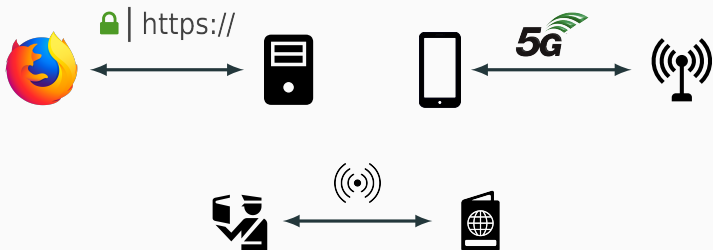
Demi-heure de Science

Adrien Koutsos Prosecco

9 November 2023

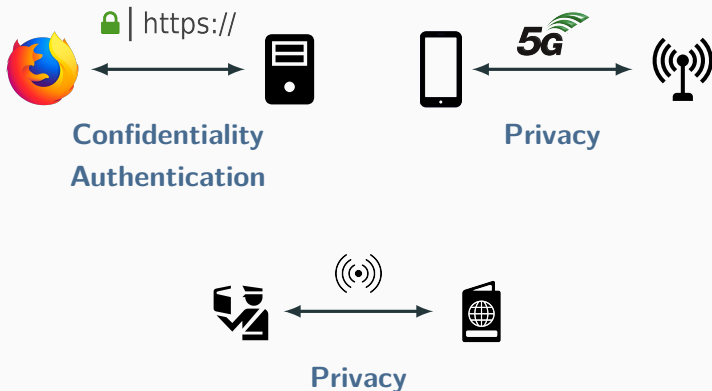
Security Protocols

- **Distributed programs** which aim at providing some **security properties**.
- Uses **cryptographic primitives**: e.g. encryption.



Context: Security Properties

There is a large variety of **security properties**.



Context: Attacker Model

Against whom should these properties hold?

- **concretely**, in the **real world**: malicious individuals, corporations, state agencies, ...
- more **abstractly**, one (or many) computers sitting on the network.

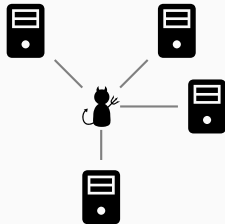
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Abstract attacker model

- **Network capabilities**: worst-case scenario: *eavesdrop*, *block* and *forge* messages.
- **Computational capabilities**: the adversary's *computational power*.
- **Side-channels capabilities**: observing the agents (e.g. time, power-consumption)
⇒ not in this talk.



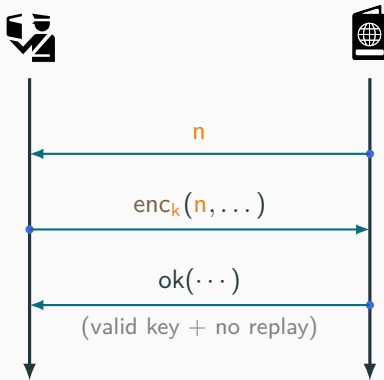
BAC Protocol (simplified)

The Basic Access Control protocol in **e-passports**:

- uses an RFID tag.
- guard access to information stored.
- should guarantee **data confidentiality** and **user privacy**.


Some security mechanisms:

- **integrity**: obtaining **key** k requires **physical access**.
- **no replay**: random nonce n , old messages cannot be re-used.



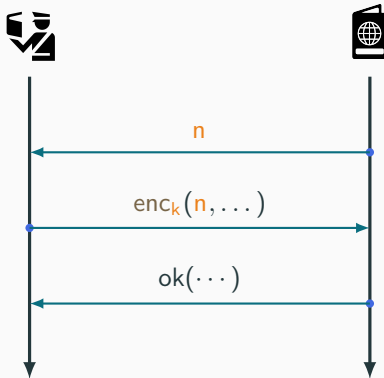
BAC Protocol (simplified)

Privacy: Unlinkability

No adversary  knows whether it interacted with a particular user, **in any context**.


Example. For two user sessions:

$$\left(\begin{array}{c} \text{cat} \\ \left(\begin{array}{c} \text{server} \\ \text{server} \end{array} \right) \end{array} \right) = \left\{ \begin{array}{c} \begin{array}{c} \text{server} \\ \text{server} \end{array}, \begin{array}{c} \text{server} \\ \text{server} \end{array} ? \\ \begin{array}{c} \text{server} \\ \text{server} \end{array}, \begin{array}{c} \text{server} \\ \text{server} \end{array} ? \end{array} \right.$$



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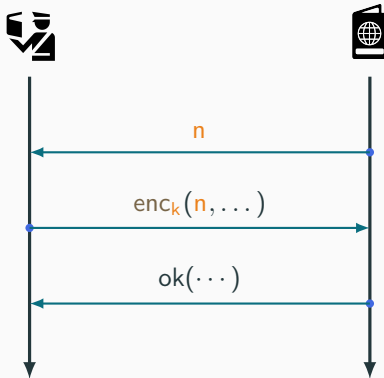
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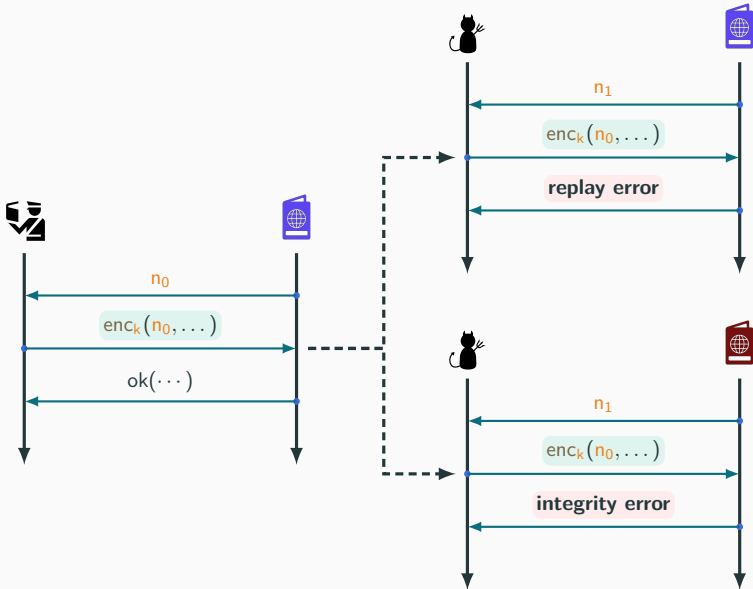
French version of BAC:

- \neq **error messages** for replay and integrity checks.

\Rightarrow **unlinkability attack.**



BAC Protocol: Privacy Attack



Take-away lessons:

- This is a **protocol-level attack**: no issue with cryptography:
⇒ cryptographic primitives are but an **ingredient**.
- **Innocuous-looking changes** can **break** security:
⇒ designing security protocols is **hard**.

BAC Protocol: Lessons

Take-away lessons:

- This is a **protocol-level attack**: no issue with cryptography:
⇒ cryptographic primitives are but an **ingredient**.
- **Innocuous-looking changes** can **break** security:
⇒ designing security protocols is **hard**.

How to get a **strong confidence** in a protocol's **security guarantees**?

High-Confidence Security Guarantees

Verification

Formal mathematical proof of security protocols:



- Must be **sound**: proof \Rightarrow property always holds
- Usually **undecidable**: approaches either **incomplete** or **interactive**.
- **Machine-checked proofs** yield a high degree of confidence.
 - **general-purpose** tools (e.g. **COQ** and **LEAN**).
 - in security protocol analysis, mostly **dedicated** tools.
E.g. **CRYPTOVERIF**, **EASYPHYPT**, **SQUIRREL**.

Research Goal

Design **formal frameworks** allowing for **mechanized verification** of **cryptographic protocols**.

- At the intersection of **cryptography** and **verification**.
- Particular verification challenges:
 - small or medium-sized programs
 - complex properties
 - probabilistic programs + arbitrary adversary

1 Cryptographic Protocol Verification

2 The SQUIRREL Prover

Mechanized Verification of Security Protocols

Cryptographic Protocol Verification

Verification

$$\forall c \in \mathcal{C}. (c \parallel \mathcal{P}) \models \Phi$$

Requires to formalize:

- the **protocol** under study \mathcal{P} .
- the **adversarial model**, i.e. a class \mathcal{C} of adversaries.
- the **security property** Φ .
- the **cryptographic arguments**.

Modeling the System

$$\forall c \in \mathcal{C}. (c \parallel \mathcal{P}) \models \Phi$$

- **Protocol**: a **concrete** concurrent program.
E.g. **imperative** or **functional** programming language, or **applied π -calculus**.
- **Adversary**: an **abstract** unknown program.
What computational capabilities?

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What computational capabilities?
 - **Computationally-bounded**: adversary is a **probabilistic Polynomial-TIME program** (PPTIME).

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 - **Quantum adversary**: adversary is a PQTM. \Rightarrow not in this talk.
 - **Computationally-bounded**: adversary is a **probabilistic Polynomial-TIME program** (PPTIME).
- **The full system**: interaction $(c \parallel \mathcal{P})$.

Modeling the System

How do we model the interaction ($c \text{ || } \mathcal{P}$)?

One **input/output** block:

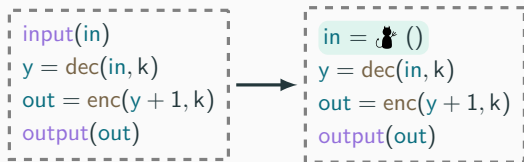
```
input(in)
y = dec(in, k)
out = enc(y + 1, k)
output(out)
```

Modeling the System

How do we model the interaction ($\mathcal{C} \parallel \mathcal{P}$)?

One **input/output** block:

- Network input \Rightarrow function call to \mathcal{C} .

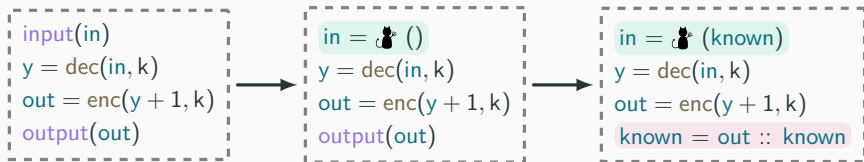


Modeling the System

How do we model the interaction ($\text{cat} \parallel \mathcal{P}$)?

One **input/output** block:

- Network input \Rightarrow function call to cat .
- Network output \Rightarrow add to cat 's knowledge.

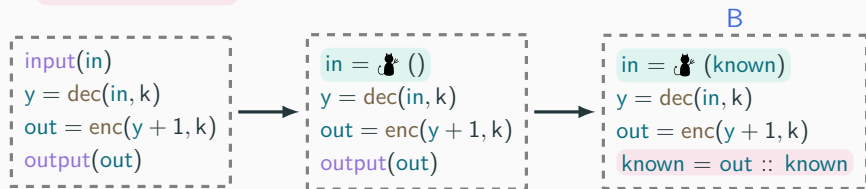


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Many **input/output** blocks, add the **time**:

$$\begin{aligned} \text{in}@B &\stackrel{\text{def}}{=} \text{cat}(\text{known}@pred(B)) \\ \text{known}@B &\stackrel{\text{def}}{=} \text{out}@B :: \text{known}@pred(B) \end{aligned}$$

Security Properties

Equivalence properties Φ

real/ideal world indistinguishability:

$$S_r \sim S_i$$

- S_r : **real-world** scenario for \mathcal{P} .
- S_i : **ideal-world** scenario for \mathcal{P} ,
where security is obvious.

$$\begin{aligned} & \text{out}@_{\text{blue}}, \text{out}@_{\text{blue}} \\ \sim & \text{out}@_{\text{blue}}, \text{out}@_{\text{red}} \end{aligned}$$

For all $\mathcal{A} \in \text{PPTIME}$:


$$\left| \Pr(S_r(\mathcal{A}) = r) - \Pr(S_i(\mathcal{A}) = r) \right| \text{ negligible}$$

Examples: strong secrecy, privacy.


Cryptographic Arguments

How to prove that **no program**  can break a protocol?

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How to prove that **no program**  can ~~break a protocol?~~
solve a problem?

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Cryptographic reduction: $(\Phi$ security property of $\mathcal{P})$

If an adversary  can break Φ

then

there exists an adversary  breaking \mathcal{H}

(with similar running time)

Hardness assumption: problem \mathcal{H} assumed *not efficiently solvable*.

- mathematical problem (e.g. [DISCRETE-LOG](#)).
- lower-level cryptographic problem (e.g. encryption is [IND-CPA](#)).

Hardness Assumption: IND-CPA


message ← key

A **symmetric encryption** function $\text{enc}(m, k)$.

Hardness assumption:

 **cannot learn anything** from an **encrypted** message (except its length).

Equivalence $S_L \sim S_R$:

-  chooses m_L, m_R of the same length
- S_L : encryption $\text{enc}(m_L, k)$
- S_R : encryption $\text{enc}(m_R, k)$

$$\left| \Pr(S_L(\img alt="cat icon" data-bbox="750 485 777 525")) = L) - \Pr(S_R(\img alt="cat icon" data-bbox="750 545 777 585")) = L) \right|$$

negligible

\mathcal{P} **secure** if:

for all  breaking \mathcal{P} there exists  breaking $S_L \sim S_R$

Game-hopping

Combines several proof-steps:

$$S_0 \sim_{\epsilon_1} \cdots \sim_{\epsilon_n} S_n \quad \Rightarrow$$
$$S_0 \sim_{\epsilon_1 + \cdots + \epsilon_n} S_n$$

Each step $S_i \sim_{\epsilon_{i+1}} S_{i+1}$ justified by:

- a **cryptographic reduction**;
- a **probabilistic argument** (e.g. small probability of guessing);
- *etc...*

Cryptographic Arguments as Reasoning Rules

- Previous slides: **cryptographers' point-of-view**.

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- A more **abstract** and **logical** presentation as **reasoning rules**:
 - **Structural**, to organize proofs:

$$\frac{U \sim W \quad W \sim V}{U \sim V}$$

- **Cryptographic**, e.g. **IND-CPA**:

$$\frac{\text{len}(m_0) = \text{len}(m_1)}{\text{enc}(m_0, k) \sim \text{enc}(m_1, k)}$$

when k correctly used in m_0, m_1

Cryptographic Arguments as Reasoning Rules

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- **Probabilities** completely **abstracted away**.
- **Application conditions** are the difficult part.

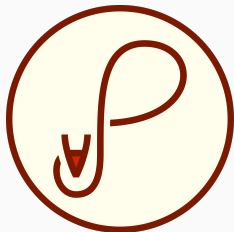
The Squirrel Prover

Mechanized Verification of Security


Protocols

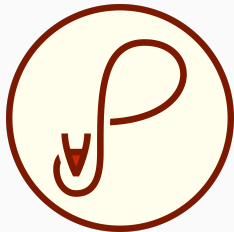
The Squirrel Prover

- Tool for verification of **security protocols**:
 - **Input language**: applied π -calculus.
 - Automatically translated as input/output blocks.
- Implements a **probabilistic logic**:
 - Supports **reachability** and **equivalence** properties.
 - **Reasoning rules** valid w.r.t. comp. attacker .
 - In the **asymptotic security** setting.



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 - **Reasoning rules** valid w.r.t. comp. attacker .
 - In the **asymptotic security** setting.
- **Proof assistant**:
 - Users prove goals using sequences of **tactics**.
 - **Crypto.** tactics, e.g. **cpa**.
 - **Probabilistic** tactics, e.g. **fresh**.
 - **Structural** tactics, e.g. **trans**.
 - **Generic** tactics, e.g. **apply**, **rewrite**.



The Squirrel Prover

```
global lemma unlinkability (t : timestamp[const]) :
  [happens(t)] -> equiv(frame@t).
Proof.
  intro Hap.
  enrich
    seq(A:index => pk( kA (A ) )),
    seq(A:index, i:index => pk( kAbis (A,i) )),
    seq(A:index => pk( kB (A ) )),
  induction t.
  (* init *)
  * auto.
  (* Case A *)
  * rewrite /* /.
  by apply IH.
  (* Case A1 *)
  * rewrite /* /.
  fa 3; fa 4; fa 4; fa 4; fa 4.
  fresh 6; 1:auto.
  by fresh 5.
  (* Case B *)
  rewrite /frame /output /exec /cond /dmess /.
  fa 3; fa 4; fa 4.
  encnp 4; 1:auto.
  ccal 4.
  + auto.
  + (* Pushing conditional underneath len(_) *)
  rewrite if_len !length_pair.
  rewrite (if_same_branch (len(nB(A,i)) ++ len(nB(A,i)))) //.
  fa 4; fa 4; fa 4; fa 4.
  fresh 5; 1:auto.
  by Fresh 4.
Qed.
```

```
[goal> Focused goal (1/2):
System: default (same for equivalences)
Variables: A,i:index[const, glob]
Hap: [happens(B(A, i))]
IH: equiv(seq(A:index=>pk (kB A)), seq(A,i:index=>pk (kAbis (A, i))),
          seq(A:index=>pk (kA A)), frame@spread (B(A, i)))
-----
forall (A0,i0:index), B(A0, i0) < B(A, i) => A <> A0 || i <> i0
```

```
U:%*- *goals* All (i,0) (Squirrel goals +1 company Helm)
```

```
Checking for side conditions on the right
Bad occurrences of kA(A) and rB(A, i) in other actions:
  rB(A, i)
  (collision with rB(A, i))
  in action B(A, i)
  in term
  enc (if (fst (dmess(A, i)@B(A, i)) = pk (kAbis (A, i))) &&
        len (snd (dmess(A, i)@B(A, i))) = len (nB (A, i))) then
    <snd (dmess(A, i)@B(A, i)),nB (A, i)>
  else <nB (A, i),nB (A, i)>, rB (A, i), pk (kAbis (A, i)))
```

```
Total: 1 bad occurrence
         0 of them are subsumed by another
         1 bad occurrence remaining
```

Open-source tool

- Development team:
Inria Paris (Prosecco), IRISA (Spicy team).

- Project web-page:

`https://squirrel-prover.github.io/`

- Documentation web-page:

`https://squirrel-prover.github.io/documentation/`

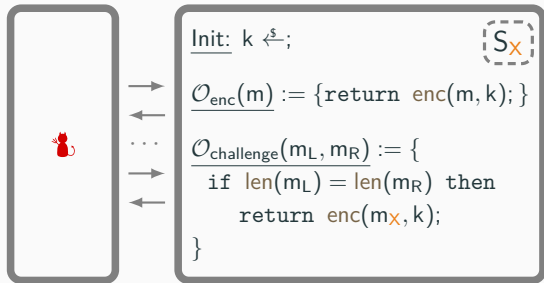


Conclusion

- **Computer-aided verification** of crypto. protocols allows for high security guarantees.
- Quick introduction to protocol verification:
 - **modeling** security properties.
 - formalizing **cryptographic arguments**.
- The SQUIRREL prover, an **interactive tool** for crypto. protocol verification.

Thank you for your attention

Hardness Assumption: IND-CPA



→ guess of the value of X

$$\left| \Pr(S_L(\text{cat}) = L) - \Pr(S_R(\text{cat}) = L) \right|$$

negligible

Security Properties

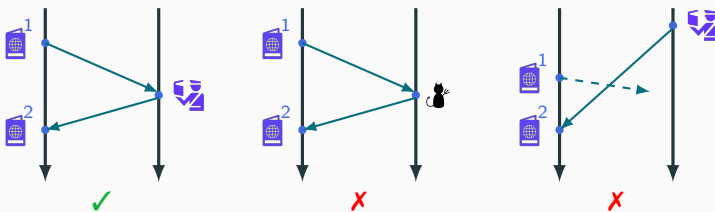
Reachability properties Φ

Directly expressed on $\mathcal{C}^* + \mathcal{P}$.

For all $\mathcal{C}^* \in \text{PPTIME}$:

$$\Pr(\text{not } \Phi(\mathcal{C}^*)) \text{ negligible}$$

Examples: authentication, injective authentication, (weak) secrecy.



Security Properties

Reachability properties Φ







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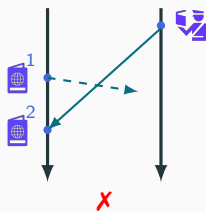
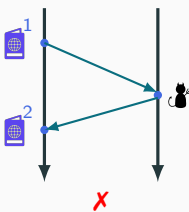
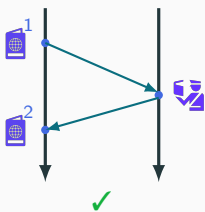
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accept@ ² \Rightarrow

$$\left(\begin{array}{l} \text{in@} \text{ ¹ < \text{in@} \text{ ² \wedge \\ \text{out@} \text{ ¹ = \text{in@} \text{ ² \wedge \\ \text{out@} \text{ ² = \text{in@} \text{ ¹ \end{array} \right)$$



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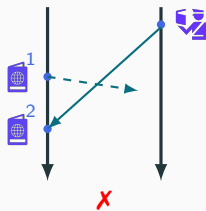
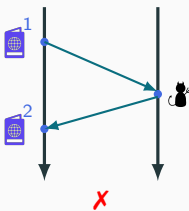
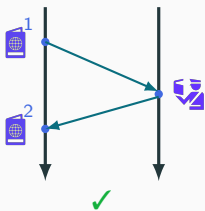
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accept@ ² \Rightarrow

$$\left(\begin{array}{l} \text{server}^1 < \text{client} < \text{server}^2 \wedge \\ \text{out@ server}^1 = \text{in@ client} \wedge \\ \text{out@ client} = \text{in@ server}^2 \end{array} \right)$$



Security Properties

Reachability properties Φ

Directly expressed on $\mathcal{C}^{\#} + \mathcal{P}$.

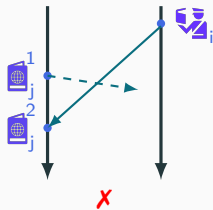
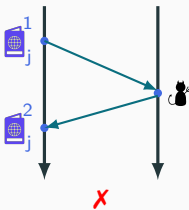
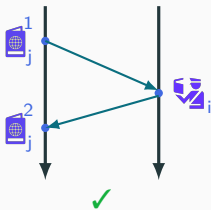
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$$\forall j. \text{accept} @ \mathbb{G}_j^2 \Rightarrow$$

$$\exists i. \left(\begin{array}{l} \mathbb{G}_j^1 < \mathbb{V}_i < \mathbb{G}_j^2 \wedge \\ \text{out} @ \mathbb{G}_j^1 = \text{in} @ \mathbb{V}_i \wedge \\ \text{out} @ \mathbb{V}_i = \text{in} @ \mathbb{G}_j^2 \end{array} \right)$$



From Hardness Assumptions to Logical Rules

Cryptographic Reduction

Cryptographic reduction:

(Φ security property of \mathcal{P})

If an adversary  can break Φ

then

there exists an adversary  breaking \mathcal{H} .

Hardness Assumption: Example

message ← | | → key

A **cryptographic hash** function $H(m, k)$.

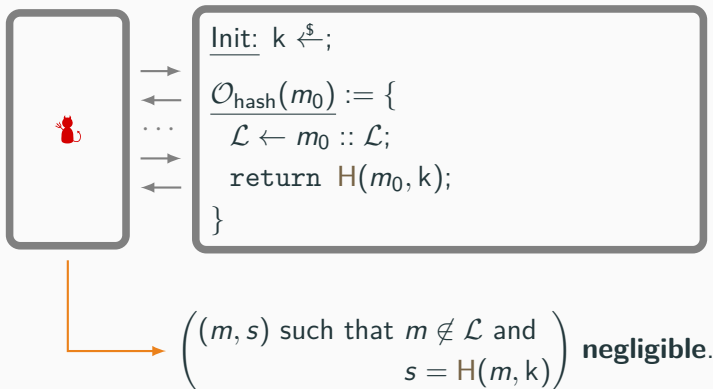
Unforgeability: cannot produce valid hashes without knowing k .

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Hardness Assumption: Example

Example

$$\Phi \stackrel{\text{def}}{=} \left(\text{cat}(H(0, k), H(1, k)) = H(m, k) \right) \Rightarrow m = 0 \vee m = 1$$

Proof by reduction

Build an adversary  against UNFORGEABILITY (UF):

- compute $w_0 \leftarrow \mathcal{O}_{\text{hash}}(0)$ and $w_1 \leftarrow \mathcal{O}_{\text{hash}}(1)$;
- black-box call: $s \leftarrow \text{cat}(w_0, w_1)$;
- compute m ;
- return (m, s) .

$$\text{Adv}_{\text{UF}}(\text{red cat}) = \text{Adv}_{\Phi}(\text{black cat}) \quad \text{black cat} \in \text{PPTIME} \text{ implies } \text{red cat} \in \text{PPTIME}$$

Hardness Assumption: Example

Example

$$\Phi \stackrel{\text{def}}{=} \left(\mathcal{A}(H(0, k), H(1, k)) = H(m, k) \right) \Rightarrow m = 0 \vee m = 1$$

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$$\text{Adv}_{\text{UF}}(\mathcal{A}) = \text{Adv}_{\Phi}(\mathcal{A}) \quad \mathcal{A} \in \text{PPTIME} \text{ implies } \mathcal{A} \in \text{PPTIME}$$

Remark: rule valid only if m computable by the adversary, e.g.

$$\exists \mathcal{A}_1 \text{ s.t. } \mathcal{A}_1() = m$$

From Hardness Assumptions to Logical Rules

Until recently:

- SQUIRREL supported a limited set of hardness assumptions (symmetric/asymmetric encryption, signature, hash, DH, ...)
- Built-in tactics for each such assumptions:

hardness assumption (imperative, stateful programs)



reasoning rules (pure, logic)

From Hardness Assumptions to Logical Rules

(recent joint work with Justine Sauvage and David Baelde)



Systematic cryptographic reductions: allows to translate hardness assumptions into cryptographic rules.

Inputs:

- an (imperative, stateful) **hardness assumption** $\mathcal{H}_0 \sim \mathcal{H}_1$.
- a (logical) **security property**, e.g. $S_0 \sim S_1$.

Goal: for any \mathcal{C} , synthesize \mathcal{R} such that
$$\left\{ \begin{array}{l} \mathcal{R}(\mathcal{H}_0) = S_0(\mathcal{C}) \\ \text{and } \mathcal{R}(\mathcal{H}_1) = S_1(\mathcal{C}) \end{array} \right.$$

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- **General framework** to add new hardness assumptions.
- **Proof system** to establish the existence of .
 - Tracking the state of \mathcal{H} : **Hoare pre- and post-conditions**.
E.g. track the set of hashed messages \mathcal{L} .
 - Correct randomness usage using (logical) **constraints**.
E.g. ensures that  does not directly use k .
 - Soundness: existence of a suitable **probabilistic coupling**.
- **Fully automated** (heuristic based \Rightarrow incomplete) procedure.
Approximate \mathcal{H} state + randomness constraints (discharged to SQUIRREL).